HOMOGENIZATION AND IDENTIFICATION OF EFFECTIVE MATERIAL PARAMETERS OF TEXTILE REINFORCED COMPOSITE

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Abstract

In the presented paper[1] the two-step homogenization procedure of effective material properties of a textile reinforced composite is developed. The homogenization is conducted within two steps. First, the effective elastic moduli of yarns in the textile composite are determined. Then obtained parameters are used to performed a numerical homogenization of the textile composite. The yarns' elastic moduli are homogenized with help of various analytical techniques of homogenization, which are based on the equivalent eigenstrain method. The main advantage of this approach is that the analytical methods deliver explicit formulae for the effective stiffness tensor and for the local stresses and strains, which reduce overall computational time. The numerical homogenization is conducted with help of Representative Volume Element (RVE) approach, which enable modelling of cyclic symmetry structures, e.g. textile reinforced composites. In this paper the FE method is used to model the RVE. The exact model of reinforced textile structure together with appropriate cyclic boundary conditions gives an efficient procedure of determining effective elastic properties of the composite. The developed homogenization model is compared to experimental data. The results were obtained in experiments with a thin textile reinforced plates i.e. tensile tests in two orthogonal directions along yarns, tensile test at an angle to yarn direction and pure shear tests. The applicability of the homogenization procedure for prediction of the elastic properties of a textile reinforced composite, details of the tests and a results are presented within the paper.

1 Introduction

The textile reinforced composites are widely used as primary structure materials in fields of aerospace, transportation and other industrial applications. The expansion of application of this composites require a significant improvement of the modelling techniques. However, the simple micromechanical models like Voigt model gives a non satisfactory prediction of out of plane elastic constants. Therefore, for a better prediction of the effective material properties of a textile reinforced composite more sophisticated micromechanical models have to be used.

One of the most popular method of modelling micromechanical behaviour of composites are homogenization techniques based on the equivalent eigenstrain method [6], which considers

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the problem of a single ellipsoidal inclusion embedded in a infinite elastic medium. Within this group several approach can be distinguished and these are: the Mori-Tanaka [7], [8] and [9] method, which takes into account the interactions between inclusions, Self Consistent method, formulated by Hill [10]. Another methods are based on concept of a unit cell and the Fourier series technique applied to estimate the overall elastic properties of materials with periodically distributed voids [11], [12]. The main advantage of analytical methods is that the method delivers explicit formulae for the effective stiffness tensor and for the local stresses and strains. However, in a case of modelling complex woven fabric structures, the analytical models give a non satisfactory prediction of effective material properties.

The alternative to the analytical models can be solution of micromechanical problem with a help of finite element (FE) method [4], [5]. The FE method enables modelling of very complex woven fabric structures and gives more accurate description of the local strain and stress inside the textile composites. Therefore, the prediction of elastic constants done with FE method provide better results then these received by use of analytical models. However, development of a fully three-dimensional finite element model of a textile composite is a time-consuming task. In addition, for each new textile structure a new three-dimensional mesh has to be generated and the new boundary value problem has to be solved numerically. In this work an averaging of material parameters method is proposed. The results of the estimated elastic parameters are compared with experimental data.

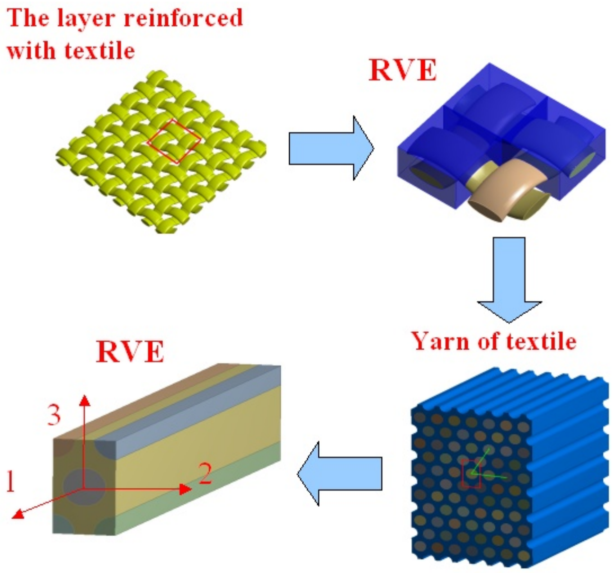

Figure 1. Two step homogenization scheme

2 Calculation Methods

In the presented paper the two-step homogenization procedure of effective material properties of a textile reinforced composite is studied. The prediction of effective material properties of a textile reinforced composite (plane-weave composite) is conducted within two steps. First the effective elastic moduli of yarns in textile composite are determined. Then obtained parameters are used to perform numerical homogenization of the composite. The numerical homogenization is conducted based on the Representative Volume Element (RVE) concept.

The yarns' elastic moduli are homogenized with help of analytical homogenization technique, which is based on the equivalent eigenstrain method [6] and with consideration of a periodic microstructure [12]. The main advantage of this approach is that the analytical method deliver explicit formulae for the effective stiffness tensor and for the local stresses and strains, which reduce overall computational time.

According to the results of microscopic study the periodic hexagonal distribution of long cylindrical fibres is assumed. Therefore, effective material of the yarns is assumed to be transversely isotropic. For such material the effective stiffness tensor is determined as

$$\bar{C}=C^m-V_f[(C^m-C^f)^{-1}-P]^{-1} \tag{1}$$

where P is polarization tensor, which describes geometry of inclusion and microstructure, V_f, is the volume fraction of the fibres and the matrix and C^f, C^m are the stiffness tensors of the fibres and the matrix, both isotropic.

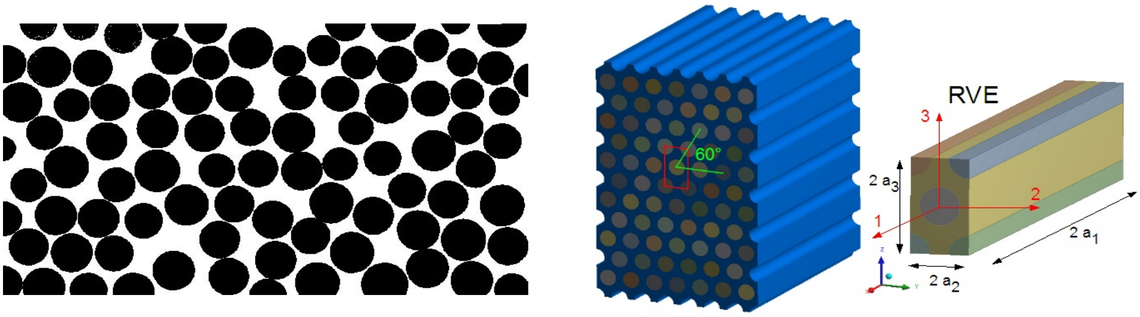

Figure 2. Micro-structure of yarns and corresponding micromechanical model

The microscopic analysis showed that the fibre volume fraction is different with respect to location of the yarns. Therefore, several effective materials, depending on the fibre volume fraction have been determined. The result of the analysis are presented in Table 1. The glass fibre properties are the following: Young's modulus 80 GPa, Poisson's ratio 0.2. The epoxy resin properties: Young's modulus 3 GPa, Poisson's ratio 0.33 respectively.

Fiber content	**E_X [GPa]**	**E_Y [GPa]**	**E_Z [GPa]**	**G_{XY} [GPa]**	**G_{XZ} [GPa]**	**G_{YZ} [GPa]**	**υ_{XY}**	**υ_{XZ}**	**υ_{YZ}**
transverse cross-section f = 82%	66.15	20.99	20.99	8.92	8.92	7.70	0.22	0.22	0.36
longitudinal cross-section f = 85%	68.46	24.51	24.51	10.43	10.43	9.12	0.22	0.22	0.34
mean f = 83.5%	67.30	22.55	22.55	9.62	9.62	8.31	0.22	0.22	0.36
transverse cross-section dense f = 92%	73.85	64.56	64.56	16.78	16.78	40.43	0.21	0.21	0.20
transverse cross-section mean f=87%	70.77	30.82	30.82	12.50	12.50	12.00	0.21	0.21	0.28

Table 1. Effective material for composite yarns (transversely isotropic)

The numerical homogenization is conducted with help of the Representative Volume Element (RVE) approach, which enable modelling of periodic structures, e.g. textile reinforced composites. In this paper the FE method is used to model the RVE. The exact model of the reinforced textile structure together with the appropriate periodic boundary conditions give the efficient procedure of determining effective elastic properties of the composite. For FE modelling the periodic boundary condition can be defined as

$$u_i^{+j}(x,y,z)-u_i^{-j}(x,y,z)=\bar{\epsilon}_{ik}\Delta x_k^j \quad . \tag{2}$$

Incorporating the periodic boundary condition, the effective elastic properties of textile composites can be determined by solving six independent boundary value problems. In each solution the components of the material stiffness tensor are determined.

The RVE is developed based on dimensions obtained from a microscopic analysis. The yarns cross section is modelled using average dimension from twenty locations (Figure 3).

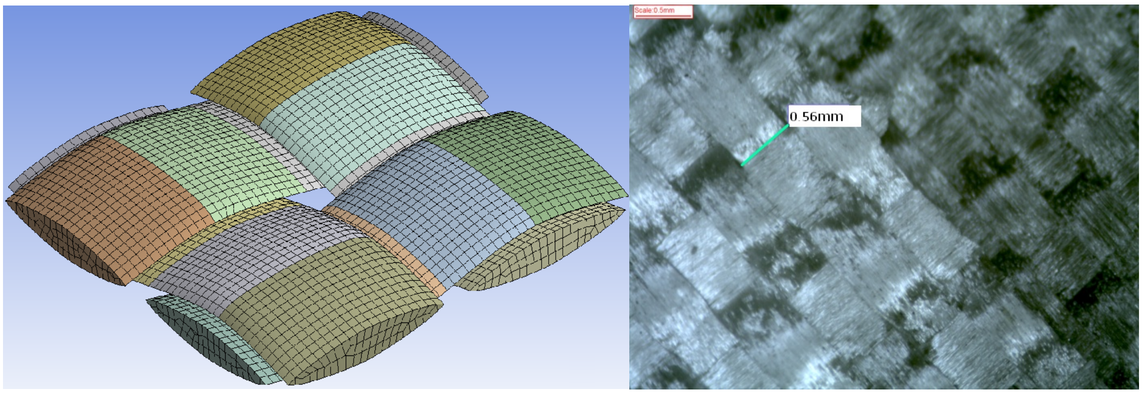

Figure 3. RVE of textile composite: FE model developed based on microscopic analysis

The FE model comprises yarns, as a transversely isotropic material (effective properties after the homogenization of fibres and epoxy resin properties) and epoxy resin in between them. The yarns and matrix are assumed to be perfectly bonded. Due to the fact that the yarns have transversely isotropic properties, the elements' coordinate systems are aligned with bend line of each yarn. The x-axis is always the axis of a yarn.

2 Material

The subject of investigation and as a test bed for numerical procedures a well-known epoxy-glass composite material was used. The material comprised a layered textile of glass fibre yarns in the epoxy matrix. The material was delivered in a form of a flat plate out of which specimens were cut out. The volume fraction of a fibre content has been estimated based on microscopic photographs of longitudinal and transverse cross-sections. Depending on the sampled region of the yarn cross-section the obtained fractions differed. The samples from the mid part of the yarn exhibit lower volume fraction of the fibres (min. 82%). In contrary, samples taken from the region of an interface between two orthogonal yarns showed a higher volume fraction (max. 95%).

The evaluation procedure of the samples included conversion to 1-bit colour images and then into 0-1 matrix to calculate number of components of particular value. The material was assumed to be linear elastic in the region being subject of interest. The elastic material model implies relevant compliance matrix form and relevant experiment. Components of the matrix were subject of the identification based on experimental results and numerical calculations.

3 Experiments

The aim of the experiments was to deliver data for identification of the compliance matrix components and further compare them with numerical calculations' results. There were two types of experiments carried out i.e. tension of a flat rectangular plate in three directions and pure shear of a circular specimen (see Figure 7). The tensile tests were conducted in two orthogonal directions along yarns and at an angle of 45°. All tests were performed by means of a PC controlled hydraulic pulser. Strains were measured with tensometers. Because of the limited number of specimen (2 for pure shear and 8 for tension) no statistical evaluation of the results has been made.

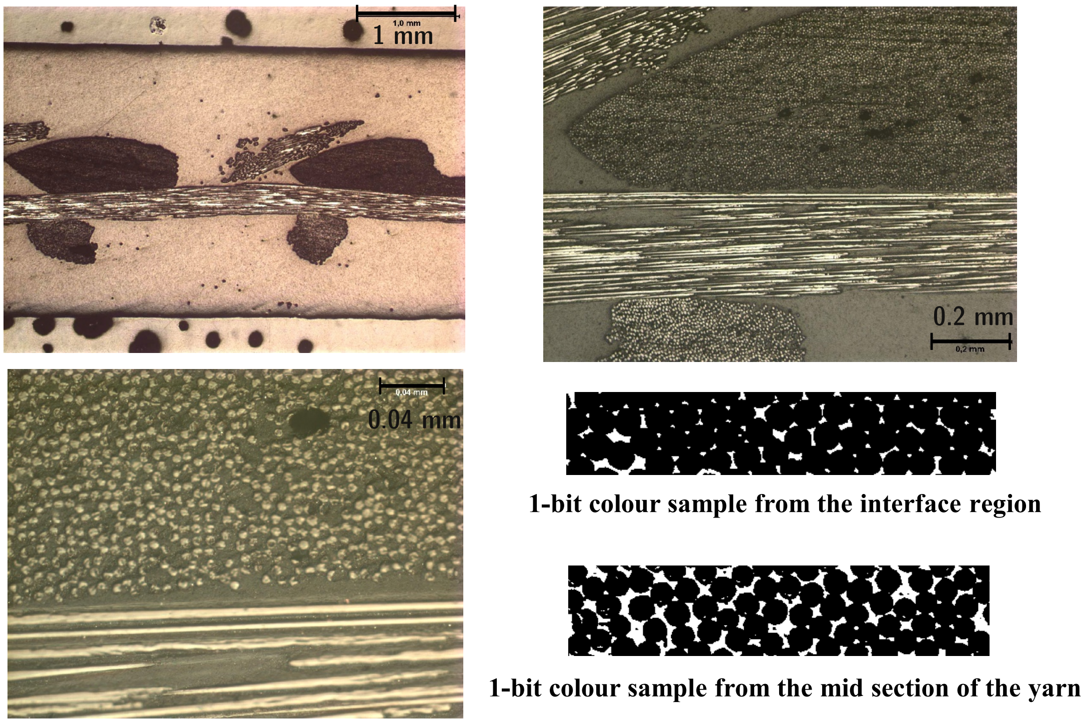

Figure 4. The examined material – microscopic photographs at specified length scales and selected yarn cross section samples used for estimation of the fibre volume fraction

4 Results of Numerical Identification

The FE simulation on RVE were performed for different fibre fractions, which were identified by numerical analysis of microscopic photographs. Additionally, calculation was performed for an average fibre fraction of 87%, averaged from different location of the RVE. The effective material properties of the textile composite obtained in the FE homogenization are presented in Table 2.

	Property											
Fibre content	E_z [GPa]	E_x [GPa]	E_y [GPa]	G_{xy} [GPa]	G_{zy} [GPa]	G_{zx} [GPa]	υ_{zx}	υ_{xz}	υ_{zy}	υ_{yz}	υ_{xy}	υ_{yx}
82%	6.92	20.96	21.06	4.14	2.28	2.28	0.36	0.12	0.36	0.12	0.15	0.15
85%	7.17	22.12	22.21	4.62	2.35	2.35	0.35	0.11	0.35	0.11	0.15	0.15
92%	8.5	28.22	28.28	6.48	2.61	2.61	0.25	0.074	0.25	0.075	0.17	0.17
87%	7.52	23.69	23.77	5.24	2.43	2.43	0.33	0.11	0.33	0.11	0.15	0.15

Table 2. The effective material properties of the textile composite obtained in the FE homogenization

5 Experimental Results

Based on the experiment described in Section 3, Young's moduli along the yarn directions 1 and 2 as well as Poisson's ratios have been identified. The identification results are presented in Figure 5 and Figure 6. For the identification of Young's moduli experimental data of the first linear region were considered. Additionally, as a result of the test at an angle, the relevant elasticity modulus has been obtained and used for estimation of the shear modulus according to Morozov formula [1]

$$G_{12}=\frac{\sin^2\phi\cos^2\phi}{1/E_x-\cos^4\phi/E_1-\sin^4\phi/E_2+(2\,\nu_{21}/E_1)\sin^2\phi\cos^2\phi}\quad . \tag{3}$$

In case of Young's moduli not varying significantly in orthogonal directions E_x can be used as obtained for $\varphi = 45^o$.

Another way of identification of the shear modulus is based on the method initially proposed in [3]. The method requires performing experiments with a flat, circular shaped specimen. It was proven that the pure shear stress state one obtains in the middle part of such a specimen. In this part of the specimen strain in three directions has to be measured with strain gauges (tensometers) offset by 45° versus each other. The maximum shear strain in the middle part is calculated as

$$\gamma_{max}=\sqrt{2}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2} \tag{4}$$

where ε_1, ε_2 i ε_3 are strains measured at 90°, 45° and 0° angles. Based on [3], it is assumed that shear stress in the middle part is calculated as a nominal stress with known tensile force referred to the cross-section of measurement area. Results of the experiments with the circular specimen are presented in Figure 7 together with the identification results.

6 Comparison and Conclusions

A comparison between results obtained in the experiments and in the numerical calculations is presented in Table 3.

		E_1 [GPa]	E_2 [GPa]	$\upsilon_{12}=\upsilon_{21}$	G_{12} [GPa]
Experimental results	Flat specimen	25.1	27.6	0.06	3.9
	Circular specimen	n/a	n/a	n/a	5.1
Numerical calculations with volume fraction of fibre	82%	20.96	21.06	0.15	4.14
	87%	23.69	23.77	0.15	5.24
	92%	28.22	28.28	0.17	6.48

Table 3. Comparison of the experimental results and numerical calculations

The differences between numerical and experimental data, especially between shear modulus identified in the direct circular flat specimen test and the tension test, are due two reasons. The numerical homogenization is conducted assuming the ideal connection between the fibre and the matrix. There are some defects in the real material and this defects influence material properties. Second reason is sensitivity of the employed numerical procedure for estimation of the volume fraction of fibres in the yarn. Thus, for the further research an improved identification method of this value needs to be employed as well as the influence of voids between fibres and matrix should be investigated. Additionally, the area of different fibre content within the yarn were identified. This fact must be also considered in FE modelling during further research. It has to be also noted that a difference is observed between shear modulus identified in the direct circular flat specimen test and the tension test.

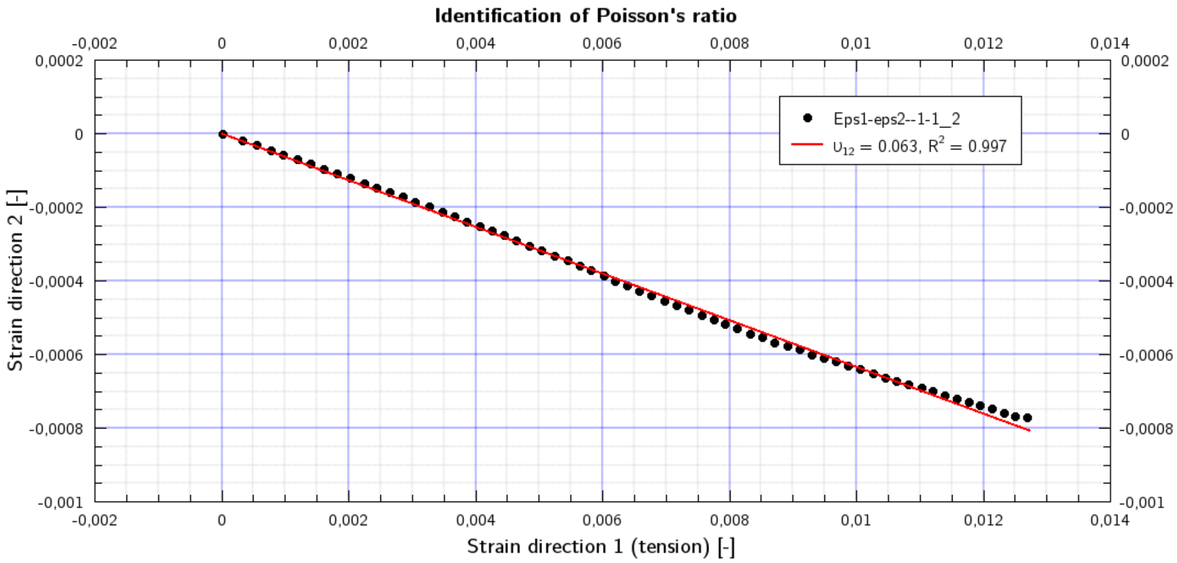

Figure 5. Identification of Poisson's ratio

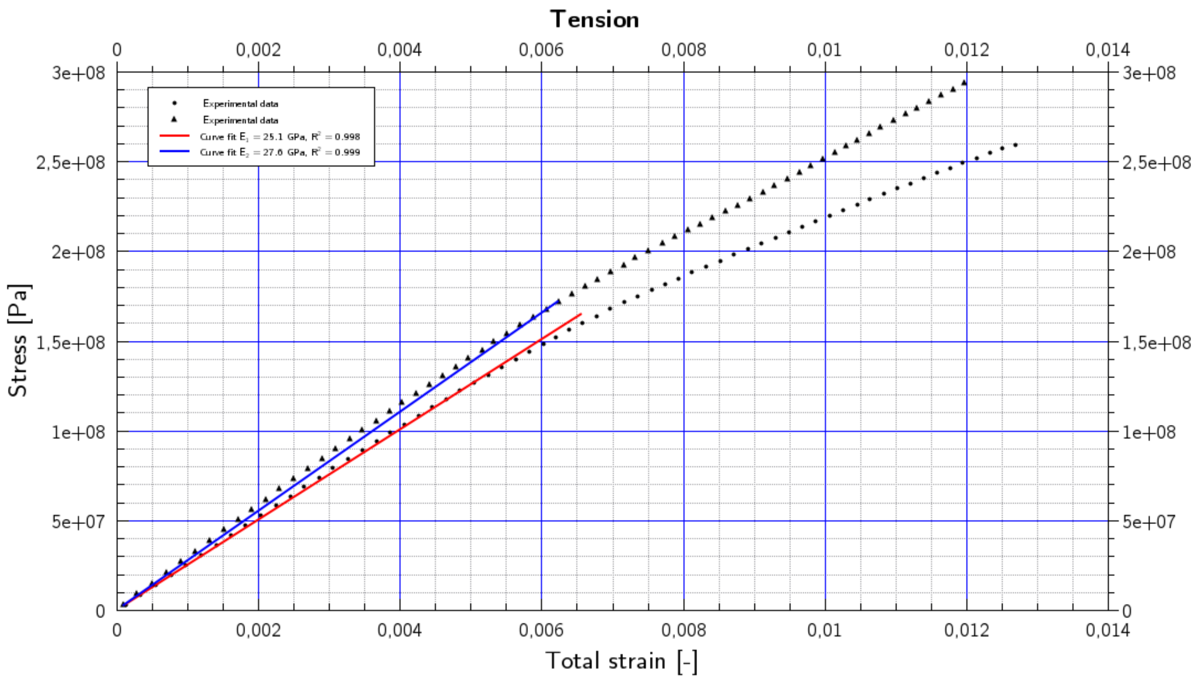

Figure 6. Young's moduli in directions along yarns – experimental results and identification

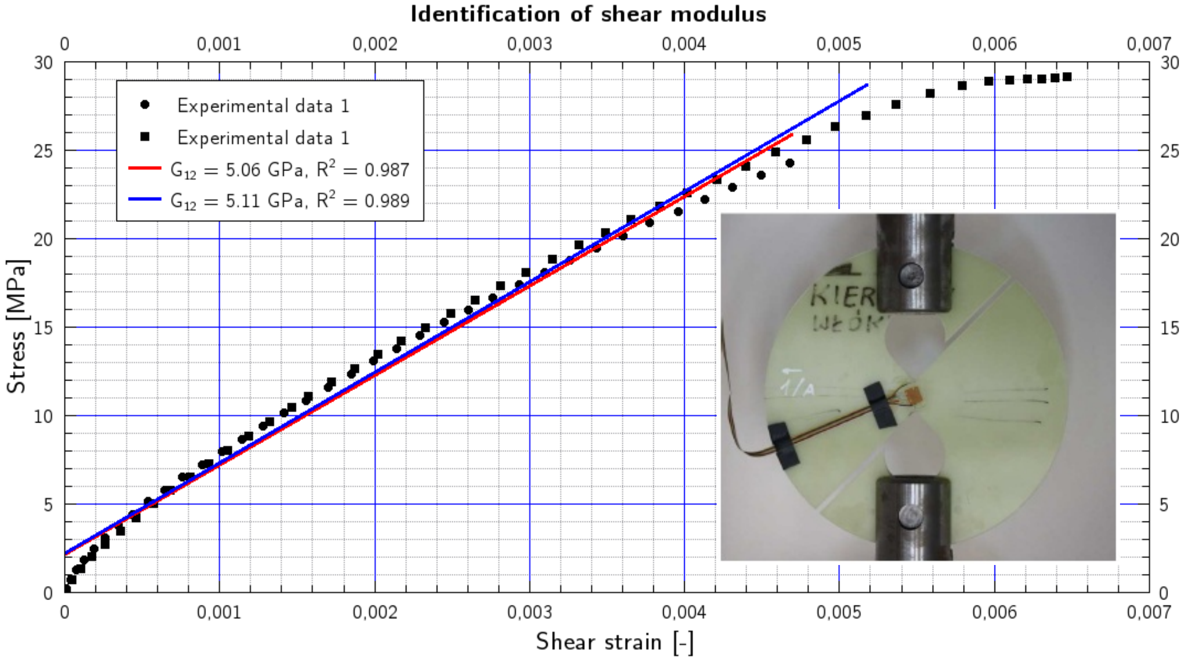

Figure 7. Identification of the shear modulus based on the experimental results for the circular flat specimen

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