DAMAGE PROGRESSION IN COMPOSITE CYLINDRICAL SHELLS

R. Massabò

Department of Civil, Environmental and Architectural Engineering, DICAT
Research Center for Materials Science and Technology, MaST
University of Genova, Via Montallegro 1, 16145, Genova, Italy
roberta.massabo@unige.it

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Abstract

A mechanical model is formulated to study delamination damage progression in laminated composite cylindrical shells subjected to dynamic loadings. The shell is modelled as an assembly of sub-shells joined by cohesive interfaces. The cohesive traction laws are defined as piecewise linear functions of the relative interfacial displacements and represent different physical mechanisms. By extending homogenization techniques formulated in the literature for intact shells and for shells with linearly elastic interfaces [1,2], the global displacement field is assumed to be piecewise linear in the thickness direction and incorporate jumps at the interfaces. The a priori imposition of interface continuity conditions then leads to a substantial reduction of the displacement unknown functions with respect to a classical discrete layer formulation, while its accuracy and efficacy in studying processes of dynamic delamination fracture are preserved .

1 Introduction

Current demand of laminate and sandwich composites in many challenging applications of the naval and aeronautical industries, where the structural components must withstand extreme loading conditions, highlights the need for a better understanding of the response of composite structures in the post-elastic regime, where different damage and failure mechanisms, may form, evolve and interact. Multiple delamination fracture at the layer interfaces is among the dominant failure mechanisms in laminated systems.

This paper deals with the problem of delamination damage progression in laminated cylindrical shells subjected to dynamic loads. Mechanical modeling of the post-elastic response of these systems can be tackled using a discrete layer approach, which approximates the shell as an assemblage of sub-shells joined by cohesive interfaces. The cohesive interfaces describe different physical mechanisms: unfailed elastic interfaces, perfect adhesion of the layers, brittle and cohesive fracture and elastic contact. This approach has been previously applied by the author to study, through semi-analytic and numerical modeling, damage progression and interaction in plane laminated and sandwich systems; the effects of damage interaction on the mechanical response and different key properties, including energy absorption and damage tolerance, have been investigated and conclusions relevant to the optimal design of these material systems drawn in [3-5].

A discrete layer model is computationally expensive due to the large number of unknowns, which depends on the number *N* of layers used to model the structure: if First Order Shear Deformation Shell Theory is used to describe the response of the layers in a shell structure, the unknown displacement 2D functions are $5 \times N$. To overcome the disadvantages of the discrete-layer approach, while keeping its accuracy and efficacy in studying processes of dynamic delamination fracture, we maintain the description of the shell as an assemblage of sub-shells with cohesive interfaces and postulate a global displacement field which is piecewise linear in the thickness direction and incorporates jumps at the layer interfaces. An homogenization technique is then applied by imposing a priori interface continuity conditions for shear and normal tractions and the constitutive laws of the cohesive interfaces. The number of unknowns of the problem gets substantially reduced and become independent of the number of sub-shells. The model extends the formulations originally proposed for intact shells in [1] and for shells with linearly elastic interfaces in [2]. By describing the cohesive tractions as piecewise linear functions of the relative displacements at the interfaces between sub-shells, the model can be used to investigate multiple delamination growth in the presence of openings or elastic contact along the delamination surfaces.

2 Mechanical Model

Consider a laminated composite cylindrical shell, with mean radius of curvature R_β and thickness h subjected to a time dependent distributed load $p(t)$ (Fig. 1). A system of curvilinear and orthogonal coordinates α, β and z is introduced, with the axis α parallel to the generator of the shell and the axis z normal to its mid-surface and measured from it. The length of an infinitesimal element of the shell at the coordinate *z* is $d\beta(z) = \left(1 + z/R_\beta\right) d\beta$. The displacement components of an arbitrary point of the shell at the coordinates (α, β, z) are v_α, v_β and w.

The shell is modelled as an assemblage of N sub-shells. The sub-shell *k*, where the index $k = 1,..,N$ is numbered from bottom to top, is defined by the coordinates z_k and z_{k+1}, has thickness ${}^{(k)}h$, mean radius of curvature ${}^{(k)}R_\beta = R_\beta + z_k^c$, with $z_k^c = (z_k + z_{k+1})/2$ the coordinate of its centroid, and unit moment of inertia with respect to its central axis of inertia parallel to the generator, I_k (Fig. 1, inset). Each sub-shell is linearly elastic, globally homogeneous and transversally isotropic, with principal material axes not necessarily coincident with the geometrical axes of the structure $\alpha - \beta$. The mass density is uniform and equal to ρ_m.

The sub-shells are joined by $n = N - 1$ cohesive interfaces which define all actual and potential delamination surfaces. Normal, T_N^k, and shear, T_α^k and T_β^k, tractions act along the surfaces of the sub-shells *k* and *k*+1 at the interface *k*. They may represent interfacial tractions in the intact portion of the beam, or contact, friction and cohesive/bridging tractions acting along the surfaces of open interfaces (delaminations); they also describe externally applied loads acting on the shell surfaces, for sub-shells *k* =1 and *k* = *n*+1. The interface tractions are related to the interfacial relative displacements (jumps), $\hat{w}^k$, $\hat{v}_\alpha^k$ and $\hat{v}_\beta^k$ by cohesive traction laws that are defined with different possible features representing different physical mechanisms. The relative displacements are:

$$\hat{w}^k = {}^{(k+1)}w(z^k) - {}^{(k)}w(z^k), \quad \hat{v}_\alpha^k = {}^{(k+1)}v_\alpha(z^k) - {}^{(k)}v_\alpha(z^k), \quad \hat{v}_\beta^k = {}^{(k+1)}v_\beta(z^k) - {}^{(k)}v_\beta(z^k) \quad \text{(1a,b,c)}$$

The interface laws are defined as piecewise linear functions with:

$$\begin{aligned}
T_N^k &= -C_N^k \hat{w}^k + c_N^k \quad \text{with} \quad \mathrm{C}_N^k = \frac{T_{NB}^k - T_{NA}^k}{\hat{w}_B^k - \hat{w}_A^k}; \;\; c_N^k = \frac{T_{NB}^k \hat{w}_A^k - T_{NA}^k \hat{w}_B^k}{\hat{w}_B^k - \hat{w}_A^k} \\
T_\alpha^k &= -C_\alpha^k \hat{v}_\alpha^k + c_\alpha^k \quad \text{with} \quad \mathrm{C}_\alpha^k = \frac{T_{\alpha B}^k - T_{\alpha A}^k}{\hat{v}_{\alpha B}^k - \hat{v}_{\alpha A}^k}; \;\; c_\alpha^k = \frac{T_{\alpha B}^k \hat{v}_{\alpha A}^k - T_{\alpha A}^k \hat{v}_{\alpha B}^k}{\hat{v}_{\alpha B}^k - \hat{v}_{\alpha A}^k} \\
T_\beta^k &= -C_\beta^k \hat{v}_\beta^k + c_\beta^k \quad \text{with} \quad \mathrm{C}_\beta^k = \frac{T_{\beta B}^k - T_{\beta A}^k}{\hat{v}_{\beta B}^k - \hat{v}_{\beta A}^k}; \;\; c_\beta^k = \frac{T_{\beta B}^k \hat{v}_{\beta A}^k - T_{\beta A}^k \hat{v}_{\beta B}^k}{\hat{v}_{\beta B}^k - \hat{v}_{\beta A}^k}
\end{aligned} \quad \text{(2a,b,c)}$$

the equations of an arbitrary branch of the functions between points A and B (inset of Fig. 1). For instance, an unfailed interface, which is used in the model to describe perfect adhesion between sub-shells is described by a single branch, with $c_\alpha^k, c_N^k = 0$ and $\mathrm{C}_N^k = T_{NB}^k / \hat{w}_B^k$, $\mathrm{C}_\alpha^k = T_{\alpha B}^k / \hat{v}_{\alpha B}^k$ and $\mathrm{C}_\beta^k = T_{\beta B}^k / \hat{v}_{\beta B}^k$ chosen to be very high to minimize errors due to the introduction of fictitious compliant surfaces in the body. A weak elastic interface [2] is described similarly with lower values of C_N^k, C_α^k and C_β^k. In cohesive fracture the initial branch of the unfailed interface is followed by a second branch, which may be hardening or softening depending on the cohesive/bridging mechanisms acting along the crack faces.

The approximate displacement field is defined as:

$$\begin{aligned}
v_\alpha(\alpha,\beta,z,t) &= v_{\alpha 0}(\alpha,\beta) + \varphi_\alpha(\alpha,\beta) z + \\
&+ \sum_{k=1}^{n} \Omega_\alpha^k(\alpha,\beta)(z - z^k) H^k + \sum_{i=1}^{nc} \hat{v}_{\alpha c}^i(\alpha,\beta) H^i + \sum_{j=1}^{nd} \hat{v}_{\alpha d}^j(\alpha,\beta) H^j \\
v_\beta(\alpha,\beta,z,t) &= v_{\beta 0}(\alpha,\beta) + \varphi_\beta(\alpha,\beta) z + \\
&+ \sum_{k=1}^{n} \Omega_\beta^k(\alpha,\beta)(z - z^k) H^k + \sum_{i=1}^{nc} \hat{v}_{\beta c}^i(\alpha,\beta) H^i + \sum_{j=1}^{nd} \hat{v}_{\beta d}^j(\alpha,\beta) H^j \\
w(\alpha,\beta,z,t) &= w_0(\alpha,\beta) + \varphi_z(\alpha,\beta) z + \\
&+ \sum_{k=1}^{n} \Omega_z^k(\alpha,\beta)(z - z^k) H^k + \sum_{i=1}^{nc} \hat{w}_c^i(\alpha,\beta) H^i + \sum_{j=1}^{nd} \hat{w}_d^j(\alpha,\beta) H^j
\end{aligned} \quad \text{(3a,b,c)}$$

where v_α, v_β and w are the displacement components of an arbitrary point of the shell. The terms on the right hand side of Eqs. (3a,b,c) denote different contributions in the displacement representation: $v_{\alpha 0}$, $v_{\beta 0}$ and w_0 are the displacement components of the reference surface of the shell and φ_α and φ_β the rotations of the normal to the reference surface (standard first order shear deformation shell theory contributions); φ_z is the deformation in the transverse normal direction, needed to capture the effect of transverse normal compressibility; the third terms, with summations on the total number n of interfaces and $H^k = H(z - z^k) = \{0,\ z < z^k; 1,\ z \geq z^k\}$, supply the zig-zag contributions, which are continuous in z but with jumps in the first derivatives at the interfaces (C_z^0) and are necessary to satisfy continuity of normal and shear tractions at the interfaces; the fourth terms, with summations on the number of cohesive interfaces nc, supply the contribution of the relative displacements (jumps) at the cohesive interfaces, while the fifth terms the relative displacement jumps at the traction free delaminations nd, where $n = nc+nd$. The displacement field defined in Eqs. (3,a,b,c) is piecewise linear in the z direction with jumps at the cohesive or traction free interfaces.

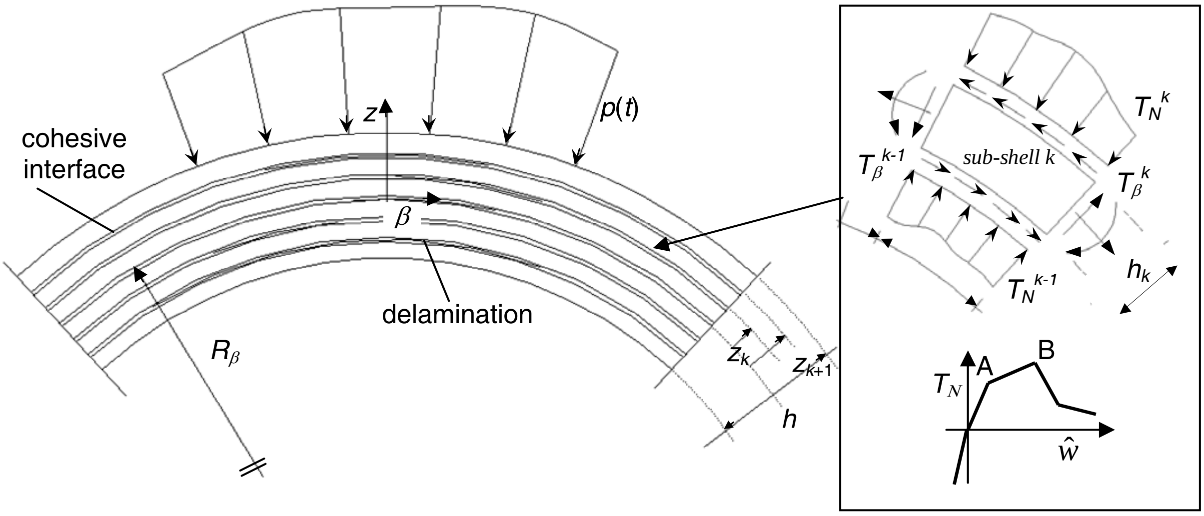

Figure 1. Cross-sectional view (perpendicular to the generator) of the laminated cylindrical shell subjected to a time dependent load; the shell is modeled as an assemblage of sub-shells joined by cohesive/contact interfaces. Inset: interfacial tractions acting on upper and lower surfaces of sub-shell k and exemplary cohesive traction law.

The transverse shear and normal strain components at the arbitrary coordinate z of the shell within sub-shell k are derived from the displacement field through the strain-displacement relations for Gaussian curvilinear coordinates:

$$
\begin{aligned}
{}^{(k)}\gamma_{\beta z}\left(1+\frac{z}{R_\beta}\right) &= \frac{\partial w_0}{\partial \beta} + \frac{\partial \varphi_z}{\partial \beta} z - \frac{v_{\beta 0}}{R_\beta} + \varphi_\beta + \sum_{j=1}^{k-1} \frac{\partial \Omega_z^j}{\partial \beta}(z - z^j) H^j + \sum_{j=1}^{k-1} \Omega_\beta^j (1 + \frac{z^j}{R_\beta}) H^j \\
&\quad + \sum_{i=1}^{(k-1)_c} \frac{\partial \hat{w}_c^i}{\partial \beta} H^i + \sum_{i=1}^{(k-1)_d} \frac{\partial \hat{w}_d^i}{\partial \beta} H^i - \sum_{i=1}^{(k-1)_c} \frac{\hat{v}_{\beta c}^i}{R_\beta} H^i - \sum_{i=1}^{(k-1)_d} \frac{\hat{v}_{\beta d}^i}{R_\beta} H^i \\
{}^{(k)}\gamma_{\alpha z} &= \frac{\partial w_0}{\partial \alpha} + \frac{\partial \varphi_z}{\partial \alpha} z + \varphi_\alpha + \sum_{j=1}^{k-1} \frac{\partial \Omega_z^j}{\partial \alpha}(z - z^j) H^j + \sum_{j=1}^{k-1} \Omega_\alpha^j H^j + \sum_{i=1}^{(k-1)_c} \frac{\partial \hat{w}_c^i}{\partial \alpha} H^i + \sum_{i=1}^{(k-1)_d} \frac{\partial \hat{w}_d^i}{\partial \alpha} H^i \quad \text{(4,a,b,c)} \\
{}^{(k)}\varepsilon_{zz} &= \varphi_z + \sum_{j=1}^{k-1} \Omega_z^j H^j
\end{aligned}
$$

As for the displacement field, the strain components are given by the superposition of different contributions: those characteristic of an extended first order shear deformation theory (which includes transverse compressibility), which depend on the displacement components of the reference surface; those characteristic of zig-zag approximations of the displacement field, with a continuous piecewise variation from layer to layer, which depends on the functions $\Omega_\alpha^k(\alpha,\beta)$, $\Omega_\beta^k(\alpha,\beta)$ and $\Omega_z^k(\alpha,\beta)$; and those related to the cohesive and traction free interfaces, which depend on the displacement jumps.

The unknown 2D functions $\Omega_\alpha^k(\alpha,\beta)$, $\Omega_\beta^k(\alpha,\beta)$ and $\Omega_z^k(\alpha,\beta)$ in the displacement equations (3,a,b,c) are determined as functions of the displacements of the reference surface by satisfying continuity conditions for shear and normal tractions across the laminate interfaces:

$$\begin{aligned}
&{}^{(k)}\sigma_{\alpha z}(z^k) = {}^{(k+1)}\sigma_{\alpha z}(z^k),\\
&{}^{(k)}\sigma_{\beta z}(z^k) = {}^{(k+1)}\sigma_{\beta z}(z^k),\\
&{}^{(k)}\sigma_{zz}(z^k) = {}^{(k+1)}\sigma_{zz}(z^k),
\end{aligned} \qquad (5,a,b,c)$$

where the stresses in the sub-shells are defined through the linearly elastic constitutive equations of each sub-shell, ${}^{(k)}\boldsymbol{\sigma} = {}^{(k)}\mathbf{C}\,{}^{(k)}\boldsymbol{\varepsilon}$, as functions of the strain components. The function $\Omega_z^k(\alpha,\beta)$, for instance, is directly determined from Eq. (5c) by assuming that the effect of $\varepsilon_{\alpha\beta}$ on σ_{zz} is negligible with respect to that of ε_{zz}, as suggested in [2]:

$$\Omega_z^k(\alpha,\beta) = {}^{(1)}C_{33}\left(\frac{1}{{}^{(k+1)}C_{33}} - \frac{1}{{}^{(k)}C_{33}}\right)\varphi_z = {}^{(1)}C_{33}\left(-\frac{{}^{(k)}\Delta C_{33}}{{}^{(k+1)}C_{33}\,{}^{(k)}C_{33}}\right)\varphi_z \qquad (6)$$

The function $\Omega_z^k(\alpha,\beta)$, as well as $\Omega_\alpha^k(\alpha,\beta)$ and $\Omega_\beta^k(\alpha,\beta)$, becomes zero if the elastic constants of the sub-shell *k* and *k+1* are the same, since no zig-zag contribution in the displacement is then needed to satisfy continuity conditions for the normal and shear tractions at the interfaces.

Once the functions $\Omega_\alpha^k(\alpha,\beta)$, $\Omega_\beta^k(\alpha,\beta)$ and $\Omega_z^k(\alpha,\beta)$ have been defined in terms of the displacement unknowns and material and geometrical properties, the relative displacements at each cohesive interface, $\hat{w}^k$, $\hat{v}_\alpha^k$ and $\hat{v}_\beta^k$ for k =1...*nc*, are defined as functions of the transverse shear and normal strains at the interfaces, Eq. 4, through the cohesive traction laws, Eq. (2a,b,c), and the constitutive equations of the sub-shells surrounding the interface.

Based on the procedure described above, the displacement field is then defined in terms of only 6 unknowns, which are $v_{\alpha 0}, \varphi_\alpha, v_{\beta 0}, \varphi_\beta, w_0$ and φ_z. The unknowns increase by $3n_{\mathrm{d}}$, namely $\hat{w}^k$, $\hat{v}_\alpha^k$ and $\hat{v}_\beta^k$ for k =1...*nd*, if traction free delaminations are present in the shell or if they are not treated as cohesive interfaces.

Work is in progress on the derivation of the equations of motion and the boundary conditions of the shell through the Hamilton principle ([1,2]). The problem will then be solved under general loading conditions using a discretization and an iterative approach, which assumes an initial state for the cohesive interfaces at the coordinate α,β and iterate until convergence is reached on interfacial displacements and tractions.

3 Conclusions

A model is formulated to study delamination damage progression in laminated composite shells subjected to dynamic loadings. The model describes the shell as an assemblage of sub-shell joined by cohesive interfaces as it is typically done in discrete layer formulations to study static and dynamic delamination fracture in beams and plates. Discrete layer formulations accurately describe the response of the system and are very convenient to study processes of dynamic delamination fracture. However, they are computationally expensive since the number of unknowns depend on the number of layers used to describe the system. In this paper a homogenization technique is applied, which assumes the global displacement field as piecewise linear in the thickness direction with jumps at the interfaces, and imposes a priori continuity conditions on shear and normal tractions at the interfaces to reduce the number of displacements unknowns of the problem. Following this approach, the unknows become independent of the number of sub-shells. Work is in progress on the derivation of the equations of motion and boundary conditions of the shell through the application of Hamilton principle of elastokinetics.

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