CONTRIBUTION TO THE ACCURATE EXPERIMENTAL MEASUREMENT OF IN PLANE AND OUT OF PLANE SHEAR PROPERTIES ON COMPOSITE MATERIALS

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Abstract

By a judicious exploitation of the elastic theory of anisotropic body (Lekhnitskii [1]), scientific community has developed methods and identified some precaution to take during the establishment of rectangular bar torsion tests. This kind of test allows simultaneous access to in plane (G12) and out of plane (G13 or G23) shear moduli. To enable the evaluation of a particular method developed by Sumsion [2], a finite element model (FEM), representative of a known material (aluminum) has been established. The study of this FEM has helped highlight the need to measure the mechanical response of the material (twist angle) on a restrained area, far from the load introduction. Under these conditions, application of the Sumsion's method allowed to find both in plane and out of plane shear moduli, which were implemented in the computer code, with great precision (maximum error less than 1.5%). Based on the findings of this preliminary study, an experimental methodology has been developed. Its originality stems on the use of stereo-correlation tool to measure accurately the angle of twist on this restrained zone. This methodology has been first applied to aluminum. Shear moduli obtained were equivalent to those measured by tensile tests (maximum error less than 2.5%). This methodology has been then extended with success to glass fiber reinforced polyester, which provides a glimpse of future applications on 3D composites.

1 Introduction

With the development of composite structures whose geometry reinforcements get more and more complex, experimental evaluation of the out of plane mechanical behavior become inescapable. However, due to limited thicknesses and highly heterogeneous reinforcement structure, experimental characterization of out of plane shear modulus is not easy to reach with conventional testing (rail shear, Iosipescu, ...). Torsion test of rectangular bar, which involves plane ($\tau 12$) and out of plane ($\tau 13$) shear stress, is a credible alternative to the tests mentioned above, by the apparent ease of its implementation. The main disadvantage of this type of test is that the load introduction changes the overall response of the material relative to the theory. Ogasawara's study [3] permit to highlight an area slightly perturbed by this phenomenon but the consequences of the mismeasurement on mechanical response do not have been mentioned.

The exploitation of Leknitskii's theory permit to determine simultaneously shear moduli G_{12} et G_{13} of an orthotropic material. This exploitation is based on the measurement of the torsional rigidity (CL / θ) of at least two sets of rectangular bar having different sections [4], [5]. Sumsion proposal is the interpolation of experimental results by the optimal adjustment of both parameters G_{12} and G_{13}. This method allows using multiple sections in order to enrich the amount of experimental data. By using the least squares method, optimal values of shear moduli G12 and G13 can be reached. This method, that was applied to laminated composite materials such as glass-epoxy, has been called into question in Davalos work [6] because it underestimated significantly the out of plane shear modulus.

The load introduction effects and the strategy chosen to get results appear to influence the relevance of the obtained moduli. The impact of these two aspects is evaluated in the first part of this study. This assessment requires the establishment of a FEM whose mechanical characteristics are the same as those of aluminum available in the laboratory.

2 Preliminary study

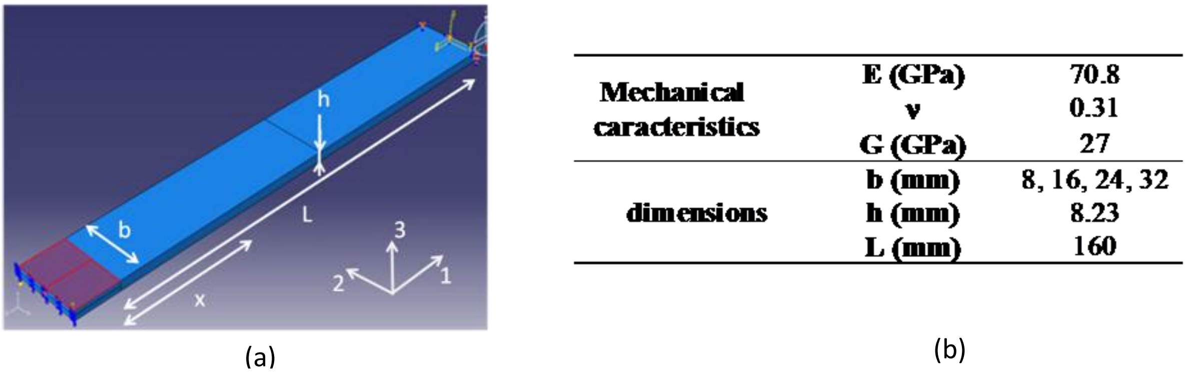
2.1 Evaluation of the effect due to load's introduction

The equation relating the applied torque to the angle of twist for torsion of an isotropic rectangular bar in the elastic range is given by Lekhnitskii's equation:

$$G_{12} = \frac{CL}{\theta b\, h^3 ß(C')} \qquad ß(C') = \frac{32C'^2}{\pi^4} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^4}\left(1 - \frac{2C'}{n\pi}\tanh\frac{n\pi}{2C'}\right) \qquad C' = \frac{b}{h}\sqrt{\frac{G_{13}}{G_{12}}} \quad (1)$$

where G_{12} and G_{13} are respectively in plane and out of plane shear moduli. b is the width, h is the thickness, and L the length of the test sample, θ is the angle of twist and C is the applied torque.

This equation does not take into account the effect due to the load introduction. To quantify this effect, FEM representative of isotropic Aluminum (**Figure 1.a**) was studied. Engineering constants implemented in the computer code ABAQUS ® are directly given by tensile tests performed on instrumented Aluminum samples. Four specimens of different widths were chosen to apply Sumsion's method of shear moduli extraction. Geometrical data are given below (**Figure 1.b**).



Mechanical caracteristics	**E (GPa)**	**70.8**
	ν	**0.31**
	G (GPa)	**27**
dimensions	**b (mm)**	**8, 16, 24, 32**
	h (mm)	**8.23**
	L (mm)	**160**

(a)

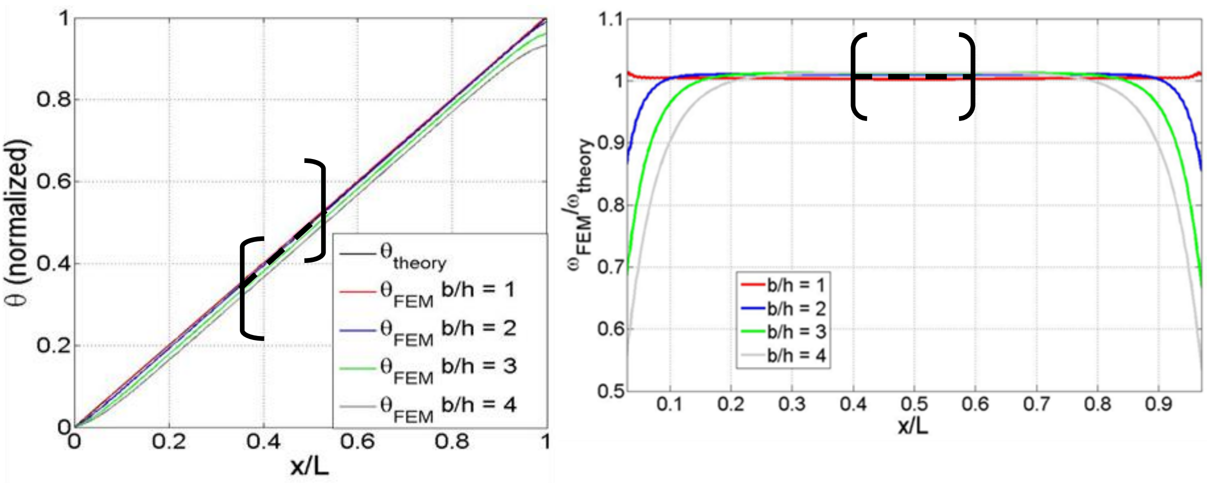
(b)

Figure 1 . FEM sample (a) and characteristics (b)

FEM Displacement field provides access to the evolution of the twist angle along the virtual specimen. This evolution admits non-linear domains near load introduction areas.

In comparison, theorical twist angle calculated from mechanical features implemented in the FEM, follows a strictly linear evolution (**Figure 2.a**). Additional calculations performed on several lengths show that this difference increases as the length of the samples decreases (length between the sample fixtures). Theoretical model cannot be applied under these conditions. This result shows that torsional rigidities experimentally measured between sample fixtures are in fact biased by the load introduction and do not reflect the intrinsic characteristics of the material tested.

The FEM twist angle evolves into a linear range on areas located far away from load introduction. The ratio between this twist angle per unit length (ω_{FEM}) and the theoretical response expected from the material (ω_{th}) gives a value close to unity when the measurement is performed on a range of between 40% and 60% of the tested sample length (**Figure 2.b).** On this limited area, Lekhnitskii's theoretical model can be applied. For the remainder of the study, the restrained area will be denoted by "RA". Direct measurement of the twist angle between the sample fixtures is equated to a measure on a total area denoted "TA".



(a) (b)

Figure 2 . Evolution of the FEM and theorical twist angle along the length of the tested sample (L=160mm) (a) and evolution of the ratio ω_{FEM} / ω_{th} (b)

This result follows work undertaken by Ogasawara[3]. To assess effectiveness of twist angle measurement on "RA" and "TA", the error calculation between each of them and the theorical twist angle expected can be estimated (**Figure 3**). For an angle measurement performed on "TA", the calculated error increases when the width increases, reaching more than 7%. This error tends to stabilize in the case of a measurement performed on "RA", reaching a maximum of 1.2%.

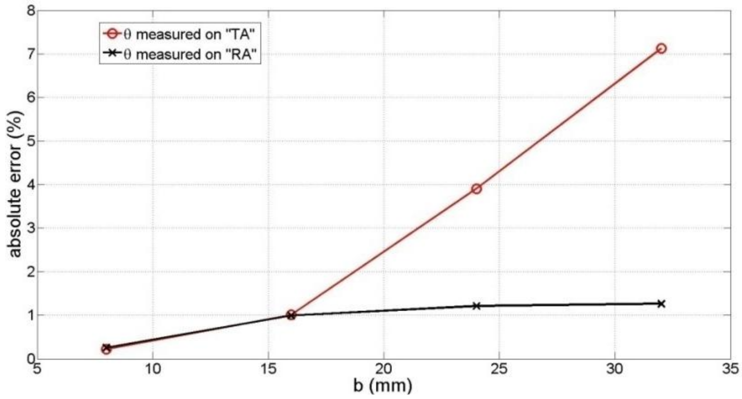


Figure 3 . Evolution of the committed error on θ measurement as a function of the specimen width

2.2 Consequences on shear moduli measurement

The simultaneous extraction process of in plane (G12) and out of plane (G13) shear moduli developed by Sumsion uses the interpolation of experimental results obtained for each tested configuration. A minimization function is calculated to get optimal values of both parameters G12 and G13. This function is presented below (2):

$$\min_{G_{12},G_{13}}\left(\sum_{i=1}^{N}\left[\frac{C_i L_i}{\theta_i b_i {h_i}^3}-G_{12}ß\left(\frac{b_i}{h_i}\sqrt{\frac{G_{13}}{G_{12}}}\right)\right]^2\right) \tag{2}$$

where i is the total number of different configuration tested, varying from 1 to N.

The method is applied to three different datasets. First one contains torsional rigidities theoretically calculated for each section according to the shear moduli implemented in the computer code (G12 = G13 = 27 GPa). The second one contains torsional rigidities issued from a twist angle measurement on "RA". The third one contains results issued from "RT". For each dataset, multi-parameter optimization is performed using the "fmincon" function available in MATLAB ® software.

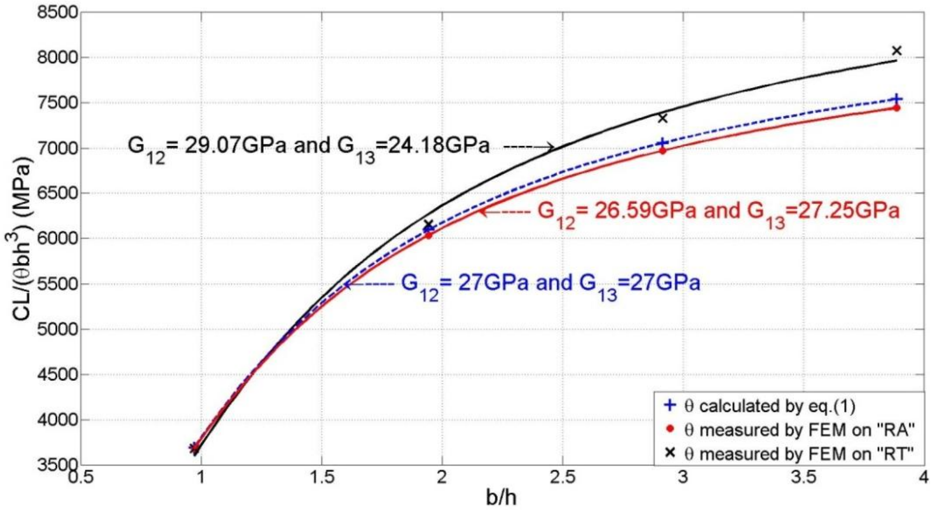


Figure 4 . Sumson's method application on three different dataset and corresponding shear moduli

Results of these different interpolations are given in Table 1. The shear moduli error is calculated by taking moduli implemented in calculation code as a reference (G12 = G13 = 27GPa). The results show that bad measurement of the twist angle causes an inaccurate estimate of the shear moduli. The phenomenon is amplified concerning out of plane shear modulus G13. In this case, the maximum error reaches more than 11%. Measuring the twist angle on "RA", coupled with the extraction method developed by Sumsion, permit to get more accurate results. In this case, the maximum error on moduli estimation reaches 1.5%.

	θ measured by FEM on "RA"	**θ measured by FEM on "TA"**
G_{12} (GPa)	26.59	29.07
error (%)	1.5	7.1
G_{13} (GPa)	27.25	24.18
error (%)	0.9	11.7

Table 1 . FEM Interpolation results performed with θ measured on "RA" and "RT" and corresponding errors

This preliminary study validates the fact that appropriate measurement of the material mechanical response (measured in "RA") combined with Sumsion's method gives remarkable accuracy concerning shear moduli estimation. It remains to validate this trend experimentally.

3. Experimental methodology: Use of stereo correlation

Stereo-correlation is a "non-contact" optical method used to measure three-dimensional position and displacement field, predominant during torsion solicitation. The detection of these three-dimensional fields is performed by matching two pictures taken simultaneously on the same object from different positions (stereovision principle). Measurement of these fields, focused on the restrained area "RA", is calculated using VIC-3D ® software. To facilitate the correlation process, a thin layer of painting giving a random pattern is applied to each sample tested.

The experimental procedure developed in this study is composed of four main steps. During torsion tests, several pictures are taken simultaneously by means of two cameras focused on the restricted area "RA" (**Figure 5 Step 1**). Then the correlation of these images provides a three-dimensional field of positions and displacements (**Step 2**). From this three-dimensional field, the data geometrical transformation permits to calculate the twist angle (**Step 3**). This angle is then attached to the torque applied and the geometry of the bar. This step allows to deduce torsional rigidity of the material in an area where Lekhnitskii’s theoretical model can be applied (**Step 4**).

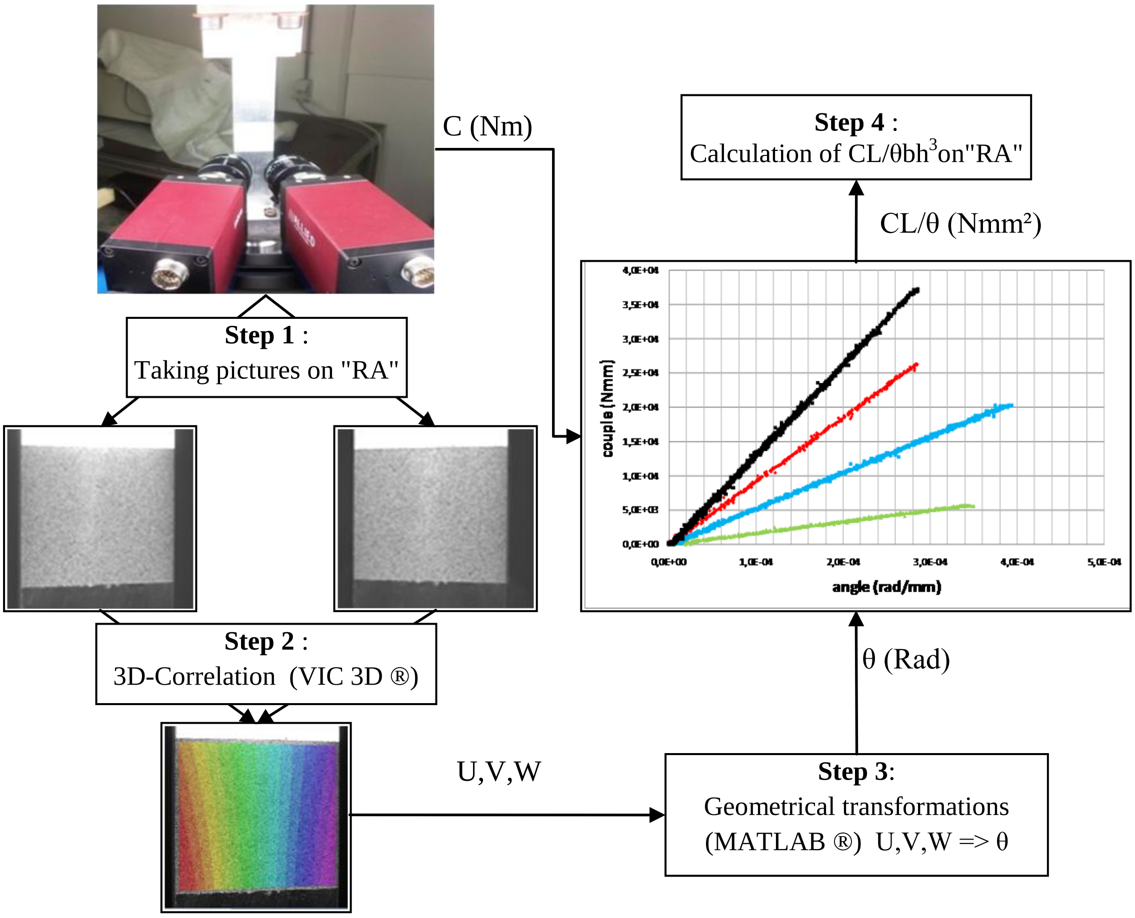


Figure 5 . Experimental methodology description

4. Validation of the experimental methodology on Aluminum

The experimental procedure is applied to four aluminum bars whose characteristics have previously been measured through instrumented tensile tests (E = 70.8GPa, ν = 0.31 and G = 27GPa). Geometrical characteristics of the samples tested are the same as those implemented in the FEM. Results issued from the procedure application are given in **Table 2**. The interpolation results of the experimental data are then carried out (**Figure 6**).

Designation		**Ep1**	**Ep2**	**Ep3**	**Ep4**
Dimensions	**L(mm)**	160	160	160	160
	b(mm)	8.03	15.98	24.04	32.01
	h(mm)	8.22	8.23	8.24	8.23
	b/h	0.98	1.94	2.92	3.89
$CL/\theta bh^3$ (MPa)		3721	5906	6886	7451

Table 2 . Experimental results issued from torsion test on Aluminum

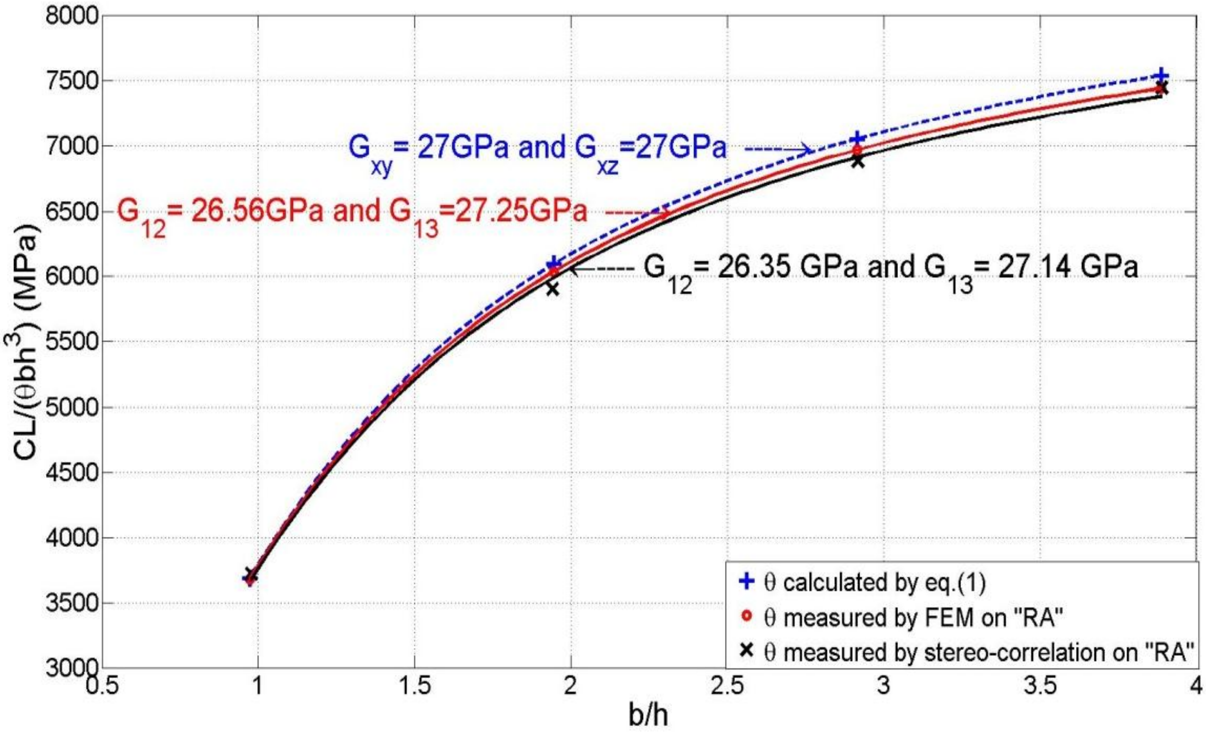


Figure 6 . Experimental data interpolation on Aluminum and corresponding shear moduli

Shear moduli calculated by the interpolation of data collected by stereo-correlation reflects the isotropy of the material. The error relative to the results of instrumented tensile test does not exceed 2.5%. The accuracy of the stereo-correlation and the effectiveness of Sumsion's method applied to a small area is therefore a robust experimental methodology to access in plane and out plane shear moduli of the material.

4 Methodology extension to the glass fiber reinforced polyester

The procedure described above is extended to a glass fiber reinforced polyester laminate made of 34 plies. Four different sections were selected for torsion tests. The averages of the specimen geometrical characteristics are given in Table 3.

Config. N° (total number of samples tested)		**Config. 1 (5)**	**Config. 2 (5)**	**Config. 3 (3)**	**Config. 4 (3)**
Mean dimensions	**L (mm)**	160	160	160	160
	b (mm)	7,39	11	17,77	24,15
	h (mm)	6,91	6,91	6,84	6,79
	b/h	1,07±0,018	1,59±0,016	2,60±0,022	3,55±0,018
$CL/\theta bh^3$ (MPa)		615±24	957±46	1275±36	1421±42

Table 3 . Geometrical characteristics of glass fiber reinforced polyester laminate

A first interpolation is based on the average values of each configuration (Figure 7). This first interpolation gives us simultaneously $G_{12\ \text{Mean}}$ and $G_{13\ \text{Mean}}$. Two other interpolations are performed, taking into account standard deviations over and under the average of experimental data for each configuration. Thus, the uncertainty on the in plane and out plane shear moduli can be estimated (**Table 4**).

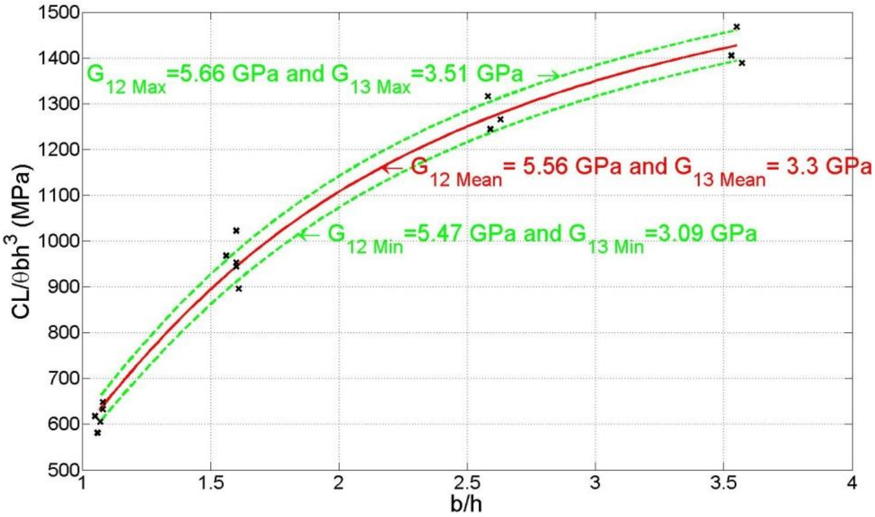


Figure 7 . Experimental data interpolation on glass fiber reinforced polyester and corresponding shear moduli

G_{12} (GPa)	**G_{13} (GPa)**
5,56 ± 1.7%	**3,3 ± 6.4%**

Table 4 . results of the interpolation on glass fiber reinforced polyester

Experimental uncertainties resulting from torsion tests significantly affect the accuracy of the measured shear moduli. However, according to the conclusions of previous sections, the combined use of stereo-correlation on a restrained area and application of Sumsion's method ensures that no significant additional bias has to be added to the given results.

5 Conclusion

Using a FEM representative of an isotropic rectangular bar under torsion loading, a preliminary study has enabled to visualize and quantify the effect of the load introduction, and its consequences on the simultaneous measurement of in plane and out plane shear moduli (G12 and G13). In addition, the analysis of this FEM permitted to identify an optimal area for the twist angle measurement. Accurate measurement of in plane and out of plane shear moduli has been next carried out (maximum error less than 1.5%). Experimentally, stereo-correlation has been used to measure the twist angle of this restrained area. Joined to the application of simultaneous extraction of the shear moduli method, originally developed by Sumsion, the methodology developed allowed to find with remarkable accuracy the aluminum in plane and out of plane shear moduli (maximum error less than 2.5%). Then, this methodology has been applied to a glass fiber reinforced polyester laminate. The reliability of this experimental methodology provides a glimpse of future applications for composites using three-dimensional architecture, whose out of plane behavior characterization becomes an important issue.

6 Bibliography

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