# STRUCTURAL PARTICLE SWARM OPTIMIZATION OF A COMPOSITE SWIMMING POOL FOR CRUISE SHIPS 

G. Tomassetti ${ }^{1 \text { * }}$, A. Pullara ${ }^{2}$, M. Codda ${ }^{3}$<br>${ }^{1}$ IMAST scarl, Piazza G. Bovio, 22, 80133, Naples, Italy<br>${ }^{2}$ CETENA S.p.A., Piazza Amendola, 5, 80053, Castellammare di Stabia (NA), Italy<br>${ }^{3}$ CETENA S.p.A., Via Ippolito D'Aste, 5, 16121, Genova, Italy<br>*giordano.tomassetti@imast.it

Keywords: composite structures, maritime, optimization, meta-modeling.


#### Abstract

Most modern cruise ships include outdoor steel swimming pools that are one of the most attractive facilities on board to capture the customers interest. Composite pools are currently being considered to avoid corrosion problems typically arising when adopting steel structures. In this frame, a study was conducted to optimize a composite pool to lighten the ship super-structure. A non-linear finite element model of a composite pool was developed. Based on this model, an optimization process was performed to determine the geometrical parameters achieving a minimal weight while reducing the maximal vertical deflections of the pool bottom side. In order to limit the computational effort necessary to deal with three objectives and ten constrained design variables, an approach based on meta-modeling was proposed, significantly reducing the total computational time.


## 1 Introduction

Multi-objective optimization (MOO) is a powerful tool to help designers to identify improved configurations optimizing multiple targets simultaneously. Differently from single-objective optimization (SOO), MOO identifies a set of optima characterized by different relative importance assigned to the various objectives. The set of optimal solutions is commonly named Pareto front.

In industrial applications, global optimization tools are available and fairly well supported for conventional problems. Typically, global optimization programs implement gradient-based or evolutionary algorithms or a combination of these. Although being numerically cost-effective, gradient-based approaches show strong limitations when objective functions are noisy, nonsmooth or when derivatives are not directly available. Also, they get easily trapped by local optima, since their exploration capabilities of the search domain are limited. On the other hand, evolutionary algorithms (EA, i.e. genetic algorithms and others) have wider exploration capabilities but require a huge number of numerical evaluations making the optimization process numerically prohibitive. A widely-used optimization algorithm is the notorious particle swarm optimization (PSO) algorithm [1]. The original algorithm is an iterative, stochastic meta-heuristics imitating birds or fishes swarm movement rules. In its basic version, PSO is not able to deal with constraints on design variables, as required by real-world engineering problems. To bypass these limits and take advantage of the fast convergence
speed of PSO algorithms, modifications to the original version of the algorithm have been proposed including penalty functions to deal with constraints. When dealing with MOO, the evaluation of the entire Pareto front involves a much larger computational cost with respect to a SOO. In fact, the optimization process is repeated in MOO for each set of relative importance assigned to each of the objectives. The large number of function evaluations required by EAs combined with the redundancy of optimizations needed to face MOO, makes the optimization process unaffordable from a computational point of view. In these cases, a common choice is to introduce a surrogate model (SM) reproducing the more expensive highfidelity model (HFM). SMs are able to supply approximations of the HFMs results of interest with a reduced numerical effort.

In this work, the MOO of a swimming pool for cruise ships was performed using SMs. The pool consists of a sandwich structure, with a PVC core and a glass fibers reinforced epoxy resin skin, conveniently stiffened in specific areas with $u$-shaped reinforcement frames.

## 2 Composite pool structure model description

The considered swimming pool consists of a sandwich composite structure made up of a PVC closed cells core and skins composed of epoxy resin reinforced with glass fibers. The pool is connected with steel ship structures by bottom longitudinal and transversal composite frames. The pool structure is $10,340 \mathrm{~mm}$ long, $6,000 \mathrm{~mm}$ wide, $1,310 \div 1,975 \mathrm{~mm}$ deep, with a transversal frame spacing of $2,840 \mathrm{~mm}$, while the transversal section of frames is $200 \times 200$ mm . The geometry of the pool is shown in Figure 1.


Figure 1. Geometry of swimming pool.
In this study the MARC/Mentat 2010.1.0 finite element (FE) software package developed by MSC was used to analyse the different configurations. Half structure was FE modeled (Figure 2) taking advantage of the geometric and loading symmetry of the pool with respect to the vertical longitudinal plane (xz).


Figure 2. F.E. model of swimming pool.
The mesh implemented is composed of the number and type of shell elements specified in Table 1:

| Number | Element Class | Element Type |
| :---: | :---: | :---: |
| 14 | TRIA (3) | 138 |
| 6,818 | QUAD (4) | 75 |

Table 1. Class and type elements of the model.
The composite structures of the pool were subdivided into four areas: bottom fore, side fore, bottom aft, side aft. The core and skin thicknesses of each area were subjected to the optimization process described in §3. Bottom longitudinal/transversal frames are constituted of a PVC core and a skin of 6 mm thicks for all the configurations analysed.
GFRP and PVC were modelled as isotropic materials with the properties shown in Table 2. Sandwich laminates were modelled as layered materials.

| Mechanical properties | GFRP | PVC |
| :---: | :---: | :---: |
| Young Modulus (MPa) | 12,500 | 130 |
| Poissont coefficient | 0.35 | 0.40 |

Table 2. Material properties.
A pressure load was applied on the wet surface of the pool and is represented as function of the z coordinate by the following equation:

$$
\begin{equation*}
P=k \rho g\left(z-z_{0}\right) \tag{1}
\end{equation*}
$$

where the factor k is assumed to be equal to 1.5 in order to consider the vertical acceleration of the ship, $\rho$ is the density of water, $g$ is the acceleration of gravity, $z_{0}$ is the coordinate of the top of the pool.

Figure 3 shows the result of a preliminary non linear simulation implementing with the large strain option activated. In Figure 3, the most stressed structural areas of the pool were easily detected in the bottom of the pool itself.


Figure 3. Displacement and Equivalent Von Mises stress of the bottom layer.

## 3 Optimization problem, methodologies and results

The aim of the optimization process carried out in this work is to simultaneously minimize bottom deflection $\delta$ and weight (per unit of surface) W of the swimming pool by varying skin $\mathrm{t}_{\mathrm{S}}$ and core $\mathrm{t}_{\mathrm{C}}$ thicknesses of the various parts of the pool itself. The problem statement clearly refers to a MOO problem because of the contradiction between minimizing weight, which means minimizing the materials employed, and minimizing deflection meaning increasing thicknesses.

In this work, two different approaches were used:

- simplified model: the skin and core thicknesses were considered to be homogeneous all over the pool;
- realistic model: different skin and core thicknesses were allowed to vary on different sides of the pool.


### 3.1 Optimization using the simplified model

Under the assumption that the different sides of the pool have all common values of skin and core thicknesses, the optimization problem was summarized by the following:

$$
\begin{cases} &  \tag{2}\\ \text { min } & {\left[W\left(t_{s}, t_{C}\right) \delta\left(t_{S}, t_{C}\right)\right]} \\ \text { subject } & \text { to: } t_{C}>6 t_{s} \\ \text { with: } & 3.0 \mathrm{~mm} \leq t_{s} \leq 10.0 \mathrm{~mm} \\ & 20.0 \mathrm{~mm} \leq t_{C} \leq 60.0 \mathrm{~mm}\end{cases}
$$

where a constraint is posed between skin $t_{S}$ and core $t_{C}$ thicknesses derived from maritime regulations [2] imposed by the Italian Maritime Authority (RINA) for composite sandwich laminates in ships design.

In problem (2), the objective $\delta$ is calculated through a FE analysis while W is derived from the following simplified expression obtained by geometrical considerations:

$$
\begin{equation*}
W\left(t_{S}, t_{C}\right)=\rho_{S} t_{S}\left(C_{1} \frac{t_{S}}{2}+C_{2}\right)+\rho_{C} t_{C}\left[C_{1}\left(t_{S}+\frac{t_{C}}{2}\right)+C_{2}\right]+\rho_{S} t_{S}\left[C_{1}\left(t_{C}+\frac{3}{2} t_{S}\right)+C_{2}\right] \tag{3}
\end{equation*}
$$

where $\rho_{\mathrm{S}}=2500 \mathrm{Kg} / \mathrm{m}^{3}$ and $\rho_{\mathrm{C}}=100 \mathrm{Kg} / \mathrm{m}^{3}$ are, respectively, skin and core densities, $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are two numerical factors expressing a simplified linear dependence of the pool surface from the thickness.

As defined by eqs. (2) and (3), the optimization problem is non-linear in the objective functions.

In this work, a hybrid particle swarm optimization algorithm [3] was used to perform the MO process. A population size of 10 individuals was used to perform the PSO optimization with a maximum number of iterations set to 100 . To avoid possible stagnation problems [4], a multistart approach was implemented re-initializing the swarm and re-running the optimization for 10 times. A convergence criterion was also set in terms of difference in the objective between two successive iterations to prematurely stop the optimization in order to avoid unnecessary calculations.

At the end of the MOO process, the optimal configurations were plotted in the design space domain and in the objectives space.


Figure 4. Pareto frontier in the design space (top) and objective domain (bottom).
In Figure 4, letters from "A" to " N " identify different optimal configurations obtained varying the relative weight of the two objectives and are both represented in terms of design variables (Figure 4, top) and the corresponding objective functions values (Figure 4, bottom). The dotted line in Figure 4 (top) is the representation of the constraint expressed in eq. (2). Point "A" (bottom) in Figure 4 refers to an optimal layout in which the relative importance of the minimization of deflection $\delta$ is maximum and no importance is assigned to the weight increase. To achieve this condition, the optimizer increases the value of the (heavier) skin thickness $t_{s}$ while reducing those of the (lighter) core thickness $\mathrm{t}_{\mathrm{c}}$. On the opposite, point " N " (bottom) in Figure 4 is relative to the condition in which the optimizer gives relevance only to the objective of weight minimization regardless the increase of the bottom deflection $\delta$. To obtain this, the optimizer increases the (lighter) core thickness $t_{C}$ and decrease the (heavier) skin thickness ts. Points from "B" to "M" in Figure 1 describe intermediate optimal conditions in terms of relative importance between the two contradictory objectives.

### 3.2 Optimization using the realistic model

The second approach takes into consideration a more realistic model of the pool in which each side of the pool may have a different thicknesses both for the skin layers and for the core. This, of course, leads to a significant increase in the number of design variables and, thus, in the optimization problem. In this case, the optimization problem is summarized by:

In this case, the expression of weight is more complicated than in eq. (3) being the sum of the contributions given by the aft bottom $\mathrm{W}_{\mathrm{AB}}$, aft side $\mathrm{W}_{\mathrm{AS}}$, fore bottom $\mathrm{W}_{\mathrm{FB}}$, fore side $\mathrm{W}_{\mathrm{FS}}$ and flange $\mathrm{W}_{\mathrm{FL}}$ components:

$$
\begin{equation*}
W=W_{A B}+W_{A S}+W_{F B}+W_{F S}+W_{F L} \tag{5}
\end{equation*}
$$

In this second and more realistic case, the following 10 design variables were considered (Table 3):

| Symbol | Meaning |
| :---: | :---: |
| $\mathrm{t}_{\text {SAB }}$ | Skin thickness aft bottom |
| $\mathrm{t}_{\text {AAB }}$ | Core thickness aft bottom |
| $\mathrm{t}_{\text {SAS }}$ | Skin thickness aft side |
| $\mathrm{t}_{\text {CAS }}$ | Core thickness aft side |
| $\mathrm{t}_{\text {SFB }}$ | Skin thickness fore bottom |
| $\mathrm{t}_{\text {CFB }}$ | Core thickness fore bottom |
| $\mathrm{t}_{\text {SFS }}$ | Skin thickness fore side |
| $\mathrm{t}_{\text {CFS }}$ | Core thickness fore side |
| $\mathrm{t}_{\text {SFL }}$ | Skin thickness flange |
| $\mathrm{t}_{\text {CFL }}$ | Core thickness flange |

Table 3. Design variables explanation.
The aim of the optimization is to minimize, at the same time, three objectives: the weight W , the bottom deflection on the aft side $\delta_{\mathrm{A}}$ and on the fore side $\delta_{\mathrm{F}}$. A part from upper and lower bounds for the design variables, the minimization must take into consideration five constraints in terms of core thickness larger than six times the respective skin thickness [2] and two more constraints on the lateral displacement induced by the structural load on the side walls of the pool both on the fore and on the aft side.

Before directly facing the optimization, some computational aspects were considered. A single FE simulation took about 132 seconds to run on an Intel Pentium 4, CPU 3.00 GHz ,
1.00 GB of RAM. Supposing to use the same number of function evaluations as done for the first approach (§3.1), this would lead to about 64,000 FE calculations considering to vary the relative importance of the three objectives on only four different values (maximum, minimum and two intermediates). Considering the 132 seconds simulation time, this leads to about 88 days to run the MOO. Such a computational time is not impossible to deal with but, of course, is not convenient and effective.

To by-pass the problem of such a long CPU time for the optimization, a meta-model was built and used to perform the optimization. Borrowing concepts from the aerospace experience in deriving fast meta-models as surrogate of complex numerical calculations, the domain space was sampled to get 500 random points values of deflections $\delta_{\mathrm{A}}$ and $\delta_{\mathrm{F}}$. These values were used to build a neural network that proved to be able to reproduce the FE results as faithfully as about $99 \%$. So, after 16 hours taken to sample the domain and train the neural network, the optimization process was able to use a meta-model with a precision of about $99 \%$ with respect to the FE one, but taking only 4 seconds to run instead of 132 seconds.

Running the optimization implementing the meta-model, 64 optimal configurations were found as represented by circles in Figure 5 in terms of optimal W, $\delta_{\mathrm{A}}$ and $\delta_{\mathrm{F}}$. These 64 configurations were then interpolated to plot the 3-D Pareto Frontier represented by the mesh in Figure 5.


Figure 5. Pareto frontier for the non-homogenous thicknesses over the pool.
By the analysis of Figure 5, it is easy to notice that larger values in vertical deflections $\delta_{\mathrm{A}}$ and $\delta_{\mathrm{F}}$ correspond to less material used to sustain the structural loads and to smaller values of weight. On the contrary, if smaller deflections are desired, a larger amount of material is needed leading to an increase in structural weight.

Representing the Pareto frontier in Figure 5 as a contour plot gives more information about the optimization. In Figure 6, the Pareto frontier is represented in the plane $\left[\delta_{\mathrm{A}}, \delta_{\mathrm{F}}\right]$ and parameterized in terms of the weight W. In Figure 6, the lines of different colors connect optimal configurations characterized by different weights. Points closer to the top-right side of the plot have a larger weight than those on the left-bottom edge. Configurations closer to the top-right corner are also characterized by a larger gradient meaning that small changes in the deflections $\left[\delta_{\mathrm{A}}, \delta_{\mathrm{F}}\right]$ imply large changes in the weight. On the contrary, on the opposite bottom-left corner, gradients are much smaller meaning that small changes in deflections give small changes in weight. Moreover, in the top-right area of the plot, a number of optimized configurations with minimal deflections are identifiable being the minimal deflection loci
when the weight varies. They correspond to the top-right point of each single iso-objective line.


Figure 6. Contour plot of the Pareto frontier.

## 4 Conclusions

In this work, starting from the development of a non-linear FE model of a composite swimming pool structure for cruise ships, a MOO was performed aimed to simultaneously minimize the weight and the bottom deflections subject to the acting loads. The MOO process identified the Pareto front which is useful to the designer to determine the best compromise layout to optimize the set of design variables giving more or less relative importance to one or another of the three considered objectives.

After estimating the conspicuous numerical effort of directly facing the MOO, a SM was developed achieving a significant reduction in the computational time. By introducing the SM of the complete FE structural model of the pool, the MOO problem was solved in about 9 days with 500 FE simulations instead of the estimated 88 days with about $64,000 \mathrm{FE}$ runs.

## 5 Acknowledgements

The activities were performed in the frame of the project "ASAP" (DM29020) funded by the M.I.U.R.

## References

[1] Eberhart, R. C., Kennedy, J. A new optimizer using particle swarm theory in "Proceedings of the Sixth International Symposium on Micromachine and Human Science", Nagoya, Japan, (1995).
[2] RINA, Rules for Pleasure Yachts, Part B, Chapter 4, Section 11 (2009).
[3] Tomassetti G. A cost-effective algorithm for the solution of engineering problems with particle swarm optimization. Engineering Optimization, Vol. 42, No. 5, pp. 471-495, (2010).
[4] Worasucheep, C. A Particle Swarm Optimization with Stagnation Detection and Dispersion in "Proceedings of the IEEE 2008 Congress on Evolutionary Computation", Hong Kong, (2008).

