FRACTURE PARAMETERS FOR THE MULTI-MATERIAL INTERFACE CORNERS

C. Hwu* and H.Y. Huang

Institute of Aeronautics and Astronautics, National Cheng Kung University, Tainan, TAIWAN, R.O.C.
** CHwu@mail.ncku.edu.tw*

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Abstract
A new unified definition of the stress intensity factors valid for all possible multi-material interface corners is proposed. The associated singular orders may be real or complex, distinct or repeated. In other words, not only the negative real power singularity but also the oscillatory and logarithmic singularities are included. The comparison and discussion between the present definition and those proposed in the literature are then presented through some analytical relations and numerical examples.

1 Introduction
It is known that singular order and stress intensity factor are two important parameters for the study of cracks, interface cracks, corners (sometimes called notches), and interface corners (sometimes called multi-material wedges, junctions, or joints). For the general multi-material interface corners, the values of singular orders depend on the corner angles as well as the elastic properties of composed materials, and may be real or complex, distinct or repeated. If the singular order is a complex value, oscillatory singular behavior occurs for the stresses near the corner tip. If the repeated root occurs for the singular orders, logarithmic singularity exists for the near tip stress distribution. If the stress intensity factors are simply taken to be the coefficients of the singular terms, they may have different units due to different values of singular orders; and hence, a direct comparison of the stress intensity factors between two different interface corners is not allowed. In other words, one cannot take the fracture toughness measured from the usual crack specimen to predict the failure of interface corners, if their stress intensity factors do not have the same definition and the same unit.

Since cracks in homogeneous materials are special cases of interface cracks with two identical materials, and interface cracks are special cases of interface corners with two flat corner angles, it is believed that a unified definition with a unified unit for the stress intensity factors is the best way to build a connection among cracks, interface cracks, corners and interface corners even their singular orders may have different types. With this consideration, some unified definitions of stress intensity factors were proposed in the literature such as [1-3]. However, in real applications it was found that most of the definitions still have different units for different interface corners and cannot cover different singular orders simultaneously. To improve this situation, recently a new unified definition valid for all possible multi-material interface corners was proposed [4]. The comparison and discussion between the present definition and those proposed in the literature were then presented through some analytical relations and numerical examples [5].

2 Definition of the stress intensity factor

2.1 A general unified definition

In terms of the polar coordinate (r,θ) with origin located at the tip of the interface corner, a unified definition of the stress intensity factors $\mathbf{k}(\theta)$ were defined as [4,5]

$$\mathbf{k}(\theta)=\begin{Bmatrix} K_{II}(\theta) \\ K_{I}(\theta) \\ K_{III}(\theta) \end{Bmatrix}=\lim_{r\to 0}\sqrt{2\pi\ell}(r/\ell)^{\mathbf{\Delta}(\theta)}\boldsymbol{\sigma}(r,\theta), \tag{1}$$

where $\boldsymbol{\sigma}(r,\theta)$ is the stress vector composed of the traction along $\theta=\text{constant}$, i.e.,

$$\boldsymbol{\sigma}(r,\theta)=\begin{Bmatrix} \sigma_{r\theta}(r,\theta) \\ \sigma_{\theta\theta}(r,\theta) \\ \sigma_{\theta 3}(r,\theta) \end{Bmatrix}, \tag{2}$$

and $\mathbf{\Delta}(\theta)$ is the *matrix of singular orders*. $K_I(\theta)$, $K_{II}(\theta)$, $K_{III}(\theta)$ are, respectively, the stress intensity factors of opening mode, shearing mode, and tearing mode. ℓ is a reference length. When $\theta=0$ which is the conventional definition for a crack in homogeneous materials, we have

$$\mathbf{k}=\lim_{r\to 0}\sqrt{2\pi\ell}(r/\ell)^{\mathbf{\Delta}}\boldsymbol{\sigma}(r,0), \tag{3}$$

where $\mathbf{k}=\mathbf{k}(0)$, $\mathbf{\Delta}=\mathbf{\Delta}(0)$. The definition given in (1) and (3) are valid for all possible cases of interface corners including the one with logarithmic singularity. Since the stress intensity factors $\mathbf{k}(\theta)$ defined in (1) will vary according to the selected direction $\theta=\text{constant}$, like the principal stress which occurs at the plane where the shear stress vanishes, the maximum stress intensity factor of opening mode - the *principal stress intensity factor* represented by K_I^p, may occur at the radial direction $\theta=\theta_p$ where

$$K_{II}(\theta_p)=0, \quad \text{and} \quad K_I^p=K_I(\theta_p)=\max\{K_I(\theta)\}. \tag{4}$$

2.2 Special case 1 – crack in homogeneous materials

When a crack is located in a homogeneous material, either isotropic or anisotropic, it has been shown that their singular orders δ_α are [4]

$$\delta_1=\delta_2=\delta_3=0.5. \tag{5}$$

With the singular orders given in (5), the matrix of singular orders $\mathbf{\Delta}$ can be shown to be 0.5**I** (**I** is an identity matrix), and the definition (3) reduces to the conventional definition

$$\mathbf{k}=\lim_{r\to 0}\sqrt{2\pi r}\boldsymbol{\sigma}(r,0). \tag{6}$$

2.3 Special case 2 – interface crack between two dissimilar orthotropic materials

When a crack is located on the interface between two dissimilar orthotropic materials, the singular orders $\delta_\alpha, \alpha=1,2,3$ and their associated eigenfunction matrix $\mathbf{\Lambda}=[\boldsymbol{\lambda}_1\ \boldsymbol{\lambda}_2\ \boldsymbol{\lambda}_3]$ for orthotropic bi-materials have been obtained by Ting [6] and Hwu [7] as

$$\delta_1 = 0.5 + i\varepsilon,\quad \delta_2 = 0.5 - i\varepsilon,\quad \delta_3 = 0.5,$$
$$\mathbf{\Lambda} = \begin{bmatrix} -i/\sqrt{2D_{11}} & i/\sqrt{2D_{11}} & 0 \\ 1/\sqrt{2D_{22}} & 1/\sqrt{2D_{22}} & 0 \\ 0 & 0 & 1/\sqrt{D_{33}} \end{bmatrix}, \tag{7}$$

where ε is the *oscillatory index* which characterizes the oscillatory behavior of the stresses near the crack tip, and D_{ij}, $i,j = 1,2,3$, are the components of the bi-material matrix $\mathbf{D}$. The explicit expressions of ε and D_{ij} for the general anisotropic bi-materials can be found in [8]. For isotropic bi-materials under plane strain condition, it has been shown that [7]

$$D_{11} = D_{22} = \frac{2(1-\nu_1^2)}{E_1} + \frac{2(1-\nu_2^2)}{E_2},\quad D_{33} = \frac{2(1+\nu_1)}{E_1} + \frac{2(1+\nu_2)}{E_2}, \tag{8}$$

where E_k, ν_k, $k = 1,2$, are Young's modulus and Poisson's ratio of material 1 and 2. With the results of (7), the matrix of singular orders $\mathbf{\Delta}$ can be obtained as

$$\mathbf{\Delta} = \mathbf{\Lambda} < \delta_\alpha > \mathbf{\Lambda}^{-1} = \begin{bmatrix} 1/2 & \varepsilon\sqrt{D_{22}/D_{11}} & 0 \\ -\varepsilon\sqrt{D_{11}/D_{22}} & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}. \tag{9}$$

The angular bracket <> used in (9) stands for a diagonal matrix in which each component is varied according to the subscript α, e.g., $< z_\alpha >= \text{diag.}\left[z_1,\ z_2,\ z_3\right]$. The matrix power function $(r/\ell)^{\Delta}$ becomes

$$(r/\ell)^{\Delta} = (r/\ell)^{1/2} \begin{bmatrix} c^*(r) & s^*(r)\sqrt{D_{22}/D_{11}} & 0 \\ -s^*(r)\sqrt{D_{11}/D_{22}} & c^*(r) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{10}$$

where

$$c^*(r) = \cos[\varepsilon \ln(r/\ell)],\quad s^*(r) = \sin[\varepsilon \ln(r/\ell)]. \tag{11}$$

The definition (3) then reduces to

$$\begin{aligned} K_I &= \lim_{r\to 0} \sqrt{2\pi r}\left\{-\sqrt{D_{11}/D_{22}}\sin[\varepsilon\ln(r/\ell)]\sigma_{r\theta}(r,0) + \cos[\varepsilon\ln(r/\ell)]\sigma_{\theta\theta}(r,0)\right\}, \\ K_{II} &= \lim_{r\to 0} \sqrt{2\pi r}\left\{\cos[\varepsilon\ln(r/\ell)]\sigma_{r\theta}(r,0) + \sqrt{D_{22}/D_{11}}\sin[\varepsilon\ln(r/\ell)]\sigma_{\theta\theta}(r,0)\right\}, \\ K_{III} &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{\theta 3}(r,0), \end{aligned} \tag{12}$$

that can be further reduced to the one proposed by Rice [9] for isotropic bi-material interface cracks whose $D_{11}/D_{22} = 1$.

3 Numerical examples

To see the difference between the present definition and that presented in our previous study [1], several numerical examples have been done. Since most of the numerical details such as path-independent H-integral, finite element meshes, integration paths, convergence test as well as geometry and loading information have been described in [1], to save the space of this paper most of the details are omitted and can be found through the following way: *Problem 1-3: cracks under uniform loading* (Example 1 of [1]). Prob.1: edge crack under uniform tension; Prob.2: edge crack under uniform shear; Prob. 3: center crack under uniform tension. *Problem 4-5: interface cracks under uniform tension* (Example 3 of [1] in which the ratio of

Young's modulus is selected to be $E^{(2)}/E^{(1)}=0.5$). Prob.4: center interface crack; Prob.5: edge interface crack. *Problem 6-7: notches under uniform tension* (Example 2 of [1]). Prob.6: center notch with $\Delta\theta=3\pi/2$; Prob.7: edge notch with $\Delta\theta=3\pi/2$, where $\Delta\theta=\theta_1-\theta_0$ denotes the notch angle. *Problem 8: interface corners under uniform tension* (Example 4 of [1]). The angle above the interface $\Delta\theta_2=\theta_2-\theta_1=\pi$, and the angle below the interface $\Delta\theta_1=\theta_1-\theta_0=5\pi/6$.

No.	ℓ (mm)	K_I [1] ($\mathrm{MPa}*\mathrm{mm}^{\delta_R}$)	K_I Present ($\mathrm{MPa}*\sqrt{\mathrm{mm}}$)	K_{II} [1] ($\mathrm{MPa}*\mathrm{mm}^{\delta_R}$)	K_{II} present ($\mathrm{MPa}*\sqrt{\mathrm{mm}}$)
		Cracks under uniform loading			
1	3.5	9.37	9.30	0.00	0.00
2	3.5	33.73	33.82	4.48	4.50
3	2.0	17.63	17.74	0.00	0.00
		Interface cracks under uniform tension			
4	1.0	17.61	17.63	0.81	0.83
5	1.0	20.84	20.68	2.06	1.96
		Notches under uniform tension			
6	2.0	22.48	23.35	0.00	0.00
7	2.0	23.15	23.79	0.00	0.00
		Interface corners under uniform tension			
8	10.0	72.77	77.62	9.15	10.15

Table 1. The comparison of stress intensity factors.

From Table 1 we see that the present results are well agreed with those presented in [1] for all crack problems (Prob.1-5). Whereas for corner problems, small difference occurs for certain cases. Detailed discussions about this difference can be found in [5]. No matter which reasons cause the small difference between two unified definitions, through the comparison shown in Table 1, it is really hard to see the benefit of new definition except the unified unit.

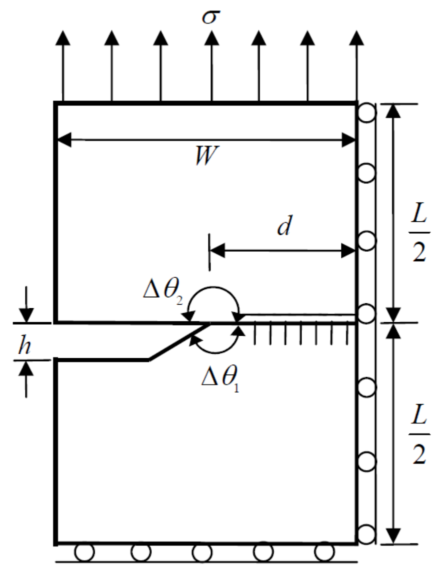

Figure 1. An interface corner between two dissimilar material ($d/W=1/3,\ h/W=1/15,\ d/L=1/18,\ \sigma=10\mathrm{MPa}$).

In order to see the necessity of the new definition, we now consider an interface corner as shown in Figure 1. The materials above and below the interface are, respectively, isotropic and orthotropic, whose properties are

$$\begin{aligned} &\text{isotropic}: \quad E = 10\text{GPa}, \ \nu = 0.2 \\ &\text{orthoropic}: \quad E_{11} = 134.45\text{GPa}, \ E_{22} = E_{33} = 11.03\text{GPa} \\ &\qquad G_{12} = G_{13} = 5.84\text{GPa}, \quad G_{23} = 2.98\text{GPa} \\ &\qquad \nu_{12} = \nu_{13} = 0.301, \quad \nu_{23} = 0.49. \end{aligned}$$

As presented in our previous study [5], the reference length ℓ is selected to be 10 mm. Figure 2 shows the first three singular orders for interface corners ranging from $\Delta\theta_1 = 20^o$ to 180^o. It is interesting to see that the singular orders meet the type change at $\Delta\theta_1 = 64.670^o$ and $\Delta\theta_1 = 140.597^o$. Figure 3 shows the results of stress intensity factors versus angle of interface corner $\Delta\theta_1$. From this Figure, we see that no abrupt change occurs in the entire region for the stress intensity factors defined by (3), represented here by the notation K_I^D and K_{II}^D, whereas the stress intensity factors defined in [1] represented by K_I^C and K_{II}^C meet an abrupt change at the transition angle $\Delta\theta_1 = 140.597^o$. Detailed explanation about the difference of their values can also be found in [5]. From the discussions given in [5], we see that the present definition (3) is really much more appropriate to be the unified definition for the stress intensity factors of interface corners.

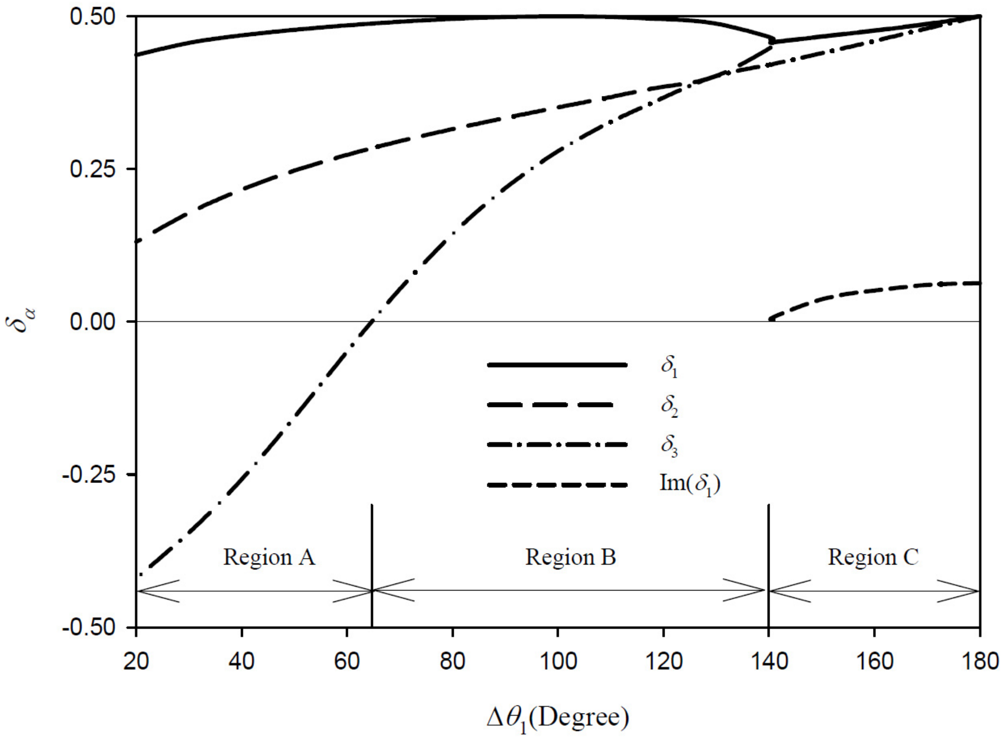

Figure 2. Variation of singular orders δ_α on corner angle $\Delta\theta_1$. [5]

4 Conclusions

A new definition of stress intensity factor is proposed in this paper for all possible cases of interface corners. To know the difference between previous definition of the stress intensity factors and the present one, several numerical examples are presented. From the numerical examples shown in this paper we see that this newly defined stress intensity factor has a unified unit ($\text{Pa}\sqrt{\text{m}}$) and will vary smoothly even when the stress singularity changes among real, complex and logarithmic types.

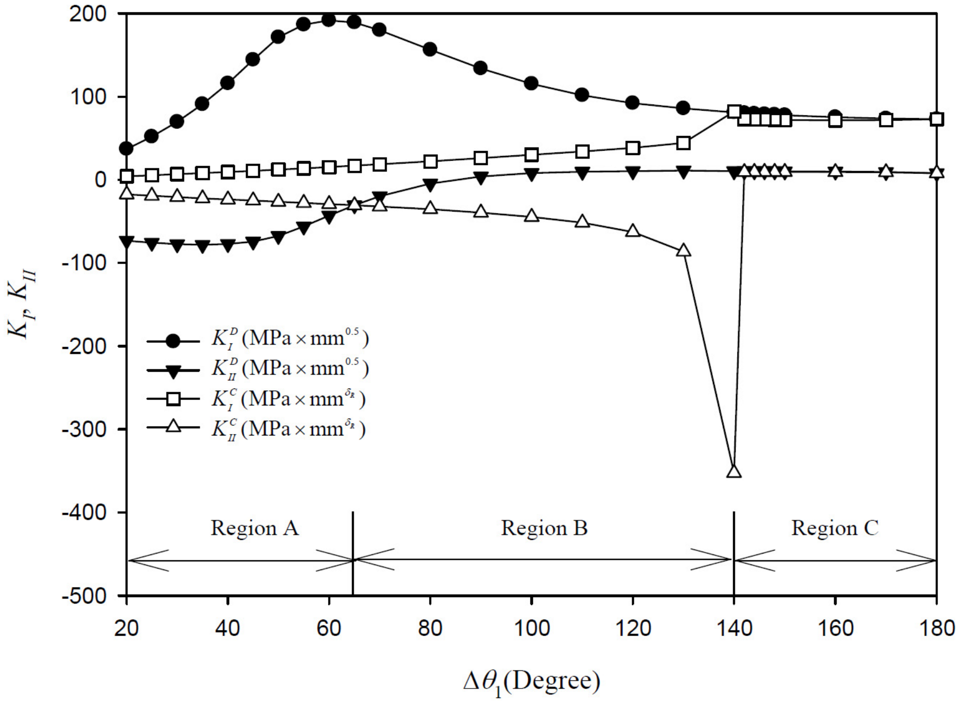

Figure 3. Variation of stress intensity factors K_I and K_{II} on corner angle $\Delta\theta_1$. [5]

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