MODELING OF THE EVOLUTION OF THE FAILURE PROBABILITY OF A SELF-HEALING CERAMIC MATRIX COMPOSITE

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Abstract

The present paper is a first step toward the prediction of the failure probability evolution of self-healing ceramic matrix composites. Two main sources of variability are identified and emphasized: fiber failure and crack density. The associated statistical data from the literature are incorporated in a multiphysic macroscopic model with microscopic enrichments. These micro enrichments allow an easy use of data at different scales. The identification of the model is explained and a first validation presented. A first simulation of the failure probability evolution map for different mechanical loading levels is also presented.

1 Introduction

Ceramic matrix composites (CMCs) are good candidates for the manufacturing of new aeronautical engine structures as they present very good specific properties at high temperatures [1]. In order to increase the lifetime of such composites, a self-healing matrix has been developed to limit the effect of fibre oxidation. Thus, a complex interaction between degradation and healing mechanisms occurs at the heart of the material. This interaction has to be accounted to have a robust prediction of the lifetime of the composite part.

A deterministic multiphysic model has been developed to predict the mechanical behaviour and the lifetime of these materials [2]. It relies on the association of a mechanical macro model describing the crack network evolution, a physico-chemical model describing the diffusion/reaction process of the healing and a model describing the subcritical cracking of SiC fibres. This model has been identified and validated using a wide range of experimental data [3].

The objective of this paper is to add a probabilistic dimension to the existing deterministic model. Indeed, this one is not able to represent the scattering that has been observed experimentally on CMCs on composite coupons [4]. The basis of the proposed modeling relies on the structure of the existing deterministic model that is given at the macro scale and includes micro enrichments. This model allows determining the evolution of the strength with time for a given loading history (stresses, temperature, oxygen and water vapor partial pressures). It is composed of three submodelings:

1. the modeling of the crack network (anisotropic damage theory, crack opening indicator) ;
2. the modeling of the subcritical cracking of fibers ;
3. the modeling of the diffusion and reaction mechanisms for the self- healing process.

From a sensitivity analysis (in a physically admissible range of values), only the two first submodels lead to a great variation of the lifetime. The third submodel is of great importance to predict the lifetime but does not seem to be for its scattering. Thus, only the two first submodels will be enhanced using the available probabilistic values from the literature i.e. crack spacing probabilities on mini-composites and subcritical cracking of fiber bundles.

The deterministic model giving the evolution of the strength with time, the probabilistic model developed here will follow and give the evolution of the strength probability with time for a given loading history.

For validation, a map presenting the evolution of the failure probability versus applied stress and applied temperature are compared to experimental data on composite coupons.

2 Sources of variability

Two major problems need to be answered in order to predict the lifetime of the self-healing CMCs. The first deals with the prediction of the crack networks and the crack openings that allow the oxygen to reach the fibres. For that an anisotropic damage theory is used, in which damage variables are attached to the different crack networks. A model crack can then be described on the basis of a crack opening indicator build using a correspondence between the damage evolution law and a shear-lag model of a micro-composite. The second deals with the healing process description. For that, a wire frame modeling of the intra-crack diffusion/reaction processes is introduced. This allows to compute the quantity of oxygen that reaches the fibres and thus to pilot the degradation of these fibres in oxidizing atmosphere. The evolution of the strength of the fibers is deduced from that pilot variable: a cumulative oxygen concentration.

Three main parts should are distinguished in that model: the modeling of the crack network, the modeling of the diffusion and reaction mechanisms for the self-healing process, the modeling of the subcritical cracking of fibers.

2.1 Crack network

Concerning the lifetime prediction, all the mechanical information is described by a crack opening indicator. The crack opening indicator has been identified as a very sensitive parameter regarding to the lifetime prediction. It allows the modeling of an intra yarn transverse crack in which the oxygen diffuses to the fibers. The expression of the crack opening indicator h reads:

$$h = d\left(K_1\, \varepsilon_{ine} + K_2\, \Delta C\, \sigma\right) \tag{1}$$

where K_1 and K_2 are coefficients identified using a micro model [5] and a lifetime test on a composite coupon [3]. ε_{ine} is the inelastic strain in the longitudinal direction and ΔC the compliance in the longitudinal direction. d is the intra-yarn transverse cracks density. Some statistical data are available on these parameters in [6] for mini-composites (gage length 75 mm). The information of the variability of ΔC is assumed to be taken into account by using directly the variability of d in the expression of h. d is modeled using a Weibull law.

For low crack densities (i.e. low mechanical loading level), one should take care of the correlation between crack location, matrix densification, local loading in the fabric. It is not considered here.

2.2 Self-healing

The studied micrographs on yarns [7,8] are very regular but do not represent the zones of crossing of warp and weft yarns where the densification may be less regular. The physico-chemistry is quite well identified from morphological analyses and the lifetime prediction is not very sensitive to the associated parameters. The self-healing part of the model is considered as deterministic.

2.3 Subcritical cracking of fibers

The subcritical cracking of fibers and bundles has been studied extensively [9] (gage length 20 mm). Probability density functions can be identified on experimental data. The choice of the author, is to use data available on bundles in order to avoid scale transition between fibers and yarns scales in the composite.

2.4 Failure scenario of a composite coupon

From the data of [4], a failure scenario of a composite coupon has been defined. A few number of fibers suffering of subcritical crack growth (about 4 on 500), located anywhere inside a yarn, leads to the failure of a yarn. Then, 7 bundle lead to the failure of the coupon (150 longitudinal bundles in a cross section). With these figures, the scattering reduction due to the scale transition should not be predominant. The study performed by [10,11] on the failure for quasi-static loadings should be extended to static fatigue loadings for the lifetime prediction. Results may be different as some physical mechanisms are different.

3 Modeling and identification

The failure criterion is based on a cumulative oxygen concentration indicator close to the fibers Θ. For given loading conditions (thermal, mechanical, environmental), the failure probability reads:

$$P(t) = P(\theta(t) > \Theta_c) \tag{2}$$

$\Theta(t)$ is a random variable representing the oxygen cumulative concentration seen by the fiber. It is an implicit function of the time and of the crack opening indicator through self-healing evolution laws. Θ_c is a random variable representing the critical cumulative concentration that leads to the failure of the bundles. It depends mainly of the bundle characteristics. The failure of a yarn is supposed to lead to the failure of the composite coupon.

The two random variables Θ and Θ_c are independent as they correspond to different physical mechanisms thus the failure probability reads:

$$P(\theta(t) > \Theta_c) = \int_0^\infty p_1(\Theta) \int_0^\theta p_2(\Theta_c)\, d\Theta_c\, d\Theta = \int_0^\infty p_1(\Theta)\, F_{\Theta_c}(\Theta)\, d\Theta \tag{3}$$

$p_1(\Theta)$ is the probability density function to have an oxygen cumulative concentration of Θ. It depends on the loading conditions and of the crack opening variability. $p_2(\Theta_c)$ is the probability density function to have a critical value of Θ_c. It depends on the failure probability of the fiber bundles. Knowing the probability density functions of each random variable, the model allows predicting the evolution of the failure probability of the composite with the time for a given loading. The problem is then to identify the two probability density functions.

The identification is conducted from data on bundles from [9] at 1500 Mpa (240 MPa equivalent loading on the composite). Note that the ratio of standard deviation over mean varies of 24% while the loading varies from 1500MPa to 1000MPa (1.14 for 1500MPa and 1.5 for 1000MPa). For the sake of simplicity, it is supposed constant in this paper. A Weibull law is considered in order to take into account scale effects and thus $F_{\Theta_c}(\Theta)$ has an analytical expression.

For a constant loading (it is the case of static fatigue tests on bundles), the identification result can be seen on Figure 1.

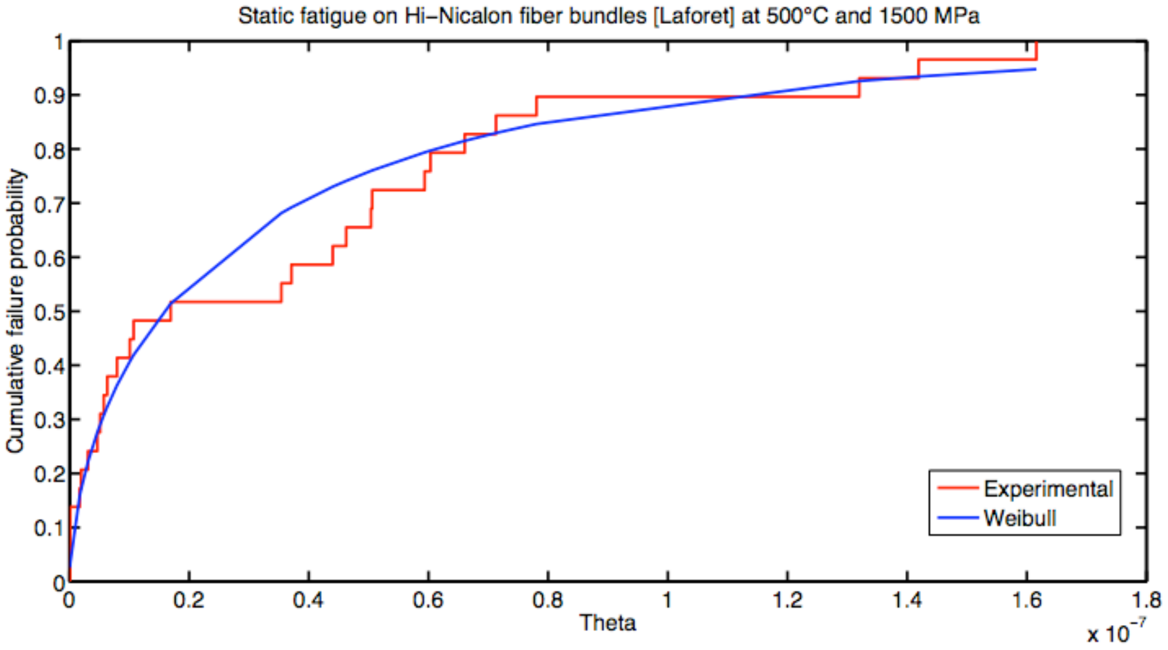

Figure 1: $F_{\Theta_c}(\Theta)$ for constant loadings on Hi-Nicalon fiber bundles [12]

From [6], the crack spacing as been modeled using a Weibull law including a size effect (experimental gage length of 75 mm, model gage length of 1 mm). The mean value is corrected in order to have a good agreement to the data from [4]. Note that it corresponds to the procedure used for the deterministic model where the crack opening was adjusted using one lifetime value on a composite coupon.

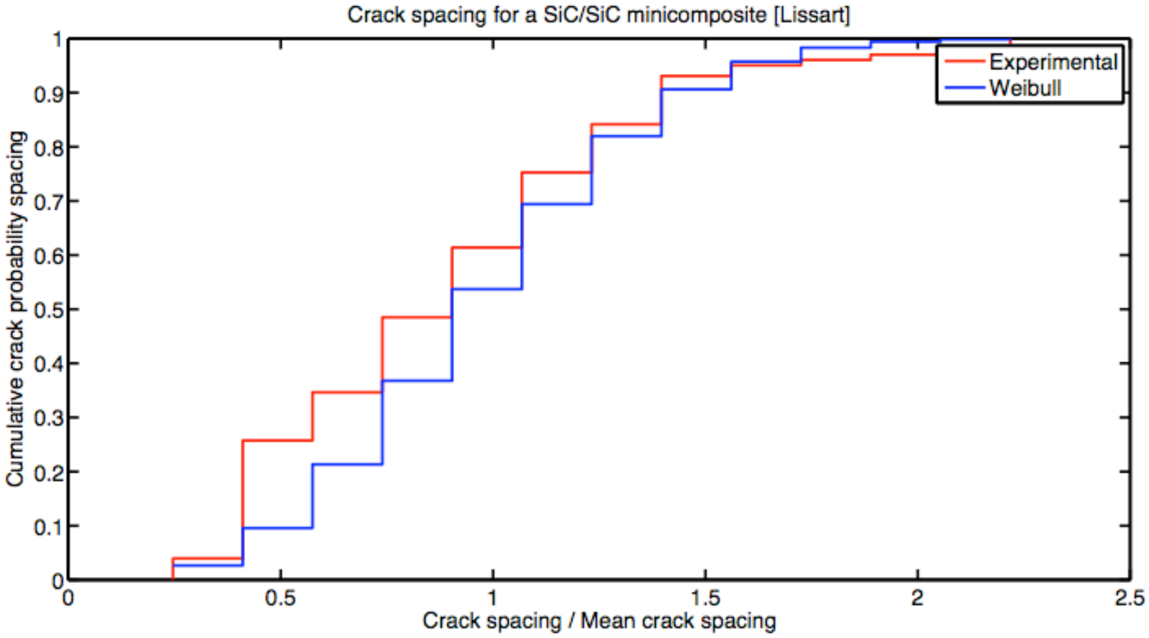

Figure 2: crack spacing over mean crack spacing ratio cumulative probability [6]

4 First validation and application

First, the data from [4] on composite coupons at 500°C and 200 MPa have been used. The failure probability evolution with time is shown on Figure 3. These first results are quite encouraging; nevertheless, the validation of that kind of response is not straightforward. In fact, the experimental data are not sufficient to estimate a statistical distribution. Concerning the simulation, the local stress distribution could be greatly improved using the work of [13]. Note also that for extreme probabilistic values, the failure may be due to a strong correlation between large crack spacing, defect location and weaving pattern; that is not taken into account in the present model.

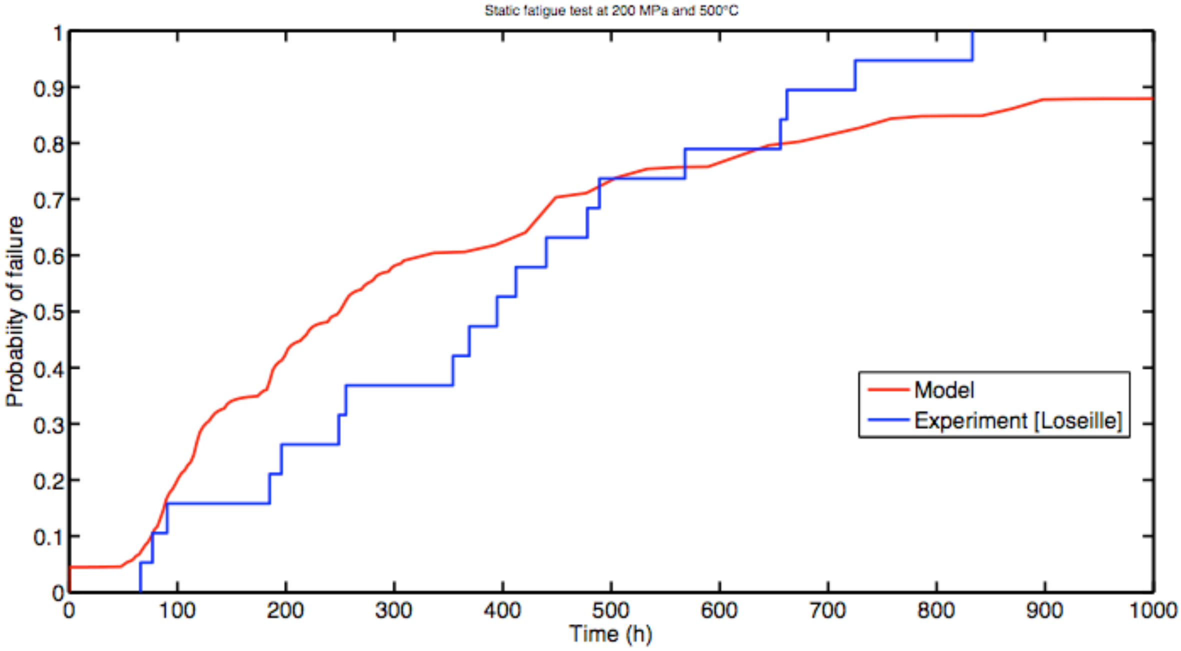

Figure 3: Failure probability versus time: comparison between numerical and experimental data from [4].

Second, failure probability maps have been computed (Figure 4). For low loading levels, initial cracks have been introduced using an initial state of damage in the computation of the crack density and opening. The damage level is low enough not to influence simulation for loading levels higher than the elastic limit. The initial damage level has almost no influence on the predicted lifetime as long lifetimes are guided by the fiber bundles in that case. It is important to note that two regimes are present depending of active or inactive healing. This leads to different failure probability laws for low stress or high stress levels.

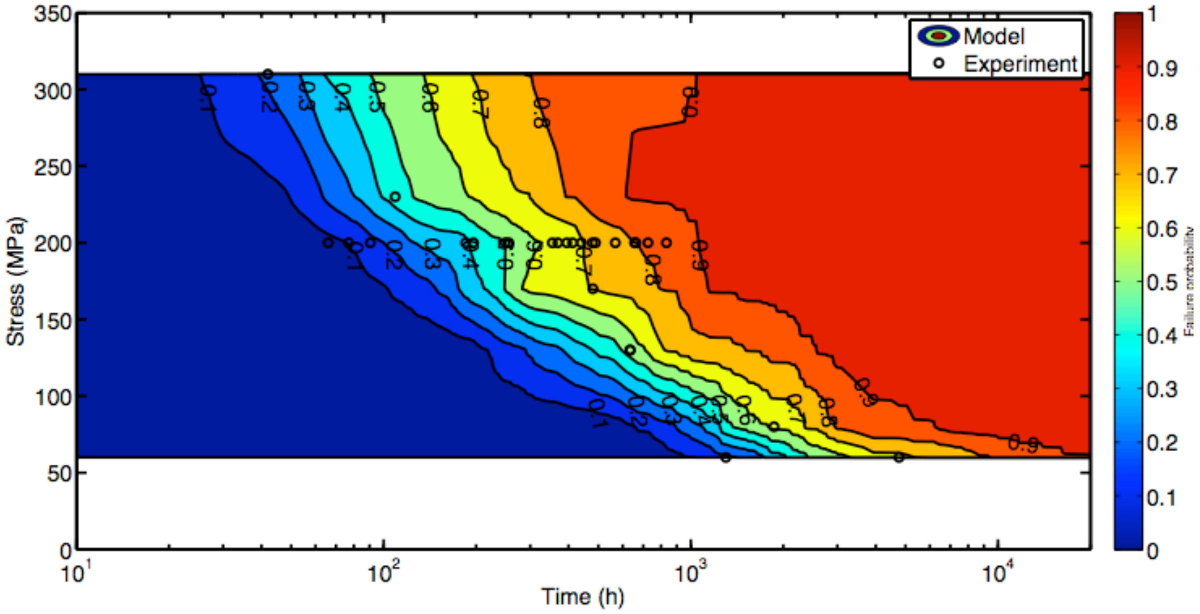

Figure 4: Evolution of the failure probability with time and applied stress.

5 Conclusion and outlook

This paper presents a first modeling attempt of the failure probability of ceramic matrix composites with self-healing matrices. It is based on a previously developed deterministic macro-model. The uncertainties have been described on the two major parameters:

- the bundles failure stress;
- the crack opening indicator.

Available experimental data on fiber bundles or mini-composites have been used to feed the model. First simulations have been performed and compared to available data on coupons. Two major problems arise. The first one concerns the validation of the model. For that, a large amount of experimental data is needed and not available at the present time. Different gaps have already been identified on the presented modeling, a richer description of the local stress distributions could partially solve that problem. For example it could help to define accurate gage factors.

The second one concerns the use of such probabilistic laws (or data) in order to size a part. For aeronautical parts, generally the flow of forces is redundant and in the present case the material is highly damage tolerant. Thus, using a minimal failure stress corresponding to x% failure probability is very conservative (x is generally very small!). A stochastic structural computation, following the [14] for deterministic predictions, should be a good way to handle this second problem of scale effect but is not straightforward.

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