FORMULATION OF 3-D HOMOGENIZATION METHOD FOR FABRIC COMPOSITE LAMINATES BY CONSIDERING THE EFFECT OF FINITE THICKNESS

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Abstract

Microstructure of fabric composite laminates can be very complex. Modeling such microstructure by using representative unit-cell requires assurance of periodicity. In that case, homogenization theory, a rigorous analytical method that considers periodicity, is often employed. Homogenization theory also considers both microscopic and macroscopic deformation of the unit-cell. The unit-cell model of fabric composite laminates is often considered to be periodic in three-dimension. However, in reality, composite laminate is very thin. Therefore, the effect of finite thickness must be taken into account. In this regard, unit-cell should be assumed to have finite thickness. It can be achieved by releasing the boundary condition in thickness direction. This paper deals with theoretical treatment of the modified 3-D homogenization method considering the effect of finite thickness.

1 Introduction

Composite material possesses a better strength-to-weight ratio compared to metal. This advantage becomes the main reason of composite application, especially in aerospace industry. In industry, the determination of equivalent thermo-mechanical properties is needed to represent the equivalent characteristic of composite materials. However, the heterogeneity in its constituent materials leads to the difficulty in the analysis of composite structure. Microstructure of composite structure could be very complex. Modeling such complex microstructure by using representative unit-cell is an efficient and accurate way to determine the equivalent thermo-mechanical properties. This modeling requires assurance of periodicity. In that case, homogenization method, a rigorous analytical method that considers periodicity [1], is often employed. This method considers both microscopic and macroscopic deformation of the unit-cell. The unit-cell model, for instance, is often considered to be periodic in three-dimension (x-, y- and z-directions). In other words, a unit-cell is assumed to be repeated infinitely in three directions. Guedes and Kikuchi [2] developed computer programs for determining the averaged elastic constants of general composite materials by using this method. The programs exclude the calculation of coefficients of thermal expansion (CTE). CTE of fiber reinforced composites are studied using finite element method by Karadeniz and Kumlutas [3]. The numerical study deals with micromechanical modeling and excludes macromechanical modeling. In this paper, thermal effects are considered to be included in the formulation of homogenization method in both microscopic and macroscopic scales. Another important aspect is that fabric composite laminates, especially for aerospace application, are very thin. Therefore, the effects of finite thickness must also be taken into account. Woo and Whitcomb [4] suggested these effects as a future study for the application in aerospace structure. In this regard, unit-cell should be assumed to have finite thickness. It can be achieved by releasing the boundary condition in z-direction. Challagulla et al [5] used homogenization method to analyze the equivalent elastic coefficients of thin composite network structure with orthotropic bars. However, formulation of homogenization method that includes both thermal effect and finite thickness is not available. This paper aims to propose the new formulation of this method for thermo-mechanical problem.

2 Preliminary Studies of Finite Thickness Effects using Finite Element Method

In ordinary homogenization method, the unit-cell model is assumed to be periodic in three-dimension. In other words, a unit-cell is assumed to be repeated infinitely in three-directions. This can be illustrated by Figure 1. The dotted lines show the periodicity direction of unit-cell.

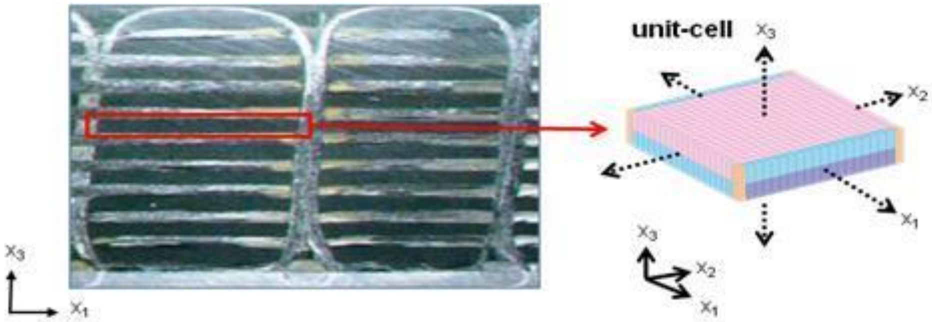

Figure 1. Unit-cell model with periodic boundary condition in three-directions.

Finite thickness influences the averaged thermo-mechanical properties of composite material. In order to have a better understanding, in this regard, a unit-cell model of 3-D orthogonal interlocked fabric composite is built and analyzed by using ABAQUS [6]. To simulate finite thickness, boundary conditions in thickness direction are excluded. In other words, the model has free traction boundaries at the top and bottom of the unit-cell surface (Figure 2).

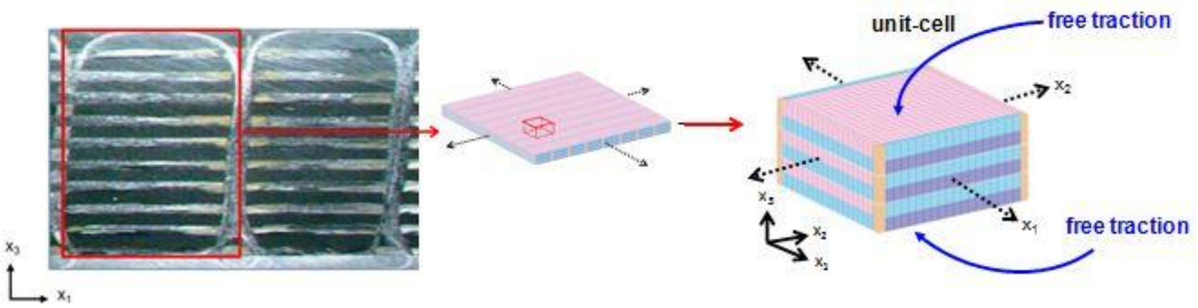

Figure 2. Unit-cell model with free traction boundaries at the top and bottom surface.

The model in Figure 2 is developed by increasing number of unit-cell stacking. The thermo-mechanical results are normalized with the results obtained by using FEM model with three-direction periodicity (Figure 1) as could be shown by Figure 3.

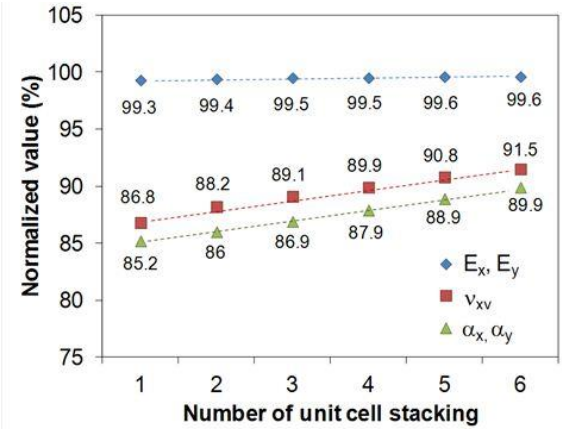

Figure 3. Normalized results of both methods against number of unit-cell stacking [6].

Figure 3 shows that finite thickness influences Poisson's ratio and coefficient of thermal expansion (CTE) significantly. However, it only reduces elastic modulus slightly.

3 Homogenization Method with Finite Thickness

Ordinary homogenization method includes all boundary conditions in three directions [2]. A rigorous formulation of this method can be found in [7]. This paper explains the new formulation of homogenization method with two-dimension periodicity. The unit-cell model used in this modified formulation can be shown in Figure 2. Following steps discuss the concept and derivation of new formulation. Consider an elastic body with heterogeneous microstructure shown by Figure 4.

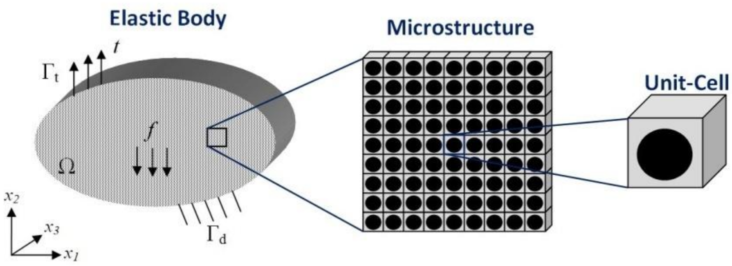

Figure 4. Elastic body with heterogeneous and periodic microstructure.

In this figure, f is body force on domain Ω, t is surface traction on boundary Γ_t, and Γ_d is boundary at which prescribed displacement is applied. The body consists of a large amount of heterogeneous and periodic microstructures as seen in Figure 5.

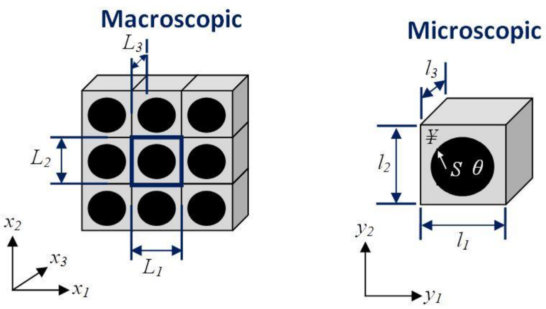

Figure 5. Unit-cell viewed from macroscopic and microscopic scale.

Heterogeneous microstructure consists of at least two parts, those are solid part (¥) and hole part (θ) incorporated in a unit-cell. In Figure 4, S is surface of θ. Unit-cell can be viewed from macroscopic and microscopic scales. Both scales are correlated by parameter ε which is the ratio of macroscopic and microscopic dimension. Periodic is represented by periodic vector function which includes macroscopic coordinate **x** and microscopic coordinate **y**. In this modified method, the laminates, without repeating the cell in the thickness direction, is considered very thin in out-of-plane direction as compared to the in-plane direction. Since the unit-cell does not have the periodicity in x_3-direction, it is necessary to modify the periodicity by excluding y_3 term, where the periodicity in thickness direction is applied.

$$g^{\varepsilon}(\mathbf{x}) = g(x_1, x_2, x_3, y_1, y_2) = g(x_1, x_2, x_3, y_1 + Y_1, y_2 + Y_2) \tag{1}$$

where $\varepsilon = \dfrac{\mathbf{x}}{\mathbf{y}} = \dfrac{L_1}{l_1} = \dfrac{L_2}{l_2} = \dfrac{L_3}{l_3}$ and **Y** is the dimension of unit-cell.

However, the use of x_3 term raises complex formulation in the derivation. Therefore, it is needed to express x_3 in terms of y_3 in order to simplify the derivation of the equations. Since there is no x_3 term, the differentiation of periodic vector function with respect to macroscopic coordinate x_i will be as follows

$$\frac{\partial}{\partial x_i}\left[g^{\varepsilon}\left(\mathbf{x}, \mathbf{y} = \frac{\mathbf{x}}{\varepsilon} \right) \right] = \frac{\partial g}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_i} \tag{2}$$

$$\frac{\partial g^{\varepsilon}}{\partial x_1} = \frac{\partial g}{\partial x_1} + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_1} \tag{3}$$

$$\frac{\partial g^{\varepsilon}}{\partial x_2} = \frac{\partial g}{\partial x_2} + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_2} \tag{4}$$

$$\frac{\partial g^{\varepsilon}}{\partial x_3} = \frac{\partial g}{\partial x_3} + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_3} = 0 + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_3} = \frac{1}{\varepsilon}\frac{\partial g}{\partial y_3} \tag{5}$$

Limit of periodic function in microscopic scale can be written as follows:

$$\lim_{\varepsilon \to 0^+} \int_{\Omega^{\varepsilon}} \Phi(\mathbf{x}, \mathbf{y}) d\Omega \to \frac{1}{|Y|} \int_{\Omega}\int_{¥} \Phi(\mathbf{x}, \mathbf{y}) dY d\Omega \tag{6}$$

where **x** = x_1, x_2; **y** = y_1, y_2, y_3; dΩ = $dx_1 dx_2$; and dY = $dy_1 dy_2 dy_3$. In this regard, the macroscopic terms (i.e. in $d\Omega$) actually consist of 2-D terms, whilst the microscopic terms still remains 3-D terms. The subsequent derivation processes follow the ordinary method [7].

The thermo-mechanical problem can generally be solved using the weak-form of principle of virtual work.

$$\int_{\Omega^\varepsilon} E^\varepsilon_{ijkl}\left(\frac{\partial u^\varepsilon_k}{\partial x_l} - \alpha^\varepsilon_{kl}\Delta T\right)\frac{\partial v_i}{\partial x_j}d\Omega = \int_{\Omega^\varepsilon} f^\varepsilon_i v_i d\Omega + \int_{\Gamma_i} t_i v_i d\Gamma + \int_{S^\varepsilon} p^\varepsilon_i v_i dS \qquad (7)$$

where u^ε_k is actual displacement, whilst v_i is virtual displacement. To solve Eq. (7), **u** should be approximated by asymptotic expansion series as follows:

$$u^\varepsilon_k(\mathbf{x},\mathbf{y}) = u^0_k(\mathbf{x},\mathbf{y}) + \varepsilon u^1_k(\mathbf{x},\mathbf{y}) + \varepsilon^2 u^2_k(\mathbf{x},\mathbf{y}) \qquad (8)$$

By substituting Eq. (8) into Eq. (7), applying the differentiation of periodic vector function, and rearranging and separating the expanded equation based on the order of ε, the following results are obtained.

- $\boldsymbol{\varepsilon}^{-2}$ terms:

$$\lim_{\varepsilon\to 0^+}\int_{\Omega^\varepsilon}\left\{\begin{aligned}&\left(E^\varepsilon_{i1k1}\frac{\partial u^0_k}{\partial y_1} + E^\varepsilon_{i1k2}\frac{\partial u^0_k}{\partial y_2} + E^\varepsilon_{i1k3}\frac{\partial u^0_k}{\partial y_3}\right)\frac{\partial v_i}{\partial y_1}\\ &+\left(E^\varepsilon_{i2k1}\frac{\partial u^0_k}{\partial y_1} + E^\varepsilon_{i2k2}\frac{\partial u^0_k}{\partial y_2} + E^\varepsilon_{i2k3}\frac{\partial u^0_k}{\partial y_3}\right)\frac{\partial v_i}{\partial y_2}\\ &+\left(E^\varepsilon_{i3k1}\frac{\partial u^0_k}{\partial y_1} + E^\varepsilon_{i3k2}\frac{\partial u^0_k}{\partial y_2} + E^\varepsilon_{i3k3}\frac{\partial u^0_k}{\partial y_3}\right)\frac{\partial v_i}{\partial y_3}\end{aligned}\right\}d\Omega = 0 \qquad (9)$$

$$\frac{1}{|Y|}\int_{\Omega}\int_{¥} E^\varepsilon_{ijkl}\frac{\partial u^0_k}{\partial y_l}\frac{\partial v_i}{\partial y_j}dYd\Omega = 0 \qquad (10)$$

Since virtual displacement is arbitrary, if **v=v(x),** the equation above will be automatically satisfied. If **v=v(y)**, by applying integration by parts, Eq. (10) can be expanded as follows

$$\frac{1}{|Y|}\int_{\Omega}\int_{¥}\left\{\frac{\partial}{\partial y_j}\left(E^\varepsilon_{ijkl}\frac{\partial u^0_k}{\partial y_l}v_i(\mathbf{y})\right)\right\}dYd\Omega - \frac{1}{|Y|}\int_{\Omega}\int_{¥}\left\{\frac{\partial}{\partial y_j}\left(E^\varepsilon_{ijkl}\frac{\partial u^0_k}{\partial y_l}\right)v_i(\mathbf{y})\right\}dYd\Omega = 0 \qquad (11)$$

Gauss' divergence theorem is applied to the first left hand side of Eq. (11), the following equation will be obtained.

$$\frac{1}{|Y|}\int_{\Omega}\left\{\int_{C_Y+\partial C_{Ya}+\partial C_{Yb}} E^\varepsilon_{ijkl}\frac{\partial u^0_k}{\partial y_l}n_j v_i(\mathbf{y})dY - \int_{¥}\frac{\partial}{\partial y_j}\left(E^\varepsilon_{ijkl}\frac{\partial u^0_k}{\partial y_l}\right)v_i(\mathbf{y})dY\right\}d\Omega = 0 \qquad (12)$$

Integrations over ∂C_{Ya} and ∂C_{Yb} will cancel each other because each boundary has opposite direction. Integration over C_Y will also be zero because free traction boundaries are applied at the top and bottom of the unit-cell (see unit-cell in Figure 2).

The remaining equation will be as follows

$$-\frac{1}{|Y|}\int_{\Omega}\int_{\yen}\frac{\partial}{\partial y_j}\left(E_{ijkl}^{\varepsilon}\frac{\partial u_k^0}{\partial y_l}\right)v_i\left(\mathbf{y}\right)dYd\Omega=0 \tag{13}$$

$\mathbf{u^0}$= $\mathbf{u^0(x)}$ satisfies Eq. (13).

- ε^{-1} terms:

$$\lim_{\varepsilon\to 0^+}\int_{\Omega^\varepsilon}\left\{\begin{aligned}&\left[E_{i1k1}^{\varepsilon}\frac{\partial u_k^0}{\partial y_1}+E_{i1k2}^{\varepsilon}\frac{\partial u_k^0}{\partial y_2}+E_{i1k3}^{\varepsilon}\frac{\partial u_k^0}{\partial y_3}\right]\frac{\partial v_i}{\partial x_1}+\left[E_{i2k1}^{\varepsilon}\frac{\partial u_k^0}{\partial y_1}+E_{i2k2}^{\varepsilon}\frac{\partial u_k^0}{\partial y_2}+E_{i2k3}^{\varepsilon}\frac{\partial u_k^0}{\partial y_3}\right]\frac{\partial v_i}{\partial x_2}\\&+\left[E_{i1k1}^{\varepsilon}\left(\frac{\partial u_k^0}{\partial x_1}+\frac{\partial u_k^1}{\partial y_1}-\alpha_{k1}^{\varepsilon}\Delta T\right)+E_{i1k2}^{\varepsilon}\left(\frac{\partial u_k^0}{\partial x_2}+\frac{\partial u_k^1}{\partial y_2}-\alpha_{k2}^{\varepsilon}\Delta T\right)+E_{i1k3}^{\varepsilon}\left(\frac{\partial u_k^1}{\partial y_3}-\alpha_{k3}^{\varepsilon}\Delta T\right)\right]\frac{\partial v_i}{\partial y_1}\\&+\left[E_{i2k1}^{\varepsilon}\left(\frac{\partial u_k^0}{\partial x_1}+\frac{\partial u_k^1}{\partial y_1}-\alpha_{k1}^{\varepsilon}\Delta T\right)+E_{i2k2}^{\varepsilon}\left(\frac{\partial u_k^0}{\partial x_2}+\frac{\partial u_k^1}{\partial y_2}-\alpha_{k2}^{\varepsilon}\Delta T\right)+E_{i2k3}^{\varepsilon}\left(\frac{\partial u_k^1}{\partial y_3}-\alpha_{k3}^{\varepsilon}\Delta T\right)\right]\frac{\partial v_i}{\partial y_2}\\&+\left[E_{i3k1}^{\varepsilon}\left(\frac{\partial u_k^0}{\partial x_1}+\frac{\partial u_k^1}{\partial y_1}-\alpha_{k1}^{\varepsilon}\Delta T\right)+E_{i3k2}^{\varepsilon}\left(\frac{\partial u_k^0}{\partial x_2}+\frac{\partial u_k^1}{\partial y_2}-\alpha_{k2}^{\varepsilon}\Delta T\right)+E_{i3k3}^{\varepsilon}\left(\frac{\partial u_k^1}{\partial y_3}-\alpha_{k3}^{\varepsilon}\Delta T\right)\right]\frac{\partial v_i}{\partial y_3}\end{aligned}\right\}d\Omega \tag{14}$$

$$=\lim_{\varepsilon\to 0^+}\int_{S^\varepsilon}p_i^{\varepsilon}v_i dS$$

If **v=v(x)**, because **$\mathbf{u^0}$= $\mathbf{u^0(x)}$**, the equilibrium of energy is automatically satisfied.

$$\frac{1}{|Y|}\int_{\yen}\int_{S^\varepsilon}p_i v_i dSd\Omega=0 \tag{15}$$

Actual displacement can be represented by the following equation.

$$u_i^{\varepsilon}\left(\mathbf{x},\mathbf{y}\right)=u_i^0\left(\mathbf{x}\right)+\varepsilon u_i^1\left(\mathbf{x},\mathbf{y}\right) \tag{16}$$

where $\mathbf{u^0}$ is macroscopic displacement and $\mathbf{u^1}$ is microscopic displacement as a function of characteristic displacement vector (***χ***) and thermal displacement characteristic (***ψ***) as follows

$$u_i^1\left(\mathbf{x},\mathbf{y}\right)=-\chi_i^{pq}\left(\mathbf{y}\right)\frac{\partial u_p^0\left(\mathbf{x}\right)}{\partial x_q}-\psi_i\left(\mathbf{y}\right) \tag{17}$$

By choosing **v**=**v**(**y**) and substituting Eq. (17) into Eq. (14), microscopic equilibrium equations are obtained as follows

Elastic problem:

$$\int_{\yen}\left\{\left[E_{ijkl}^{\varepsilon}-E_{ijpq}^{\varepsilon}\frac{\partial\chi_p^{kl}}{\partial y_q}\right]\frac{\partial u_k^0\left(\mathbf{x}\right)}{\partial x_l}\right\}\frac{\partial v_i\left(\mathbf{y}\right)}{\partial y_j}dY=\int_S p_i v_i\left(\mathbf{y}\right)dS \tag{18}$$

Thermal problem:

$$\int_{\yen}\left\{E_{ijkl}^{\varepsilon}\left(\frac{\partial\psi_k}{\partial y_l}-\alpha_{kl}^{\varepsilon}\Delta T\right)\right\}\frac{\partial v_i\left(\mathbf{y}\right)}{\partial y_j}dY=0 \tag{19}$$

- $\boldsymbol{\varepsilon}^0$ terms:

$$\lim_{\varepsilon\to 0^+}\int_{\Omega^\varepsilon}\left\{\begin{aligned}&\left[E^\varepsilon_{i1k1}\left(\frac{\partial u^0_k}{\partial x_1}+\frac{\partial u^1_k}{\partial y_1}-\alpha^\varepsilon_{k1}\Delta T\right)+E^\varepsilon_{i1k2}\left(\frac{\partial u^0_k}{\partial x_2}+\frac{\partial u^1_k}{\partial y_2}-\alpha^\varepsilon_{k2}\Delta T\right)+E^\varepsilon_{i1k3}\left(\frac{\partial u^1_k}{\partial y_3}-\alpha^\varepsilon_{k3}\Delta T\right)\right]\frac{\partial v_i}{\partial x_1}\\&+\left[E^\varepsilon_{i2k1}\left(\frac{\partial u^0_k}{\partial x_1}+\frac{\partial u^1_k}{\partial y_1}-\alpha^\varepsilon_{k1}\Delta T\right)+E^\varepsilon_{i2k2}\left(\frac{\partial u^0_k}{\partial x_2}+\frac{\partial u^1_k}{\partial y_2}-\alpha^\varepsilon_{k2}\Delta T\right)+E^\varepsilon_{i2k3}\left(\frac{\partial u^1_k}{\partial y_3}-\alpha^\varepsilon_{k3}\Delta T\right)\right]\frac{\partial v_i}{\partial x_2}\\&+\left[E^\varepsilon_{i1k1}\left(\frac{\partial u^1_k}{\partial x_1}+\frac{\partial u^2_k}{\partial y_1}\right)+E^\varepsilon_{i1k2}\left(\frac{\partial u^1_k}{\partial x_2}+\frac{\partial u^2_k}{\partial y_2}\right)+E^\varepsilon_{i1k3}\frac{\partial u^2_k}{\partial y_3}\right]\frac{\partial v_i}{\partial y_1}\\&+\left[E^\varepsilon_{i2k1}\left(\frac{\partial u^1_k}{\partial x_1}+\frac{\partial u^2_k}{\partial y_1}\right)+E^\varepsilon_{i2k2}\left(\frac{\partial u^1_k}{\partial x_2}+\frac{\partial u^2_k}{\partial y_2}\right)+E^\varepsilon_{i2k3}\frac{\partial u^2_k}{\partial y_3}\right]\frac{\partial v_i}{\partial y_2}\\&+\left[E^\varepsilon_{i3k1}\left(\frac{\partial u^1_k}{\partial x_1}+\frac{\partial u^2_k}{\partial y_1}\right)+E^\varepsilon_{i3k2}\left(\frac{\partial u^1_k}{\partial x_2}+\frac{\partial u^2_k}{\partial y_2}\right)+E^\varepsilon_{i3k3}\frac{\partial u^2_k}{\partial y_3}\right]\frac{\partial v_i}{\partial y_3}\end{aligned}\right\}d\Omega \quad (20)$$

$$=\lim_{\varepsilon\to 0^+}\int_{\Omega^\varepsilon} f^\varepsilon_i v_i\, d\Omega+\int_{\Gamma_t} t_i v_i\, d\Gamma$$

By choosing **v=v(x)** and substituting Eq. (17) into Eq. (20), macroscopic equilibrium equation is obtained as follows

$$\begin{aligned}&\int_\Omega E^\varepsilon_{ijkl}(\mathbf{x})\frac{\partial u^0_k(\mathbf{x})}{\partial x_l}\frac{\partial v_i(\mathbf{x})}{\partial x_j}d\Omega=\int_\Omega \tau_{ij}(\mathbf{x})\frac{\partial v_i(\mathbf{x})}{\partial x_j}d\Omega+\int_\Omega \sigma^t_{ij}(\mathbf{x})\frac{\partial v_i(\mathbf{x})}{\partial x_j}d\Omega\\&+\int_\Omega b_i(\mathbf{x})v_i(\mathbf{x})d\Omega+\int_{\Gamma_t} t_i(\mathbf{x})v_i(\mathbf{x})d\Gamma\end{aligned} \quad (21)$$

i,k,p,q=1,2,3;
j,l=1,2;
where:

i. Macroscopic homogenized elastic constants:

$$E^0_{ijkl}(\mathbf{x})=\frac{1}{|Y|}\int_{¥}\left(E^\varepsilon_{ijkl}-E^\varepsilon_{ijpq}\frac{\partial \chi^{kl}_p}{\partial y_q}\right)dY \quad (22)$$

$$E^0_{ijk1}=\frac{1}{|Y|}\int_{¥}\left(E^\varepsilon_{ijk1}-E^\varepsilon_{ijpq}\frac{\partial \chi^{k1}_p}{\partial y_q}\right)dY \quad (23)$$

$$E^0_{ijk2}=\frac{1}{|Y|}\int_{¥}\left(E^\varepsilon_{ijk2}-E^\varepsilon_{ijpq}\frac{\partial \chi^{k2}_p}{\partial y_q}\right)dY \quad (24)$$

Elastic modulus in thickness direction E^0_{ijk3} cannot be obtained as a result of macroscopic terms changing into 2-D terms.

ii. Averaged stresses due to internal forces:

$$\tau_{ij}\left(\mathbf{x}\right)=\frac{1}{|Y|}\int_{\text{¥}}\left(E_{ijkl}^{\varepsilon}\frac{\partial\psi_k}{\partial y_l}\right)dY \tag{25}$$

iii. Averaged thermal stresses:

$$\sigma_{ij}^{t}\left(\mathbf{x}\right)=\frac{1}{|Y|}\int_{\text{¥}}\left(E_{ijkl}^{\varepsilon}\alpha_{kl}^{\varepsilon}\Delta T\right)dY \tag{26}$$

iv. Averaged body forces:

$$b_i\left(\mathbf{x}\right)=\frac{1}{|Y|}\int_{\text{¥}}f_i dY \tag{27}$$

4 Conclusions

Theoretical treatment of modified 3-D homogenization method for composite material has been developed. Several conclusions could be drawn from the formulation:

1) The new method includes:
   a. Effect of finite thickness, whereby out-of-plane periodicity is omitted.
   b. Effect of thermal stresses in the unit cell model.
2) Excluding thickness effect in the derivation of the 3-D homogenization formulation leads the new formulation could not obtain the averaged thermo-mechanical properties in thickness direction because x_3-direction terms are omitted.

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