DETERMINATION OF MATERIAL PARAMETERS OF TEXTILE REINFORCED CEMENTITIOUS COMPOSITE EXPOSED TO HIGH TEMPERATURES USING AN INVERSE METHOD

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Abstract
In this paper the textile reinforced concrete under study is essentially composed of two brittle materials. Using a textile reinforcement results in a fibre volume fraction which is exceeding the critical. If so strain hardening occurs. The material parameters are computed by using an inverse method. The method is based on a bending model which uses an iterative process for the determination of the model parameters by minimizing the cost function of likeness between the computed and the measured force - displacement. The material parameters are derived from the model parameters. The material parameters obtained from the inverse method are compared with the experimental values.

1 Introduction
Textile reinforced concrete (TRC) is essentially composed of two brittle materials. In this case a random distributed glass fibre textile is used in combination with an inorganic phosphate cement (IPC) matrix. IPC has been developed at the "Vrije Universiteit Brussel". The major benefits of IPC compared to other cementitious materials is the non-alkaline environment of IPC after hardening. For this reason, ordinary E-glass fibres are not attacked by the matrix and can be used as reinforcement. By using a fibre volume fraction which exceeds the critical fibre volume fraction, the fibres can ensure strength and stiffness at applied loads far exceeding the range in which matrix multiple cracking is developed, which results in a real strain hardening behaviour in tension. Since the combination of IPC and E-glass fibres leads to a composite, which is entirely made of inorganic materials, it is well suited for utilisation in structures which are exposed to high temperatures. The material itself is characterised by a linear behaviour in compression and a marked nonlinear tensile behaviour. Over the years efforts have been directed into the modelling of this tensile and bending behaviour without taking in to account the effect of temperature loading. However, to be able to design structures which are exposed to high temperatures simple and effective calculation methods are needed. In this paper the stress strain behaviour in tension will be modelled using a modified Aveston-Cooper-Kelly theory [1] (ACK). According to the ACK theory, three distinct stages can be detected in the stress-strain curve. The aim of this paper is two folded: first a calculation method is presented to derive the load displacement curves of textile reinforced cementitious composite beam which is exposed to a high temperature (300°C) [2,3,4,5]. The method uses data from experimental tensile tests of similar specimens, exposed

to the same temperature, to calibrate the material stress – strain model (ACK). In the second stage an inverse method is developed for the determination of the material parameters starting from a bending test which is less complicated to execute in comparison with a tensile test. The inverse method developed within the scope of this paper uses an iterative process for the determination of the model parameters (η_{matrix}, η_{fibre}, σ_{mu}) by minimizing the cost function of likeness between the computed and the measured force - displacement. The material parameters (E_{c1}, σ_{mu}, E_{C3}) are derived from the model parameters.

2 Development of an analytical model

2.1.TRC in tension

According to the ACK theory, three distinct stages can be detected in the stress-strain curve of a unidirectional reinforced brittle matrix composite. In the first stage the material behaves linear elastic. In this first stage a perfect "elastic" bond between matrix and fibres is assumed. The failure strain of the matrix is lower than the failure strain of the fibres. At the ultimate matrix strain the composite will crack. If the fibre volume fraction is higher than the critical fibre volume fraction, the fibres will be able to sustain the additional loading. According to the ACK theory this stage is called the "multiple cracking" stage. In the third stage, called "post cracking" the matrix is completely cracked. The fibres will carry the load in this final stage, until failure. In the linear elastic stage (stage I), according to the ACK theory, the stiffness of the composite E_{c1} is a function of the fibre volume fraction V_f, the volume fraction of the matrix V_m, the stiffness of the fibres E_f and the stiffness of the matrix E_m. The matrix-fibre interface bond is assumed to be elastic and the composite stiffness E_{c1} can be determined by the law of mixtures:

$$E_{c1} = E_f V_f + E_m V_m \tag{1}$$

The ACK theory is modified: due to imperfect matrix-fibre adhesion, warping or misalignment of the unidirectional fibres, inclusion of air voids, etc. the fibre volume fraction has to be lowered with a fibre efficiency factor η_f. Also the stiffness of the matrix has to be reduced with a matrix efficiency factor η_m. The modified law of mixtures can be written as follows (2). In this model the efficiency factors are assumed to be constant in all stages.

$$E_{c1} = E_f V_f^* + E_m V_m^* \tag{2}$$

The effective fibre volume fraction can be calculated by using the following equation:

$$V_f^* = V_f \eta_f \tag{3}$$

Taking the efficiency of the matrix in account results in the following equation:

$$V_m^* = V_m \eta_m \tag{4}$$

At a certain unique composite stress σ_{mc}, multiple parallel cracks are introduced in the matrix. When the first crack appears and reaches a fibre, debonding of the matrix-fibre interface occurs and further matrix-fibre interaction occurs through friction. The composite multiple cracking stress (σ_{mc}) can be determined by the following equation (5) with (σ_{mu}) defined as the ultimate matrix stress:

$$\sigma_{mc} = \frac{\sigma_{mu} E_{c1}}{E_m} \tag{5}$$

When the first crack appears and reaches a fibre, debonding of the matrix-fibre interface occurs and further matrix-fibre interaction results from friction. The frictional interface stress is assumed to be constant along the debonding interface. The debonding length δ_o can be calculated from the equilibrium of the forces along the crack. In case of circular fibres with a radius (r) the following expression (6) can be used to derive the debonding length:

$$\delta_0 = \frac{\sigma_{mu}.r.V_m^*}{2.\tau_0.V_f^*} \tag{6}$$

Increasing the load will lead to multiple cracking, if the fibre volume fraction is above the critical volume fraction. According to the ACK theory, at a certain unique composite stress σ_{mc} multiple parallel cracks are introduced in the matrix. Cracks are introduced until saturation is reached. The distance between neighbouring cracks is situated between δ_o and $2\delta_o$, with an average of $1.337\delta_o$. After multiple cracking, the strain of the composite $\varepsilon_c^{\text{stage II}}$ can be calculated as follows (7) from integration of the strain field between two cracks with distance $1.337\delta_o$

$$\varepsilon_c^{stageII} = (1 + 0.66\alpha)\varepsilon_{mu} \tag{7}$$

with

$$\alpha = \frac{E_m V_m^*}{E_f V_f^*} \tag{8}$$

Once full multiple cracking has occurred, only the fibres further contribute to the stiffness in stage III (post-cracking stage). The stiffness of the composite E_{c3} in this stage is thus:

$$E_{c3} = E_f V_f^* \tag{9}$$

Figure 1 illustrates a theoretical (ACK) stress-strain curve with these three distinct stages: linear elastic stage (I), multiple cracking stage (II) and post cracking stage (III).

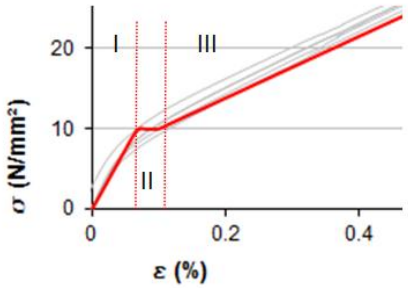

Figure 1. experimental and theoretical stress-strain curve, according to the ACK theory [3]

2.2 TRC beam element in bending

Since the behaviour of TRC in tension shows three stages, three expressions are needed to define the equilibrium of forces and moments if a cross section is loaded in bending. By expressing the equilibrium of forces and moments, the position of the neutral axis (a) and the maximum occurring tensile strain (ε_t) in the section can be calculated for each cross section along the beam. Subsequent integration of the bending stiffness for each differential beam element with height (h) and a unit width, over the total length, will lead to a force

displacement diagram. For each cross section one of the following sets of equilibrium equations can be used depending on the value of the maximum tensile stress. As long as the composite behaves linear elastic along the whole beam section the following expressions can be used, with M_e defined as external moment:

$$M_e = \frac{2}{3} a^2 E_{c1} \varepsilon_t \qquad (10) \qquad \text{With:} \qquad a = h/2 \qquad (11)$$

Once the composite maximum tensile strain is situated in the multiple cracking stage the equilibrium equations can be based on the internal strains and stresses in figure 2.

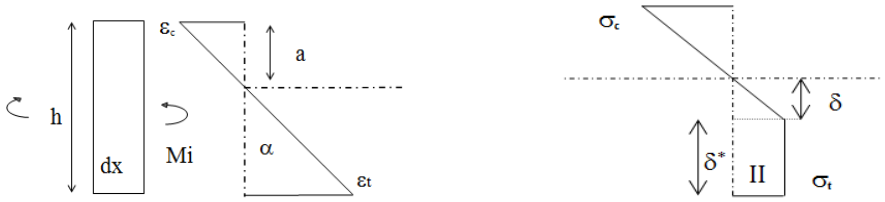

Figure 2: distribution of stress strain over cross section in the multiple cracking stage

By expressing the equilibrium of forces (12) and moments (13), the position of the neutral axis and the maximum occurring tensile strain (εt) in the section can be calculated for each cross section along the beam.

$$E_{c1}\varepsilon_t \frac{a^2}{2(h-a)} = \delta \frac{\sigma_{mc}}{2} + \delta^* \sigma_{mc} \qquad (12)$$

$$E_{c1}\varepsilon_t \frac{a^3}{3(h-a)} + \delta^2 \frac{\sigma_{mc}}{3} + \sigma_{mc}\delta^* (\delta + \frac{\delta^*}{2}) = M_e \qquad (13)$$

with $\delta = \frac{\varepsilon_{mu}}{\varepsilon_t}(h-a)$ and $\delta^* = h - a - \delta$

Finally the composite maximum tensile strain will be situated in the post-cracking stage.

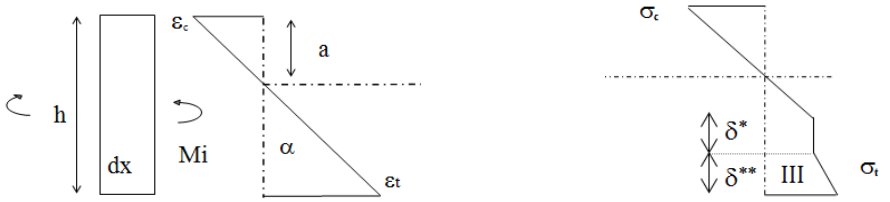

Figure 3: distribution of stress strain over cross section in the post cracking stage

By expressing the equilibrium of forces (14) and moments (15), the position of the neutral axis and the maximum occurring tensile strain (εt) in the section can be calculated for each cross section along the beam.

$$E_{c1}\varepsilon_t \frac{a^2}{2(h-a)} = \delta \frac{\sigma_{mc}}{2} + \delta^* \sigma_{mc} + (\varepsilon_t - \varepsilon_c^{stageII}) E_f V_f^* \frac{\delta^{**}}{2} \qquad (14)$$

with $\delta^{**} = h - a - \frac{\varepsilon_c^{stageII}}{\varepsilon_{mu}}\delta$

$$E_{c1}\varepsilon_t \frac{a^3}{3(h-a)} + \delta^2 \frac{\sigma_{mc}}{3} + \sigma_{mc}\delta^* (\delta + \frac{\delta^*}{2}) + (h - a - \frac{\delta^{**}}{3}) \frac{\delta^{**}}{2} (\varepsilon_t - \varepsilon_c^{stageII}) E_f V_f^* = M_e \qquad (15)$$

Once the equilibria of all sections are established, the deflection in any section can be determined by double integration of $M_e / EI_{section}$ along the length of the beam. $EI_{section}$ is the bending stiffness as calculated for each section of the beam.

2.3 Inverse Method

The inverse method developed within the scope of this paper allows the user to determine a set of model parameters (η_{matrix},η_{fibre},σ_{mu}) with a certain degree of accuracy [6]. The set of model parameters contains the efficiency of the matrix (η_{matrix}), the efficiency of the fibres (η_{fibre}) and the ultimate matrix tensile stress (σ_{mu}). At first the experimental force deflection curve obtained from a four point bending test will be loaded in the program. The inverse method will use the bending model based on the ACK theory. The program will generate for each set of three parameters (η_{matrix},η_{fibre},σ_{mu}) an analytical force displacement curve.

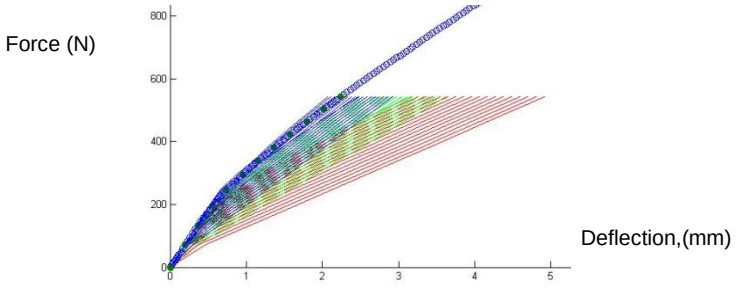

Figure 4: Analytical force displacement curves generated for each set of three model parameters (η_{matrix},η_{fibre},σ_{mu}).

The process is repeated until the difference between the experimental and analytical force displacement curve is minimized, using the coefficient of determination (R^2) [7].

3 Materials and testing methods

The experimental part was performed to obtain force deflection curves from 4 point bending test which were used in the inverse method as input data. To verify the proposed inverse method, tensile tests were also performed in order to determine the material parameters (E_{c1},E_{c3}, σ_{mu}).

3.1 Specimen preparation

One standard IPC mixture is chosen in this work, without use of any fillers or retarding or accelerating components. All IPC laminates are made by hand lay-up and cured in ambient conditions for 24 hours. Post-curing is performed at 60°C for 24 hours. Like most cementitious mixtures, the strength of IPC increases with time. This effect can however be accelerated by the post-curing at 60°C. During the curing and post-curing process, both sides of the laminate are covered with plastic to prevent early evaporation of water. The E-glass fibre reinforcements used in the specimens was chopped glass fibre mats (“2D-random”) with a fibre density of 300g/m² (Owens Corning M705-300). The dimension of the plate is 50cmx50cm. The plates were produced with an average matrix consumption of 800 g/m² for each layer. The plates were cut with a water cooled diamond saw. About five cut strip specimens had a width of 25mm and five others had a width of 50mm. The specimens with a width of 25mm are used for tensile testing and the specimens with a width of 50mm were tested in bending.

3.2 Bending test

For each laminate, several specimens were loaded in a four-point bending test, Figure 5. Two supports were placed with a span of 200 mm, the crosshead was equipped with horizontal bars

were two cylinder were mounted with a distance of 100 mm. The force on the crosshead and the displacement in the centre of the laminate were measured. The testing machine (Instron 5885 H) was displacement controlled; the loading rate was set to 1 mm/min.

Figure 5: experimental setup four-point bending test

3.3 Tensile testing

The tensile test was carried out to obtain the ACK-model parameters for all tested laminates. The stress strain curve data was generated using a tensile testing machine (Instron 5885 H) with a capacity of 100 kN. The rate of crosshead displacement was set at 1 mm/min. The strain was measured with an extensometer. The resulting stress-strain curves for a 6 layers random reinforced TRC are plotted and discussed here.

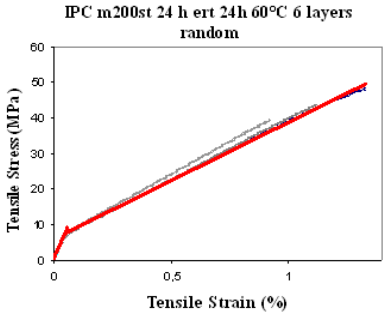

Figure 6: shows the stress-strain curves obtained on the specimens from plate 2 (with 6 layers of fibres)

The average stiffness (5 specimens) in the first (E_{c1}) and the third stage (E_{c3}) can be obtained from the experimental data by determination of the slope of the curves. The average values (5 specimens) of E_{c1} and E_{c3} are listed below in table 1. The model parameters (η_{matrix},η_{fibre},s_{mu}) can be derived from the experimental data.

	E_{C1}		E_{C3}		t_c		V_f		η_f		η_m	
	GPa		GPa		mm							
4L IPC ST ert 60	14.82	1.53	3.86	1.09	2.03	0.14	0.23	0.02	0.23	0.02	0.80	0.12
4L IPC ST ert 60-300	7.95	0.43	4.20	0.31	1.97	0.08	0.24	0.01	0.24	0.03	0.27	0.01
6L IPC ST ert 60	14.41	0.96	4.19	0.55	3.05	0.07	0.23	0.01	0.25	0.04	0.74	0.04
6L IPC ST ert 60-300	7.08	0.18	3.06	0.38	2.94	0.23	0.24	0.02	0.18	0.02	0.30	0.03
8L IPC ST ert 60	15.74	0.43	3.83	0.11	4.86	0.07	0.19	0.00	0.27	0.01	0.82	0.02
8L IPC ST ert 60-300	6.17	0.62	2.87	0.53	4.51	0.10	0.21	0.00	0.19	0.03	0.23	0.02
12L IPC ST ert 60	14.57	2.19	3.65	0.16	7.05	0.17	0.20	0.00	0.25	0.01	0.76	0.15
12L IPC ST ert 60-300	6.34	1.70	3.17	0.42	6.38	0.40	0.22	0.01	0.20	0.02	0.23	0.04

Table 1. Overview of the experimental data obtained from tensile tests

By knowing the thickness (t_c) the fibre volume fraction (V_f) can be computed. The fibre efficiency (η_{fibre}) can be derived by using the following equation (16). Also the matrix

efficiency can derived from the experimental data. As shown in equation (17). In this case a theoretical value is chosen for the matrix an fibre stiffness (E_f= 72 GPa, E_m=18GPa).

$$\eta_{fibre} = \frac{E_{c3}}{E_f V_f} \quad (16)$$

$$\eta_{matrix} = \frac{E_{c1} - E_f V_f \eta_{fibre}}{E_m (1 - V_f)} \quad (17)$$

In table 1 the efficiencies derived from the experimental data is presented in the grey shadowed cells. The data of the bending test was used in the inverse method to obtain the model parameter. For each set of 5 specimens with the same amount of layers the average of model parameters are plotted in the table 2 below.

Specimen	tc		Vf		ηf		η m		EC1		EC3	
	mm								GPa		GPa	
4L IPC ST ert 60	2.3	0.18	0.21	0.02	0.31	0.02	0.79	0.01	15.77	0.34	4.55	0.3
4L IPC ST ert 60-300	2.51	0.05	0.19	0.02	0.23	0.02	0.27	0.01	7.06	0.26	3.11	0.26
6L IPC ST ert 60	3.24	0.06	0.22	0.01	0.22	0.01	0.74	0.01	14.00	0.1	4.32	0.29
6L IPC ST ert 60-300	3.42	0.26	0.21	0.02	0.16	0.03	0.29	0.01	6.66	0.39	3.47	0.24
8L IPC ST ert 60	4.5	0.11	0.21	0.02	0.25	0.01	0.82	0.01	15.37	0.1	3.71	0.17
8L IPC ST ert 60-300	4.13	0.44	0.23	0.03	0.19	0.05	0.23	0.01	6.40	1.01	3.25	1.01
12L IPC ST ert 60	6.36	0.08	0.15	0.03	0.33	0.05	0.74	0.02	14.89	0.66	3.49	0.51
12L IPC ST ert 60-300	7.2	0.18	0.13	0.06	0.23	0.02	0.17	0.02	5.10	0.14	1.57	0.18

Table 2. Overview of generated model parameters using the data of the bending test

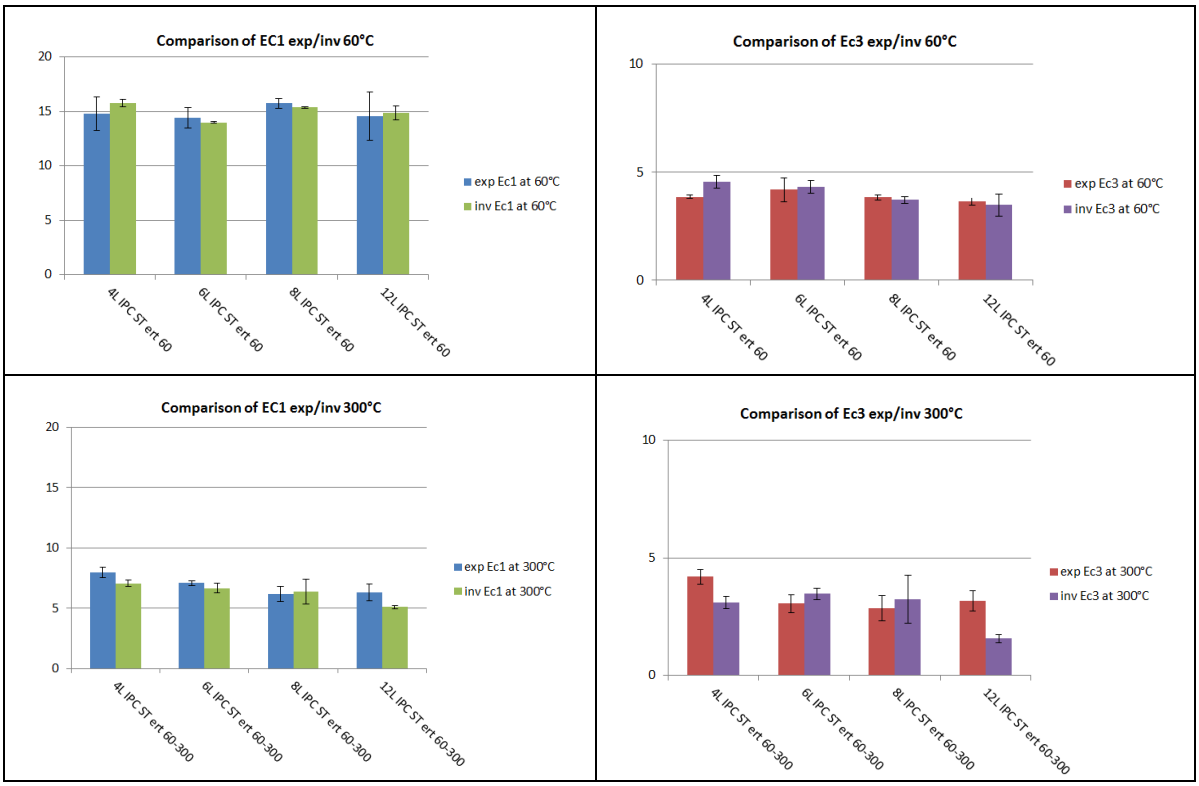

Figure 7. Comparison of the material parameters obtained from the experimental and inverse method

4. Discussion

The temperature load (300°C) causes a significant decrease of the stiffness E_{c1}. Due to the temperature load the matrix will be full of micro cracks. The stiffness E_{c3} is less affected. The stiffness of the uncracked (E_{c1}) and cracked composite (E_{c3}) can be derived with accuracy in case of specimens cured at room temperature and post cured at 60°C. When comparing the experimental and analytical values for the material parameters it can be seen that the difference is smaller than the standard deviation. If the specimens are exposed to a high temperature (300°C) it is still possible to compute the stiffness of the uncracked composite (E_{c1}). The method will start to loose accuracy when computing the stiffness of the heated (300°C) specimens. We can assume that using the modified ACK model is not valid any more.

5. Conclusion

The inverse method developed within the scope of this paper allows the user to determine a set of model parameters (η_{matrix},η_{fibre},σ_{mu}). The parameters can be used to derive the material parameters (E_{c1}, σ_{mu}, E_{C3}). By comparing the experimental and analytical values of the material parameters it is clear that the composite stiffness in the uncracked stage can be computed with good accuracy. Determination of the stiffness of the cracked composite is accurate for non-heated specimens. The proposed method starts to lose accuracy when calculating the stiffness of the cracked heated composite.

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