EFFECTS OF MATRIX CRACKING ON THE ESTIMATION OF OPERATIONAL LIMITS OF FRP LAMINATES

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Abstract

Probabilistic failure analyses of structures are performed in order to determine the loading operational limits with respect to different degrees of safety. To guarantee an accurate prediction by numerical simulation, it is necessary to account for all the major sources of uncertainty and natural variation. Most probabilistic analyses disregard the degradation of the laminate's properties caused by matrix cracking. The objective of this investigation is to determine the effects that matrix cracks have on the estimation of operational limits through probabilistic analyses. The analyses compare the predictions of different models that consider some of the effects of the development of matrix cracks during loading. The importance of the methodology for taking into account matrix cracking in probabilistic failure analyses is discussed.

1 Introduction

One of the main challenges of structural design is guaranteeing the right safety level. In engineering, a structural safety level is commonly understood as a combination of the consequences of structural failure and its probability. If one considers the consequences to be fixed, an 'unreasonably safe' structure would have a too small probability of failure; such a structure can be considered to be inefficient since it will be heavier and/or more expensive than it actually needs to be. For structures made out of Fibre Reinforced Plastics (FRPs) this issue is especially important since these materials are commonly used in weight-critical designs and considered to be expensive. Therefore, it is vital to have accurate methods for estimating the probability of the different types of failure of FRP structures.

Probabilistic analyses are used to estimate the operational limits of FRP laminates and structures. Sriramula and Chryssanthopoulos [1] presented a thorough review of modelling approaches. One of these approaches is to model the FRP material uncertainties at the ply level, meaning that each one of the material properties of the ply is a stochastic variable with a particular distribution type, mean value and Coefficient of Variation (CV). The Ultimate Limit State (ULS) of the material is considered to be first ply failure and is predicted with one of the many available failure criteria for FRPs, such as Tsai-Wu, Tsai-Hill or Maximum Stress. This approach was applied in the analyses presented by Lin [2], Jeong and Shenoi [3], Frangopol and Recek [4] and Lekou and Philipiddis [5]. An expansion of this approach is to account for the degradation of the plies due to matrix cracking, when a matrix strength criterion is satisfied in a ply, the ply is considered to be damaged and some of its stiffness properties are set to zero or a percentage of their original value. This extension was used by Chen and Soares [6], Hwang et al. [7] and Sánchez et al. [8]. Clearly this consideration of the ply degradation in the stochastic analysis is intended to provide more accurate operational limits; however, one big consideration is that matrix cracking is considered to be a discrete fully developed event. Several investigations [9,10] have shown that matrix cracking is in fact a progressive event and that the degradation of the ply properties depends on the development of the crack density in the plies.

The objective of this investigation is to determine the effects that matrix cracks have on the estimation of operational limits through probabilistic analyses. Three approaches for the probabilistic analyses are compared: linear elastic, linear elastic with full degradation, and linear elastic with progressive degradation. Each approach was used to calculate the operational limits of two carbon/epoxy and two glass/epoxy laminates for a given safety level.

2 Laminate Response Models

In the Linear Elastic model (LE) the responses of the laminates were calculated by means of a one-dimensional analysis [11] where the effects of the transverse Poisson's contractions are considered to be negligible. This simplification was performed in order to have the same accuracy in the calculation of the stresses for all the models. In the LE model the degradation effects caused by matrix cracking were disregarded. The one-dimensional analysis was also used for the Linear Elastic with Full Degradation (LE-FD) model, in this model however, the laminates were considered to be fully degraded due to matrix cracks. The transverse stiffness (E_2) and the transverse thermal expansion coefficient (α_2) of the 90° plies were set to 1% of their original value.

The Linear Elastic with Progressive Degradation (LE-PD) model utilized the fracture mechanics variational analysis, developed by Nairn and colleagues [11,12,13,14], to predict the formation, accumulation and effects of matrix cracks in the 90° plies. The analysis estimates the laminate average crack density by predicting the formation of a matrix crack in the middle of a periodic laminate unit cell. A new matrix crack is expected to occur between two cracks when the energy release rate due to the formation of such a crack (ΔG_m) is equal to the critical energy release rate, referred to in this text as the matrix fracture toughness of the material (G_{mc}). The ΔG_m of $[0_m/90_n]_s$ and $[90_n/0_m]_s$ cross-ply laminates subjected to a displacement control loading are estimated by means of a variational analysis and expressed as a function of the undamaged stress state in the 90º plies ($\sigma_{x,90°}$) and the unit load energy release rate $G_{m,unit}(D)$

$$\Delta G_m = (\sigma_{x,90°})^2 G_{m,unit}(D) \tag{1}$$

$G_{m,unit}(D)$ is a function of the crack density (D) and its formulation on the type of laminate, either $[0_n/90_m]$s or $[90_m/0_n]$s. The x-direction stresses in the 90° plies of a cross-ply laminate are expressed as

$$\sigma_{x,90°} = k_{m,90°}\sigma_X + k_{th,90°}T \tag{2}$$

where σ_X is the applied laminate stress, T is the difference between the ambient and stress free temperature of the laminate, and k_m and k_{th} are the mechanical and thermal stiffness constants of each ply calculated through the same one-dimensional stress analysis used for the LE and LE-FD models. At low crack densities ΔG_m is independent of the crack spacing; however at higher crack densities its value decreases due to the influence of the neighbouring cracks. In the calculations the laminate is assumed to present periodic micro crack intervals, in reality, however, this is not the case. Matrix cracks tend to form in intervals larger than the average one since in them the energy release rate is higher. To account for this effect, the adjustable parameter f is introduced in the calculation of $G_{m,unit}$. An expression relating crack density to laminate stress is found by considering that matrix cracks occur when $\Delta G_m = G_{mc}$, substituting equation (2) in (1) and solving for σ_X:

$$\sigma_X = \frac{1}{k_{m,90°}}\left(\sqrt{\frac{G_{mc}}{G_{m,unit}(D)}} - k_{th,90°}T\right) \tag{3}$$

Expressions for the damaged laminate stiffness (E_X^d) and the thermal expansion coefficient (α_X^d) as a function of the crack density can be found as well through the fracture mechanics variational analysis [12,13]. Utilizing both expressions the strain state in the 0º ply when the laminate is damaged can be calculated as:

$$\varepsilon_{x,0°}^d = \frac{\sigma_X}{E_X^d(D)} + \left(\alpha_X^d(D) - \alpha_{x,0°}\right)T \tag{4}$$

In all the models fiber fracture in the 0° plies was regarded as the ULS of the laminate, and it was predicted with the Maximum Strain criterion. The LE-PD model was used also to predict the laminate's crack density when the ULS was reached.

3 Study Cases

Four cross-ply laminates were investigated: two made out of AS4/3501-6 carbon/epoxy prepreg, $[90/0/90]_T$ and $[0_2/90_2]_S$; and two out of VICOTEX NVE 913/28%/192/EC9756 glass/epoxy prepreg, $[0/90_2]_S$, $[0_2/90_2]_S$. Both materials were chosen due to the availability of published material properties and crack density measurements [11,12,15,16]. Table 1 presents the material properties used in this investigation. For the carbon/epoxy ply, the expected values were obtained from [12] with the exception of the ultimate tensile strain (ε_{u1T}) which was assigned the values presented in [17]. In the case of the glass/epoxy plies, the expected values are presented in [16]; however, ε_{u1T} is not available, thus the value for another glass/epoxy ply presented in [17] was chosen as an approximation. Additionally, the expected values for f, G_{mc} and T were obtained by fitting the calculated laminate stress vs. crack density curve to the experimental data given in [15]. The probability distributions and CV were set to reasonable types and values considering conservative engineering judgment and published values [1,18]. Additionally, any correlations between the input variables were disregarded. All the laminates are considered to be subjected to a unidirectional displacement-controlled tension load in the 0° direction. The fit of the crack density curves against experimental data for two of the studied laminates are presented in Figure 1. The curves were calculated with the LE-PD model and the expected values presented in Table 1.

Property	C/E	G/E	CV	Distribution
Longitudinal modulus, E_1 (GPa)	130	42.50	5%	Norm
Transverse modulus, E_2 (GPa)	9.70	13.30	5%	Norm
In-plane shear modulus, G_{12} (GPa)	5	5.80	5%	Norm
Out-of-plane shear modulus, G_{23} (GPa)	3.6	4.68	5%	Norm
In-plane Poisson's ratio, ν_{12}	0.3	0.29	5%	Norm
Out-of-plane Poisson's ratio, ν_{23}	0.5	0.42	5%	Norm
Long. thermal expansion coeff., α_1 (10^{-6}/°C)	-0.09	8.60	5%	Norm
Trans. thermal expansion coeff., α_2 (10^{-6}/°C)	28.80	22.10	5%	Norm
Microcracking fracture toughness, G_{mc} (J/m^2)	2 0	650	5%	Wbl/Norm
Ambient - stress free temperature, T (°C)	-95	-90	-	-
Ultimate longitudinal tensile strain, ε_{u1T} (%)	1.38	2.20	10%	Wbl/Norm
Ply thickness, t_{ply} (mm)	0.15	0.13	5	Norm
Average crack formation interval/Average crack spacing ratio, f	1.2	1.3	-	-

Table 1. Material properties of the carbon/epoxy (C/E) and glass/epoxy (G/E) plies.

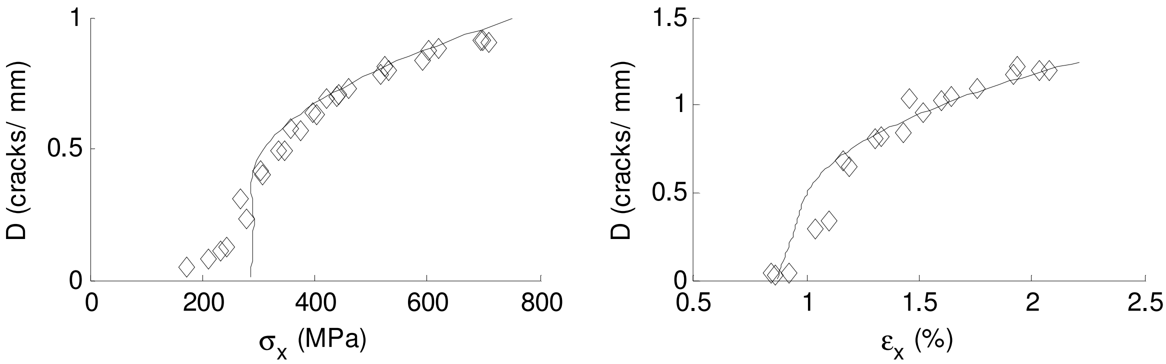

Figure 1. Calculated crack density curves (–) vs. experimental measurements (⍰) [11,15] of the carbon/epoxy (left) and glass/epoxy (right) $[0_2/90_2]_S$ laminates.

4 Probabilistic Analyses

For each laminate a probabilistic analysis utilizing the different laminate response models was performed. The probabilistic analyses consisted of two types of Monte Carlo simulations. In the first one, labelled 'all Normal', all the stochastic input variables were considered to follow a Normal distribution, while in the second one, 'Normal & Weibull', the variables G_{mc} and ε_{u1T} were instead considered to follow a Weibull distribution. These two types of simulations were performed to assess how the choice of distribution affects the evaluations of the operational limits. Structures are expected to have low probabilities of failure [19]. In this investigation a laminate stress that gave a probability of reaching the ULS (P_f) of 10^{-5} was chosen to determine the operational limits of the laminates. The rule of thumb for calculating cumulative distribution functions accurately, through Monte Carlo simulations, states that the number of simulations should be of an order of magnitude of $100/P_f$, therefore, 10^7 laminate response analyses were performed for every Monte Carlo simulation.

5 Results

The probabilistic distributions of the states, predicted with the different laminate response analyses, are presented with boxplots. Figure 2 shows a description of the information depicted with the boxplots. The left and right whiskers indicate the quantiles where the state has a probability of occurrence equal or less than 10^{-5} and 0.999, respectively. The sides of the box indicate the 0.25 and 0.75 quantiles while the white circle marks the location of the median. Figures 3 and 4 present the probabilistic ULS of the studied laminates. For describing the results, letters are used to refer to the lowest stress quantiles where the ULS has a probability of occurrence of 10^{-5}, as predicted with the different types of input ('all Normal' or 'Normal & Weibull') and laminate response analyses. Only the quantiles of one laminate are marked in Figures 3 and 4 for clarity. For each type of input and layup, the LE-PD quantiles are "bounded" by the LE and LE-FD quantiles (B<C<A, B*<C*<A*). The factor that affects the most the estimation of the quantiles is clearly the type of input (A*>A, B*>B, C*>C). To compare more effectively the results, Table 2 contains a summary of the lowest stress quantiles, normalized with respect to the predictions made with the LE-PD model, since this model is the most accurate. Figure 5 presents the probabilistic crack densities at the ULS of the laminates, as predicted by the LE-PD model. Clearly, the type of input plays an important role on the estimation of the quantiles.

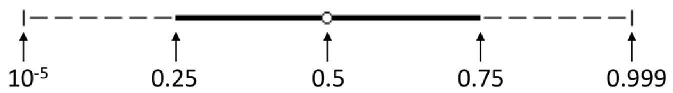

Figure 2. Explanation of the quantiles represented through the boxplots.

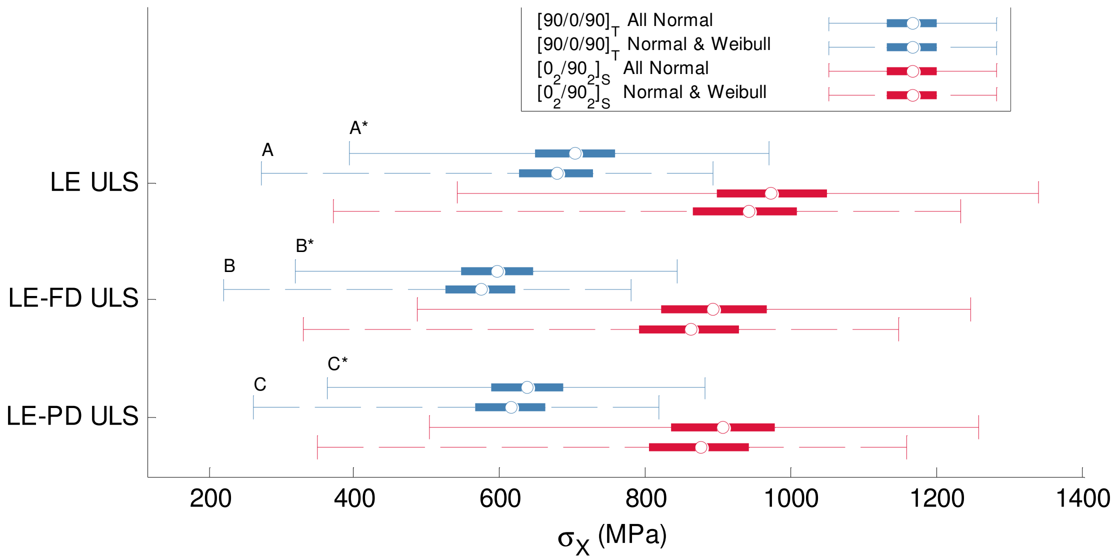

Figure 3. ULS probabilistic predictions for the $[90/0/90]_T$ & $[0_2/90_2]_S$ carbon/epoxy laminates.

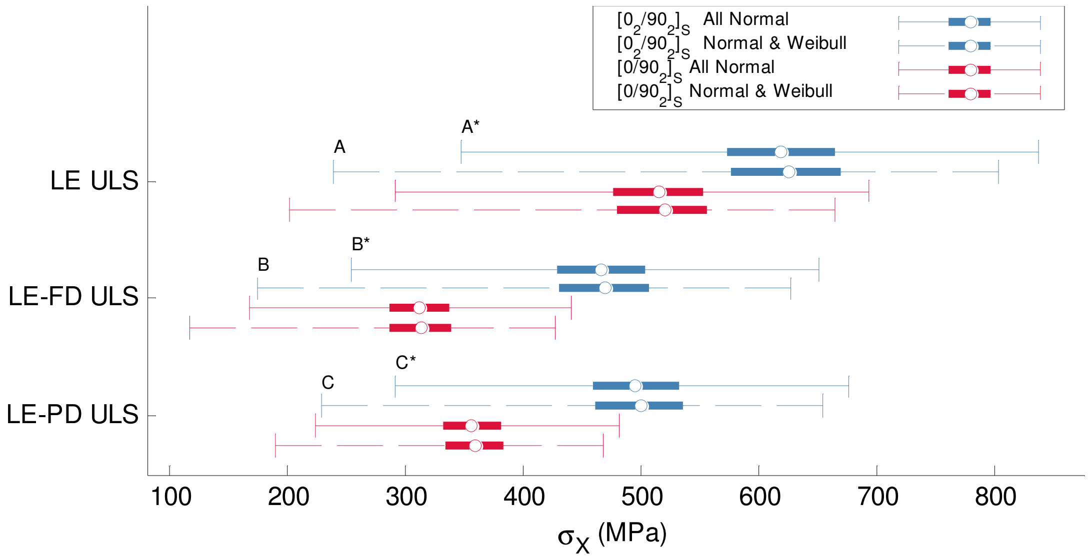

Figure 4. ULS probabilistic predictions for the: $[0_2/90_2]_S$ & $[0/90_2]_S$ glass/epoxy laminates.

	carbon/epoxy		glass/epoxy	
	$[0_2/90_2]_S$	$[90/0/90]_T$	$[0_2/90_2]_S$	$[0/90_2]_S$
A/C	1.06	1.04	1.05	1.06
A*/C*	1.08	1.08	1.19	1.28
B/C	0.94	0.85	0.77	0.62
B*/C*	0.97	0.88	0.87	0.77
(A – B)/C	0.12	0.20	0.28	0.44
(A*- B*)/C*	0.11	0.20	0.32	0.51

Table 2. Summary of the lowest stress quantiles presented in Figures 3 and 4. The symbol '*' indicates 'All Normal' input distributions.

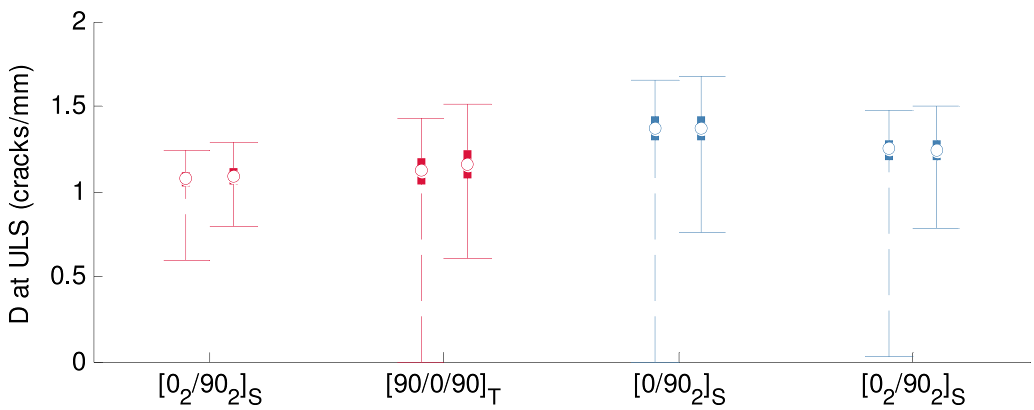

Figure 5. Probabilistic predictions of the laminates' crack densities at their ULS; see Figure 4 for the definition of the legends.

6 Discussion and Conclusions

From Figures 3 and 4, and Table 2, it is clear that the loss of stiffness due to matrix cracking has a significant influence on the estimation of the operational limits for glass/epoxy laminates, while for carbon/epoxy laminates the influence is small. An example of this is that the normalized difference between the LE and LE-FD quantiles (Table 2: (A-B)/C and (A*-

B*)/C*) is considerably larger for glass/epoxy than for carbon/epoxy laminates. This is due to the ratio between the plies' longitudinal and transverse stiffness, glass/epoxy plies have a ratio one order of magnitude smaller than the one of the carbon/epoxy laminate, most of the load is carried by the 0º plies in carbon/epoxy laminates while in glass/epoxy laminate the 90º plies carry a larger fraction of the load. From these results it seems reasonable to consider the LE-FD model appropriate for probabilistic analyses of carbon/epoxy laminates. The estimation of the operational limits would be on the conservative side without significantly under predicting the capabilities of the material. For glass/epoxy laminates, operational limits estimated with the LE-FD model will most likely be too conservative and lead to an inefficient use of the material.

The most critical factor in the estimation of the operational limits is the choice of distributions for the input variables, not the loss of laminate stiffness due to matrix cracking. Sensitivity analyses, which were not included in this paper, indicate that the shape of the tail of the distribution used for ε_{u1T} is of most importance. The true shape of the distribution for ε_{u1T} is a very uncertain issue. The characterization of the distribution's tail shape through experimental testing is impractical due to the cost and time required to test a number of specimens large enough. The tensile strength of a brittle fibre is considered to follow the weakest link theory, and therefore it is modelled with a Weibull distribution. Further on, the tensile strength of a bundle of equally loaded parallel fibres is considered to follow a Normal distribution [20]. The bundle tensile strength is equal to the mean strength of the fibres, which according to the centre limit theorem should have a Normal distribution. In reality the fibres in the ply are not equally loaded and when one of the fibres breaks its load is not equally distributed among the other fibres. Several models with damage accumulation have been proposed to predict the tensile strength of FRPs [20,21]; unfortunately, they require difficult to measure micro-mechanical properties and significant simplifications of the failure process. The conservative choice is therefore to model longitudinal ply strength with the Weibull distribution.

In this investigation the only considered effects of matrix cracking are the degradation of the 90° plies' transverse stiffness and thermal expansion coefficients. In reality, however, matrix cracks might act as starting points for other types of damage, such as delamination. The probabilistic predictions shown in Figure 5 indicate that all the laminates will most definitely present a crack density of at least 0.5 cracks/mm by the time they reach the defined ULS. Such a high crack density could therefore cause laminate failure before the predicted ULS. It is therefore important to investigate what would a "maximum allowable crack density" be for FRP laminates.

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