PREDICTION OF CRACK PROPAGATION IN SANDWICH BEAMS UNDER FLEXURAL LOADING

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Abstract
A sandwich beam under three point bending containing a crack in the core material very close to the upper skin interface and parallel to the longitudinal beam axis is investigated. In this paper a numerical study of the fractured sandwich beam is presented with the objective to characterize the fracture very close to upper skin interface under mixed-mode loading conditions. A cohesive damage model was employed to simulate crack propagation and kinking into the core taking into consideration that the crack propagation on the compression side of the core is mainly subjected in shear. The crack considered, is analyzed with static non-linear two- dimensional finite element analyses.

1 Introduction

Sandwich structures are more and more implemented in modern industrial applications such as ship hulls and wind turbine blades where low weight and high rigidity is very important. A sandwich material consists of a core material with thin laminate on each side. The laminate could be any combination of fibres and matrices, or even of a metallic material. The core is usually made of PVC, wood or a honeycomb material. The most interesting problems appeared in sandwich structures are, the failures modes in the core and the faces materials, the fracture of the core material and the interaction between the fractured core and the face materials. In sandwich structures the foam is typically the weakest part and is the first to fail under static or cyclic loading because it transfers the applied loads as shear stresses. In addition a very critical problem in sandwich structures is the debonding problem between the face and core materials [1-5]. Unstable cracking propagation and kinking into the core material represents one of the weakest failure modes in sandwich composites. The fracture behavior in sandwich composites has been directed toward the understanding of crack propagation, and at the same time toward improving the durability of composites against fracture. A crack flaw may be introduced during processing or subsequent service conditions [1-3]. It may result from low velocity impact, from eccentricities in the structural load path, or from discontinuities in structures, which induce a significant out-of-plane stress.

Cohesive crack models are widely used to simulate crack growth and kinking phenomena. The basic concept of cohesive zone was introduced by Barenblatt [6] and Dugdale [7] but Xu and Needlemen [8] introduced the cohesive surface network. In order to develop numerical methodologies to simulate crack propagation in composite structures, cohesive damage models have attracted much interest [9-14] due to their well established advantages compared to the stress based and fracture mechanics methods. But the application of cohesive damage models in sandwich structures for the numerical simulation of the crack propagation is very limited [12-14].

The nature of this paper is to develop a cohesive damage model to simulate crack propagation in the core of sandwich structures very close to the upper skin interface under mixed mode loading conditions. It is the first time that a cohesive model will propose to simulate crack propagation and kinking in the core of sandwich structures. Results from the numerical analysis combined with the experimental data [2] predict the crack growth behavior under flexural loading. The simplicity of the proposed procedure and the numerical model developed, make possible the prediction of the crack propagation for various types of sandwich beams under flexural loading.

2 Cohesive damage model

A cohesive mixed mode damage model based according to [9, 12] on interface finite elements was proposed to simulate damage onset and growth. A linear constitutive relationship between stresses $\boldsymbol{\sigma}$ and relative displacements $\boldsymbol{\delta}$ between homologous points of the interface elements with zero thickness is considered. At first developing the pure-mode model, the constitutive equation before damage starts to grow, is given by:

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\delta} \tag{1}$$

where $\mathbf{D}$ is the diagonal matrix containing the penalty parameters (d) in each mode. The values of the penalty parameter must be quite high in order to hold together and prevent interpenetration of the element faces. According to a considerable number of numerical simulations [9, 12], it was determined $d = 10^6 \text{N/mm}^3$. For this value the produced results converge and numerical problems during the nonlinear procedure have been avoided.

After the peak stress $(\sigma_{u,i})$ is reached a gradual softening process between the stress and the relative displacement is observed and is defined as:

$$\boldsymbol{\sigma} = (\mathbf{I} - \mathbf{E})\mathbf{D}\boldsymbol{\delta} \tag{2}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{E}$ is a diagonal matrix containing the damage parameter:

$$e = \frac{\delta_{u,i}(\delta_i - \delta_{o,i})}{\delta_i(\delta_{u,i} - \delta_{o,i})}, \quad i = \mathrm{I}, \mathrm{II} \tag{3}$$

$\delta_{o,i}$, is the displacement corresponding to the onset of damage, and δ_i is the current relative displacement. In pure-mode loading, the strength along other directions is abruptly cancelled. The maximum relative displacement, $\delta_{u,i}$ for which complete failure occurs, is obtained by equating the area under the softening curve to the respective critical fracture energy:

$$G_{c,i} = \frac{1}{2}\sigma_{u,i}\delta_{u,i} \tag{4}$$

In thick composite structures under flexural loading such as in ship hulls, failure is more likely to occur under a mixed-mode situation. Therefore, a formulation for interface elements having zero thickness should include a mixed-mode damage model, which, in this case, is an extension of the pure mode model described previously. Damage initiation may be predicted by using the following criterion [9, 12]:

$$\begin{aligned} &\left(\frac{\sigma_{I}}{\sigma_{u,I}}\right)^{2}+\left(\frac{\sigma_{II}}{\sigma_{u,II}}\right)^{2}=1 && \text{if} \quad \sigma_{I}>0 \\ &\sigma_{II}=\sigma_{u,II} && \text{if} \quad \sigma_{I}\leq 0 \end{aligned} \tag{5}$$

where $\sigma_{u,I}$, $\sigma_{u,II}$ represent the ultimate normal and shear stresses, respectively and it is assumed that normal compressive stress does not induce damage. Providing an equivalent mixed-mode displacement:

$$\delta_{e}=\sqrt{\delta_{I}^{2}+\delta_{II}^{2}} \tag{6}$$

and a mixed-mode ratio:

$$\beta=\frac{\delta_{II}}{\delta_{I}} \tag{7}$$

and taking into considering Equation (1), we have from Equation (5):

$$\left(\frac{\delta_{om,I}}{\delta_{o,I}}\right)^{2}+\left(\frac{\delta_{om,II}}{\delta_{o,II}}\right)^{2}=1 \tag{8}$$

where $\delta_{om,i}(i=I,II)$ are the relative displacements at damage initiation, which correspond to the critical interface stresses $\sigma_{um,i}$ (Figure 1). Combining Equations (6)-(8), the value of the equivalent mixed-mode displacement leading to damage initiation (δ_{om}) results:

$$\delta_{om}=\frac{\delta_{o,I}\delta_{o,II}\sqrt{1+\beta^{2}}}{\sqrt{\delta_{o,II}^{2}+\beta^{2}\delta_{o,I}^{2}}} \tag{9}$$

The mixed-mode damage propagation is simulated considering the linear fracture energetic criterion:

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \tag{10}$$

The released energy in each model at complete failure can be obtained from the area of the triangle (Figure 1):

$$G_i = \frac{1}{2}\sigma_{um,i}\delta_{um,i} \tag{11}$$

being $\delta_{um,i}(i = I, II)$, the relative displacement in each direction for which complete failure occurs. From Equations (1), (4), (6), (7), (10) and (11), the mixed mode relative displacement leading to total failure (δ_{um}) can be obtained:

$$\delta_{um} = \frac{2(1+\beta^2)}{e\delta_{om}}\left[\frac{1}{G_{Ic}} + \frac{\beta^2}{G_{IIc}}\right] \tag{12}$$

The values of δ_e, δ_{om} and δ_{um} are introduced into Equation (3), instead of δ_i, $\delta_{o,i}$ and $\delta_{u,i}$, thus setting the damage parameter under mixed mode. The mixed mode model proposed is general and it can be applied under any combination of modes.

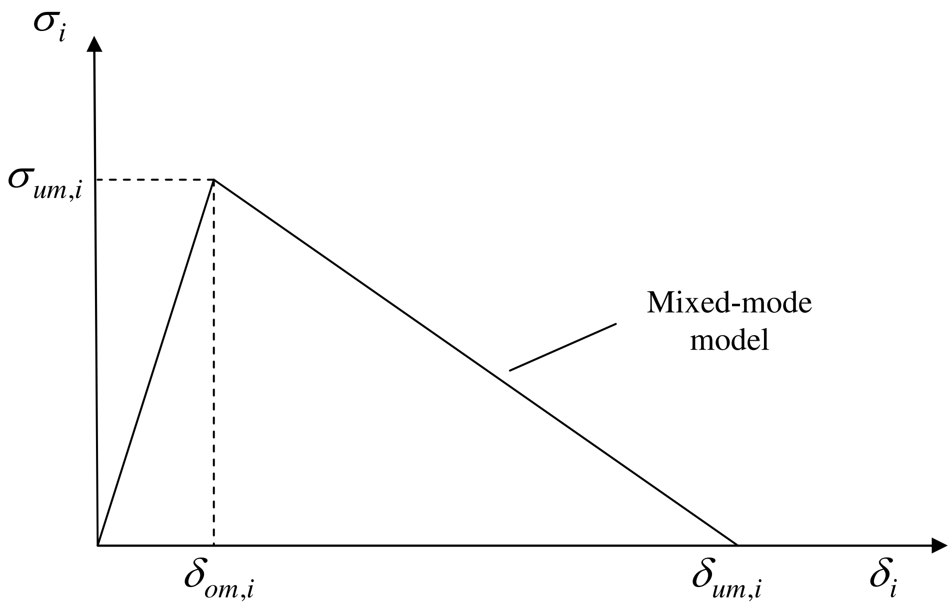

Figure 1. The linear softening law for mixed-mode cohesive damage model. $\sigma_{um,i}$, stress component leading to damage initiation in mixed mode, $\delta_{om,i}$ damage onset relative displacement in mixed mode; $\delta_{um,i}$, ultimate relative displacement in mixed mode.

2 Simulation of the crack propagation

In the sandwich beam and into the core material an initially very small crack of about 2mm is considered. The position and the orientation of this initially small flaw of the body has been chosen to be about 1.5mm below the upper boundary of the core material, the right crack tip lays at the midspan of the beam and the crack is oriented parallel to the longitudinal axis of the beam according to the experimental investigation [2]. The dimensions of the test specimen were $L = 228.6$ mm (support span) and $b = 63.5$ mm (width). The core thickness was t =12.7 mm and the face sheet thickness was $t_1 = 2.28$ mm. The overall thickness was 17.26 mm (Figure 2). The experimental data [2] show that the crack propagation occurs parallel to the beam axis until the crack length reaches an approximate value between 65mm and 75mm. The direction of the crack is then suddenly changed and the crack is kinking into the core material. The sandwich beam at the same time loses completely its stiffness and is being collapsed.

Materials	**Face sheets S-2 Glass 240F Epoxy SC 15**	**PVC Foam R75**
E_1 (MPa)	16300	80
E_2 (GPa)	16300	80
ν_{12}	0.3	0.4

Table 1. Mechanical properties of the materials used in the experimental investigation [2].

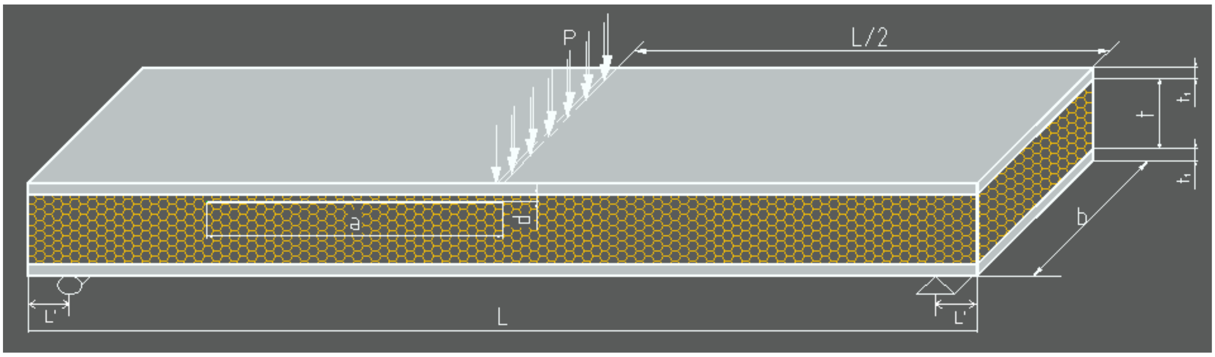

Figure 2. A cracked sandwich beam under flexural loading.

The model shown in Figure 2 is introduced in Abaqus [15] considering the data from Table 1. By this way we are able to adjust the model according to the demands of the specific analysis. For example we can create various models with different materials different crack lengths, different crack positions and orientations and so on. The element type used in the proposed analysis is the four-node two dimensional plane strain elements CPE4 [15]. Different mesh configurations were used in the vicinity of the crack tip and the cohesive layer in order the convergence of the solution to be succeeded. The cohesive layer is placed over the entire plane of crack propagation (Figure 3) by implementing the procedure given in Abaqus [15].

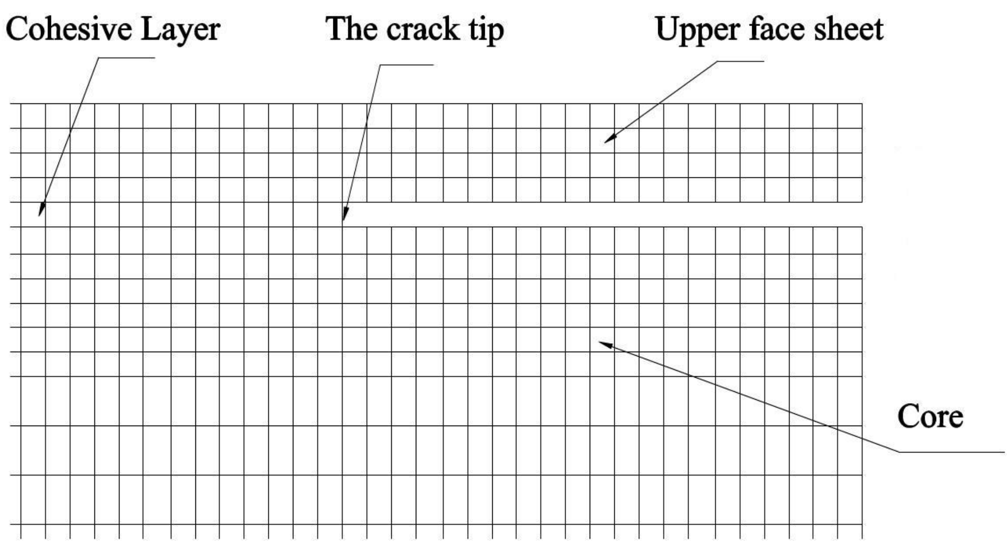

Figure 3. The finite elements mesh at the crack tip.

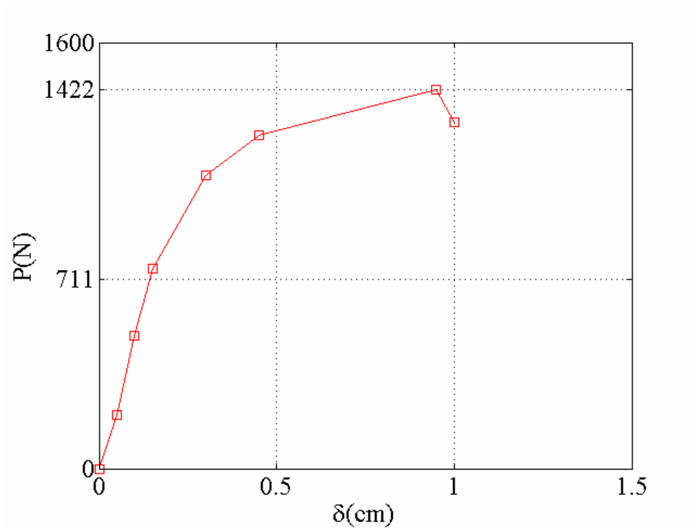

Figure 4. Numerical and experimental results for mixed mode loading

The proposed modeling of the sandwich beam is transformed gradually and imposes the discretion of the areas regarding the mesh density and quality in three different regions according to the length of the crack as it has already been applied in [5]. This discretion is a prerequisite in order to avoid the worthless consumption of computational resources and time. The model shown in Figure 2 corresponds to the maximum crack length (a=70mm) in which about 22500 elements have been used in the finite element analysis. The experimental data and the numerical predictions according to the proposed cohesive law are shown in Figure 4. At the load at failure ($P \simeq 1422N$) kinking of the crack into the core and catastrophic failure was observed.

3 Conclusions

In this paper a procedure based on the cohesive damage model is developed to simulate crack propagation and kinking into the core of sandwich structures very close to the upper skin interface under mixed mode loading conditions. The objective is to obtain a mixed mode cohesive law characterizing the crack propagation and kinking inside the core but very close to the face sheet in terms of cohesive parameters. The damage model was implemented in a finite element model. The numerical application analyzed was in good agreement with the experimental results. The above numerical investigation gives a first indication of the crack growth behavior in sandwich beams under flexural loading. Comparison with experimental results, different cohesive laws and three dimensional finite element analyses are further needed in order to verify the numerical implementation of the crack propagation.

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