AN INTEGRATED APPROACH FOR THE PROGRESSIVE DAMAGE ANALYSIS OF V-NOTCHED GLASS/EPOXY COMPOSITE LAMINATES

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Abstract

This work proposes an integrated approach for the progressive damage analysis of V-notched glass/epoxy composites with layups $[90/0]_s$ and $[45/90/-45/0]_s$. The material property degradation method (MPDM) is used to model the intra-lamina failure whereas cohesive elements (CE) are employed to account for the delamination at the interfaces. Different failure theories are considered in the MPDM-CE approach and a comparative study of these failure theories is presented. The predictions are compared with the experimental results reported by Hallett and Wisnom [1]. Good agreement between experiment and simulation is obtained, showing that the MPDM-CE approach can effectively predict the progressive failure in V-notched glass/epoxy composite laminates.

1 Introduction

Recently, there has been a great increase in the use of advanced composites as primary structural materials, especially in the aircraft and wind turbine industries. More substantial parts of composites are being used for the building of wind turbines and new generation airliners such as Airbus 380 or Boeing 787. Most of composites in general contain notches as defects or as circular and semi-circular cutouts for easy access or fastening applications. Unfortunately, the presence of notches in composites significantly influences the performance of composite structures, especially for sharp notches. A study of notch effects on composite structures is therefore important and needs to be investigated. Some researchers have analyzed the failure of particular notched structures and proposed methodologies to predict the failure of these structures [2-5]. Nevertheless, the failure of notched composites has not been fully understood in general due to the complex failure mechanisms involving the matrix cracking, fiber failure, fiber kinking, fiber/matrix debonding, delamination, etc.

In order to account for the complex failure mechanisms in notched composites, a progressive failure analysis is performed to enable the prediction of crack initiation and propagation in composite structures. A progressive failure analysis comprises a damage initiation predicted by a failure theory and a material damage model to simulate a loss in the load-carrying capability of the part and advances the progression of damage. The results of failure analysis are dependent on the choice of the failure criterion and associated damage modeling technique. It is therefore important to employ reliable failure theories and damage modeling techniques for the progressive failure analysis to mirror the complex mechanisms in notched composites. In this article, an integrated MPDM-CE approach for the progressive damage modeling in composites is presented.

2 Description of the MPDM-CE approach for progressive failure analysis

The MPDM describes a material stiffness degradation scheme that the stiffness matrix of an element is degraded to reflect the damage of the element. For the failure analysis of composites, the state of damage usually depends on either partial failure (matrix failure) or complete failure (fiber failure) predicted. If fiber-dominated failure is predicted, the MPDM will fully degrade all the elastic moduli E_{11}, E_{22}, E_{33} and shear moduli G_{12}, G_{13}, G_{23} of composite materials to very small values (10^{-6} of the original values). When matrix-dominated failure is predicted, the MPDM will keep the longitudinal modulus E_1 (fiber direction) but degrade all the other properties of the material, meaning that the damaged matrix can only sustain the longitudinal load and cannot transfer any load in transverse and shear directions.

When MPDM is used in the failure analysis, failure initiation in an element is usually determined by a failure criterion and progressive failure will be carried out by the MPDM scheme. In this article, the MPDM is integrated with three fracture mechanics criteria including Tsai-Wu criterion [6], Christensen criterion [7], micromechanics of failure (MMF) criterion [8] and a continuum damage mechanics (CDM) criterion [9]. In addition, modified versions of Christensen criterion and CDM are also introduced to better account for fiber failure progression. The fracture mechanics criteria are summarized in Table 1. For these criteria, composite material or ply strengths (longitudinal tensile strength X_T and compressive strength X_C, transverse tensile strength Y_T and compressive strength Y_C, and in-plane shear strength *S12* and out-of-plane shear strength S23) are usually used to predict the complex failure of composites under general loading conditions. On the other hand, for continuum damage mechanics CDM, strain energy E_D and damage variables are used to describe the damage in composites. The damage variables associated with the strain energy density release rate are derived in Equation 1 and damage evolution laws is then determined by d_1, d_2, d_F in Equations 2 to 4. Details of the CDM criterion such as derivation of its parameters can be found in the paper of Ladeveze [9].

$$\begin{cases} Y_F = \left.\dfrac{\partial E_D}{\partial d_F}\right|_{\sigma,d_F} = \dfrac{1}{2(1-d_F)^2}\left[\dfrac{\sigma_1^2}{E_1^0} - \sum_{\substack{i,j=1\\ i<j}}^{3}\left(\dfrac{\nu_{ij}^0}{E_i^0} + \dfrac{\nu_{ji}^0}{E_j^0}\right)\sigma_i\sigma_j\right], \\ Y_{d1} = \left.\dfrac{\partial E_D}{\partial d_1}\right|_{\sigma,d_1} = \dfrac{1}{2(1-d_1)^2}\left(\dfrac{\sigma_2^2}{E_2^0} + \dfrac{\sigma_3^2}{E_3^0}\right), \\ Y_{d2} = \left.\dfrac{\partial E_D}{\partial d_2}\right|_{\sigma,d_2} = \dfrac{1}{2(1-d_2)^2}\left(\dfrac{\sigma_{12}^2}{G_{12}^0} + \dfrac{\sigma_{23}^2}{G_{23}^0} + \dfrac{\sigma_{13}^2}{G_{13}^0}\right) \end{cases} \tag{1}$$

$$d_i' = \left\langle \sqrt{Y} - Y_{0i} \right\rangle_+ \Big/ Y_{ci}, \quad (i = 1, 2) \tag{2}$$

$$d_i = \begin{cases} d_i', & \text{if } d_i' < 1 \text{ and } Y_{di} < Y_s \\ 1, & \text{if } d_i' \geq 1 \text{ or } Y_{di} \geq Y_s \end{cases}, \quad (i = 1, 2) \tag{3}$$

$$d_F = \begin{cases} 0, & \text{if } Y_F < Y_{f1} \text{ when compression or } Y_F < Y_{f2} \text{ when tension} \\ 1, & \text{otherwise} \end{cases} \tag{4}$$

Criterion	Formulations	Paramters
Tsai-Wu	$F_i\sigma_i + F_{ij}\sigma_i\sigma_j \leq 1$	$F_i, F_{ij} (i, j = 1, 2, \ldots, 5, 6)$
Christensen	Matrix failure: $\left(\frac{1}{Y_T} - \frac{1}{Y_C}\right)(\sigma_2 + \sigma_3) + \frac{1}{Y_T Y_C}(\sigma_2 + \sigma_3)^2 + \frac{1}{S_{12}^2}(\sigma_{12}^2 + \sigma_{13}^2) + \frac{1}{S_{23}^2}(\sigma_{23}^2 - \sigma_2\sigma_3) \leq 1$ Fibre failure: $-X_C \leq \sigma_1 \leq X_T$	Ply strengths: $X_T, X_C, Y_T, Y_C, S_{12}, S_{23}$
MMF	Matrix failure: $\sigma_{VM}^2 + (C_m - T_m)I_1 - C_m T_m \leq 0$ Fibre failure: $\sigma_{1f}^2 + (C_f - T_f)\sigma_{1f} - C_f T_f \leq 0$	Constituent strengths: T_m, C_m, T_f, C_f

Table 1. Three fracture mechanics criteria for composite materials.

Overall, all of the above four criteria have different modeling strategies for matrix-dominated and fiber-dominated failures. When Tsai-Wu, Christensen or MMF criterion are used with MPDM to model the progressive failure, a sudden stiffness degradation scheme is usually adopted for failed elements due to matrix or fiber failure. The CDM criterion, in contrast, can describe a gradual stiffness degradation of the failed elements due to matrix failure but a sudden stiffness degradation due to fiber failure. It can be seen that no gradual stiffness degradation scheme due to fiber failure is considered in the traditional failure theories such as Tsai-Wu, Christensen, MMF and CDM, where the stiffnesses of failed elements due to fiber failure are suddenly degraded to zero (Figure 1a). In fact, whenever a fracture develops, a certain amount of energy or fracture energy is dissipated to form new surfaces. That means the stiffnesses of the failed elements are gradually degraded so that the total energy dissipation will be equal to the fracture energy. This fracture energy can be described by the area under the softening curve as shown in Figure 1. It can be seen that the conventional failure theories do not take into consideration of the fracture process after the fiber failure. Therefore, modified versions of the traditional failure theories are introduced by modeling the fracture process after the initiation of fiber failure until the ultimate failure.

Considering failure at element level, once fiber damage is predicted, the modified failure criterion will gradually degrade the stiffnesses of the failed elements by a gradual degradation factor d_f so that the total energy dissipation is equal to the fracture energy. The fracture energy is related to the fracture toughness of the material and can be obtained from the experiment. If ε_0 is defined as the damage strain when fiber damage initiates and ε_f as critical strain when fiber is completely failed, with assuming a linear softening stress-strain response after the initiation of fiber damage (Figure 1b), the damage variable d_f accounting for the linear degradation scheme of fiber can be expressed by:

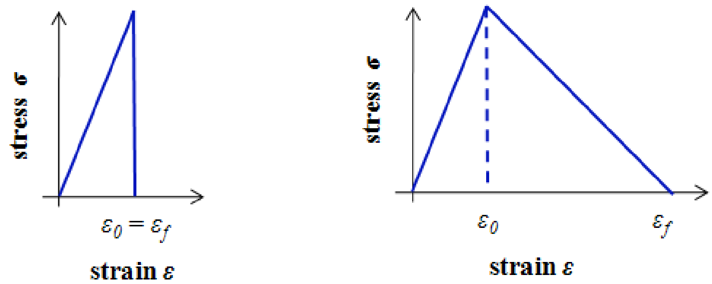
$$d_f = 1 - \frac{\varepsilon_0}{\varepsilon_1} \cdot \frac{\varepsilon_f - \varepsilon_1}{\varepsilon_f - \varepsilon_0} \tag{5}$$

and

$$\varepsilon_0 = \frac{X_T}{E_1}, \ \varepsilon_f = \frac{2G_{\mathrm{Ic}}}{X_T l_e} \tag{6}$$

where G_{IC} and l_e fracture energy and characteristic length of the failed element, respectively.

For simplicity but without loss of generality, the CDM and Christensen failure criteria with the above gradual fibre damage scheme, denoted as MCDM and MChristensen, respectively, are considered in this article.



a. b.

Figure 1. Fiber modelling strategy for different failure theories: (a) traditional failure theories and (b) modified failure theories

For prediction of the delamination initiation by cohesive elements, a stress-based quadratic criterion proposed by Hou et al [10] is selected. According to this delamination criterion, the initiation of interface damage is controlled by the normal interface stress t_n and two shear interface stresses t_s and t_t, which can be written as:

$$\left(\frac{t_n}{S_n}\right)^2 + \left(\frac{t_s}{S_s}\right)^2 + \left(\frac{t_t}{S_t}\right)^2 = 1 \tag{7}$$

where S_n, S_s and S_t are the cohesive strengths. Their values are estimated from a study by Brewer and Lagace [11] where N = 39 MPa and $S = T$ = 89 MPa.

For modeling the delamination propagation, a fracture mechanics-based criterion is used:

$$\frac{G_n}{G_{\mathrm{Ic}}} + \frac{G_s}{G_{\mathrm{IIc}}} + \frac{G_t}{G_{\mathrm{IIIc}}} = 1 \tag{8}$$

where G_n, G_s and G_t are the work done by the tractions and their relative displacements in the normal and shear directions, respectively; G_{IC}, G_{IIC} and G_{IIIC} are the critical strain energy release rates (SERRs) corresponding to mode I, mode II and mode III fractures, respectively.

The SERR values G_{IC} = 0.25 N/mm and G_{IIC} = G_{IIIC} = 1.08 N/mm are referred to Petrossian and Wisnom [12] for glass/epoxy composite material

The double-notched glass/epoxy models are built having the same geometry and dimensions as the specimens of Hallett and Wisnom's experiment. The FE models of notched composite laminates are constructed with 8-node three-dimensional (3D) continuum shell elements (SC8R) and eight-node hexahedral (COH3D8) cohesive elements at the interface between plies. Comparative study of different failure theories by the MPDM-CE approach is carried out by the implicit solver of Abaqus.

3 Results and conclusions

The experimental results by Hallett and Wisnom [1] show complex failure mechanisms in V-notched $[90/0]_s$ glass/epoxy. Simulation results of V-notched $[90/0]_s$ laminate show the initiation and propagation of longitudinal splitting in 0^0 plies and transverse matrix cracking in 90^0 plies, followed by fiber failure in 0^0 plies and delamination at the $[90/0]_s$ interface. Only the progressive failure patterns predicted by the MChristensen model are shown in Figures 2 to 4 since the progressive failure patterns by Tsai-Wu, Christensen, MMF and CDM models follow those of MChristensen closely. The failure loads predicted by all models are plotted in Figure 5. It can be seen that the failure loads predicted by Christensen, Tsai-Wu, MMF and CDM models are lower than the experiment because these models consider a complete loss of element's stiffness due to fiber failure. The MCDM and MChristensen models predict the experimental failure load better because they consider that the damaged fibers still have residue stiffness to carry additional load until final failure. It is also noted that a discontinuity in the predicted curves has been observed. This may be due to a substantial stiffness loss of 90^0 plies before the fiber failure in 0^0 plies occurs.

Simulation results for the quasi-isotropic glass/epoxy laminate also show good agreement with the experimental ones when transverse cracking in $\pm45^0$ and 90^0 plies, longitudinal splitting and fiber failure in $\pm45^0$ and 90^0 plies and delamination at the interfaces are all predicted. The predictions by MCDM and MChristensen models appear to agree better with the experiment than the other models. The progressive failure patterns predicted by the MChristensen are shown in Figures 6 and 7. The ultimate loads predicted by all model are plotted in Figure 8. It is observed that a discontinuity in the load *vs.* displacement curves by the Tsai-Wu, Christensen and MMF models is found while the predicted curves by the CDM, MCDM and MChristensen models are continuous. This is because the CDM, MCDM and MChristensen models degrade the stiffness of the failed elements gradually while the other models quickly and completely degrade the stiffness of failed elements. Therefore, when a large number of elements fail in Tsai-Wu, Christensen and MMF models, discontinuities in their predicted load-displacement curves may not be avoided.

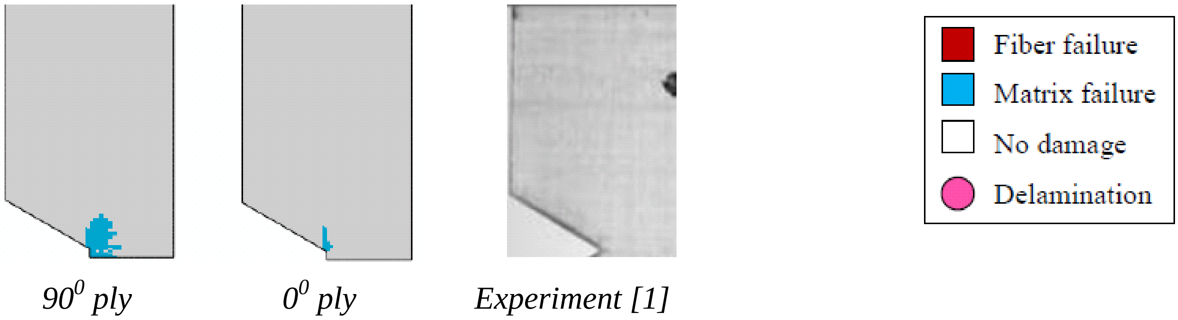


Figure 2. MChristensen: Predicted damage patterns in comparison with Hallett and Wisnom's experiment (25% maximum load).

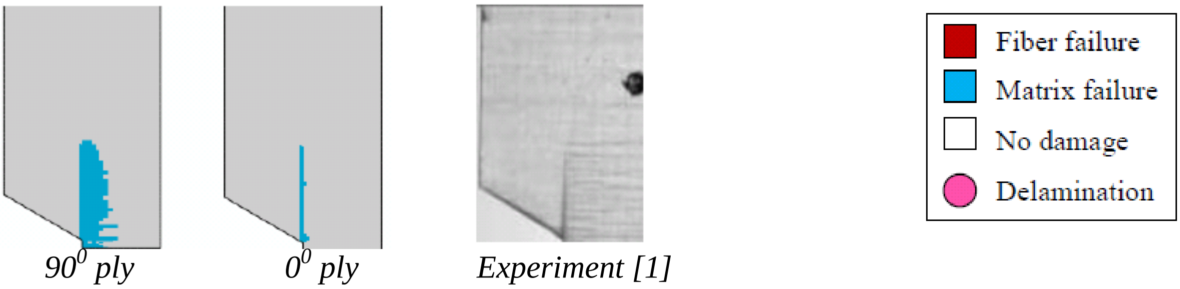


Figure 3. MChristensen: Predicted damage patterns in comparison with Hallett and Wisnom's experiment (65% maximum load).

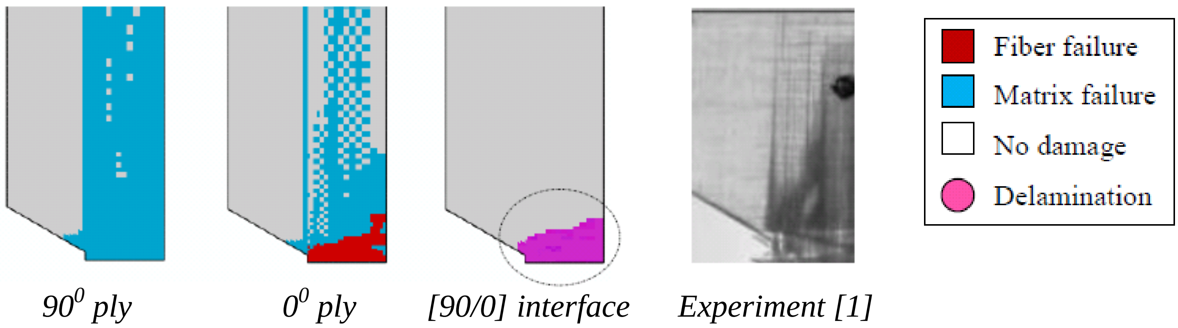


Figure 4. MChristensen: Predicted damage patterns in comparison with Hallett and Wisnom's experiment (final failure).

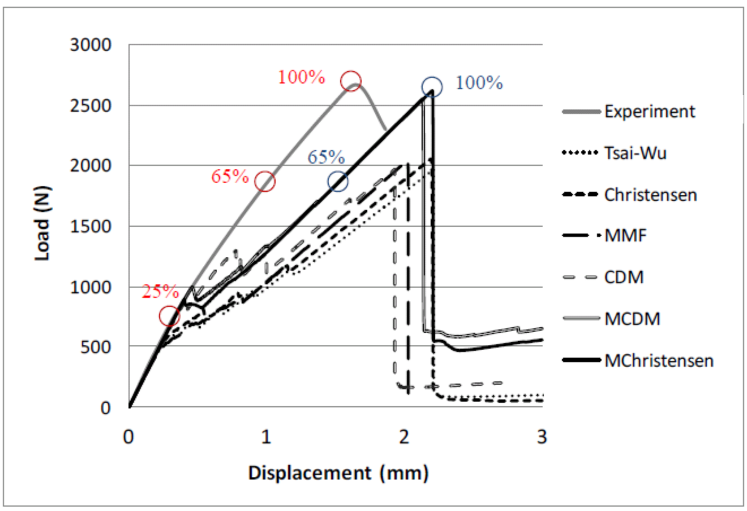


Figure 5. Predicted load-displacement curves and comparison with the experiment for the [90/0]s glass/epoxy laminate.

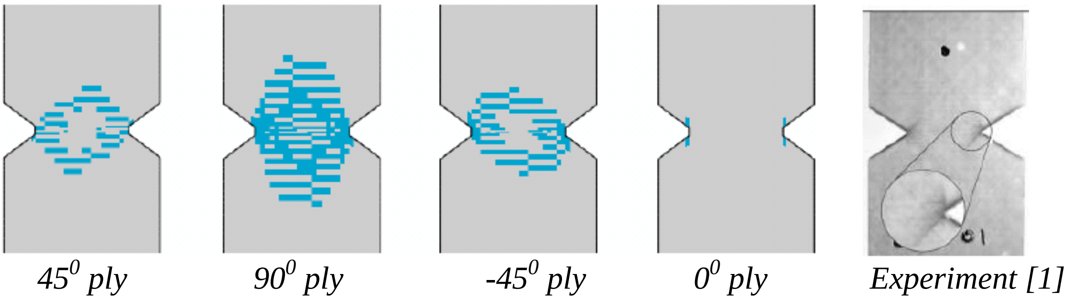


Figure 6. MChristensen: Predicted damage patterns in comparison with the experiment (6% maximum load).

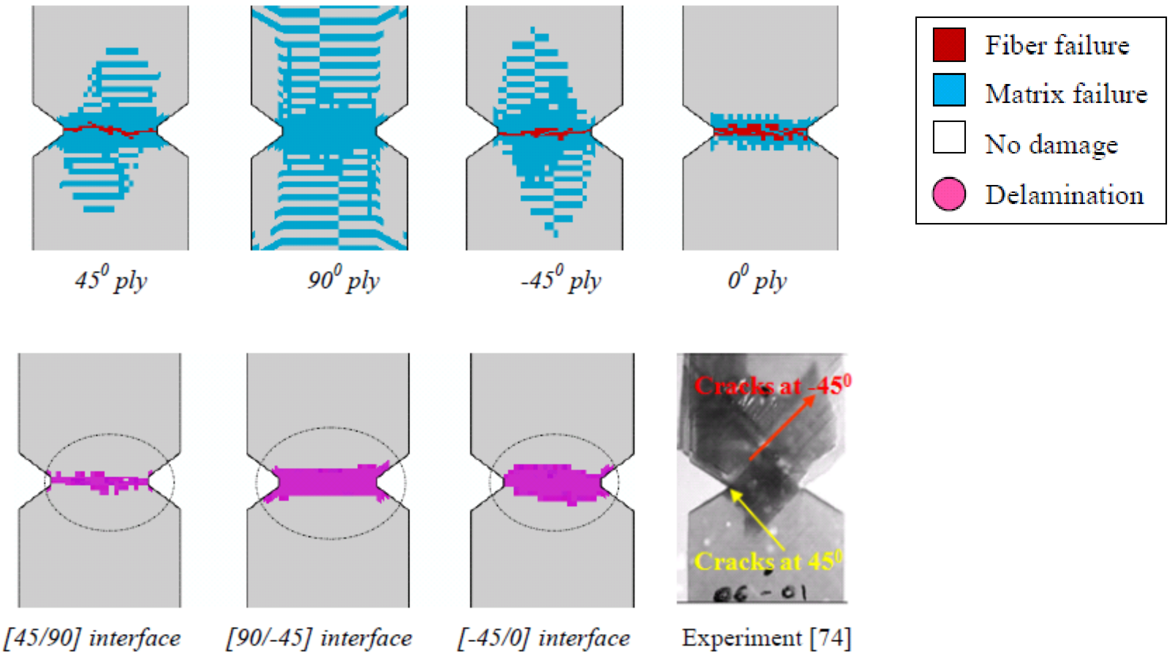


Figure 7. MChristensen: Predicted damage patterns in comparison with the experiment (final failure).

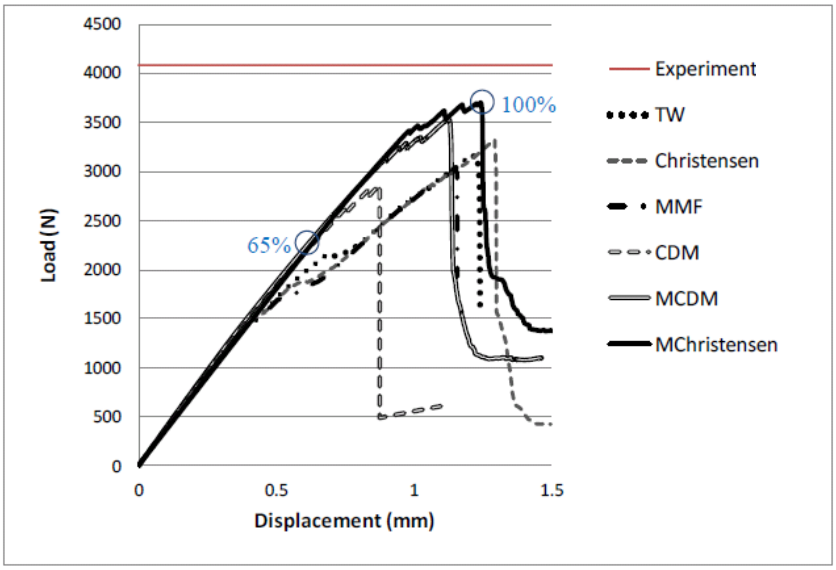


Figure 8. Predicted load-displacement curves and comparison with the experiment for the [45/90/-45/0]s glass/epoxy laminate.

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