FAILURE OF NOTCHED COMPOSITE LAMINATES SPECIMEN USING A COUPLED CRITERION

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Abstract

The aim of this paper is to present a method to predict the strength of notched specimen. The approach is based on a coupled criterion. A crack of length l could be initiated if two criteria are fulfilled: (i) an energy based criterion and (ii) a stress criterion. The computation of these two criteria is based on elastic calculation using 2D Finite Element modeling. This approach has been applied to predict the failure of open-hole laminates subjected to tensile loading. The results are in quite good agreement with the experimental data. It permits moreover to predict the length at onset of failure. It is shown that this length both depends on the diameter of the hole and the fracture properties of the laminate. A correction of the usual point stress method can thus be obtained.

1 Introduction

The use of laminate CRFP composites to manufacture large structures, such aeronautical or wind turbines for instance has drastically increased over the past few years. However, composite structures can be weakened by the introduction of geometrical singularities, such as holes or notches. It is well known that simply applying a stress criterion to predict the failure of notched laminates leads to underestimate drastically the final failure. This is the reason why some alternative approaches have been proposed in the literature to overcome this difficulty. The main idea consists in applying a failure criterion using stresses calculated at a given distance d_{ps} from the hole [1] or stresses averaged on a given volume (a cylinder for instance defined by its diameter d_{fcv} [2][3]). This kind of approach permits to predict failure of notched composite with a low computational cost. Moreover, the physical meaning of these critical distances is arguable and thus the extension of these approaches to other cases is very limited. Experimental results indicate that this length must be related to the notch size and the specimen geometry to obtain accurate predictions [1][3]). This dependence must be identified with the help of a large experimental campaign.

This is the reason why, the aim of the present paper is to propose a method to predict the strength of notched laminates based on physical consideration, with a limited time of computation, easy to implement, and with a limited cost in term of identification (it means that only data from the ply scale are required). To attain this goal, a coupled criterion based on an energetic condition and a stress criterion [3] is proposed. This approach is here applied to predict the failure of tensile loaded open-hole laminates with different geometry and stacking sequence. Section 1 is devoted to the presentation of the coupled criterion. Two parameters are needed to apply this approach: the strength σ^c and the toughness G^c of the laminate. The identification procedure used to determine these parameters as a function of data obtained from the ply are also presented. The results are presented in section 3. Two cases with two different materials are under investigation. The first one concerns the influence of the size of the hole on the strength of open-hole specimens. The second one is aimed at predict the failure of multiple holes specimens.

2 Presentation of the model

2.1 The coupled criterion

As explain in the introduction, classical method used an internal length to predict the failure of notched specimens. It has been demonstrated in [4] that it is necessary to use a criterion which combines energy and stress conditions to describe a crack nucleation mechanism in the vicinity of a stress concentration. This approach has already been used to describe the failure due to delamination initiated form edge in laminate composites. The aim of this paper is to apply this coupled criterion to predict the failure of notched specimen and especially open-hole specimens subjected to tensile loading. The diameter of the hole is d, the width w and the length L. When the load reaches the failure load (noted σ^*) two symmetric cracks of length a^* are initiated form the hole.

The energetic condition compares the change of the potential energy ΔW and the toughness G^c of the laminate. ΔW is expressed as a variation between the final state (with a crack of length a) and the initial state (without a crack) at a fixed macroscopic loading (imposed displacement ΔL). This criterion is written as

$$G^{inc}(a) = \frac{\Delta W}{\Delta S} = \frac{W(a) - W(0)}{\Delta S} = A(a)\varepsilon^2 E^{eq} d \geq G^c \qquad (1)$$

where $G^{inc}(a)$ is the incremental energy release rate in which the infinitesimal energy rates of the classical Griffith approach are replaced by finite energy increments. Under the assumption of plane elasticity, the area ΔS=d*h is where d is the crack length and h the thickness of the plate. A(a) is a dimensionless parameter depending only on the geometry. ε is the global strain of the open-hole specimen ($\varepsilon=\Delta L/L$). E^{eq} is defined by [6]:

$$E^{eq} = \frac{\sqrt{2E_y E_x}}{\sqrt{\sqrt{\frac{E_y}{E_x}} + \frac{E_y}{2G_{xy}} - \upsilon_{yx}}} \qquad (2)$$

where E_x, E_y are repectively the Young modulus of the laminate in the x and y directions, G_{xy} is the shear modulus of the laminate and ν_{xy} is the Poisson's ratio (expressed in the (x,y) axis).

ΔW being a decreasing function of the crack length a, the coefficient A(a) is null for a=0 and increases as a function of a. The energetic balance (1) thus provides a lower bound of the crack increment (a) for a given value of the applied loading (ΔL).

The stress condition assumes that the tensile normal stress $\sigma_{yy}(x)$ reaches the tensile strength σ^c along a distance at least equal to the nucleation length

$$\sigma_{yy}(x) = k_{yy}(x)E^{eq}\varepsilon \geq \sigma^c \text{ for } R \leq x \leq R + a \tag{3}$$

where $k_{yy}(x)$ is a dimensionless coefficient. Due to the stress concentration near the hole, k is obviously a decreasing function of x. Thus the stress condition (3) provides an upper bound of the crack increment (a) for a given value of the applied loading.

The energetic condition (1) provides a lower bond of the crack increment while the stress criterion (2) provides a upper bond. Increasing the loading reduces the lower bound but increases the upper bound. Finally, for a monotonic and increasing applied loading, the increment of crack nucleation at a * corresponds to the increment of crack for which both conditions (1) and (3) are fulfilled. It leads to

$$\frac{A(a^*)}{k_{yy}^2(a^*)} = \frac{L^c}{d} \tag{4}$$

where L^c is a characteristic fracture length of the material defined as $L^c = E^{eq}G^c / (\sigma^c)^2$.

It is worth mentioning that (4) always admits a solution since A(a*)/k2yy (a*) is an increasing function of a* and is equal to 0 for a*=0. Once the initiation length a* is determined, the initiation strain ε^c is given using (1) by

$$\varepsilon^c = \sqrt{\frac{G^c}{E^{eq}dA(a^*)}} \tag{5}$$

It is worth mentioning that ε^c could be determined using (3) and leads to the same result.

2.2 Practical application of the criterion

In order to apply the coupled criterion presented in section 2.1, it is necessary (i) to calculate the dimensionless parameters A(a) and $k_{yy}(x)$ and (ii) the fracture properties of the laminate (strength and toughness).

The dimensionless parameters involved in the coupled criterion (see (1) and (3)) are computed using Finite Element computation. It is worth mentioning that for a unixial loading and an open-home specimen with a central hole, some approximate analytical solutions exists [7][8] . However, this paper represents a first step and will be extended to multiaxial loadings. Moreover, in the second part of this paper, the case of a laminate with multiple holes is investigated. This is the reason why a Finite Element model (that is more general than the analytical models) has been chosen. A 2D FE model (with linear triangular elements) is used to model the laminate with plane stress assumptions. The symmetry of the problem (geometry, loading and homogenized orthotropic behavior) allows to consider only a quarter of the plate. The mesh and the boundary conditions are given in Figure 1. It is worth mentioning that a particular attention has been paid to the size of the mesh near the hole in order to capture in a correct manner the stress gradient.

The plate is subject to an applied displacement ΔL ($\varepsilon=\Delta L/L$). In a first step the plate is modeled without any crack (see Figure 1). It permits to calculate the stress in the ligament

($\sigma_{yy}(x)$ in (3)) and the energy at the initial state (W(0) in (1)). In a second step, a crack of length (a) is incrementally inserted in the mesh by releasing the boundary condition on the ligament (see Figure 2). The finite element solution gives the energy W(a) as a function of the length of the crack length (a). Using (1) and (3), the dimensionless parameters are given by

$$\begin{aligned} A(a) &= -\frac{1}{\varepsilon^2 E^{eq} d}\frac{W(a) - W(0)}{\Delta S} \\ k_{yy}(x) &= \frac{\sigma_{yy}(x)}{E^{eq}\varepsilon} \end{aligned} \qquad (6)$$

Finally, these dimensionless parameters are used to calculate the crack nucleation length a* (eq. 4) and the initiation strain ε^c (5). The failure load is calculate by

$$\sigma^c = E^{lam}\varepsilon^c \qquad (7)$$

where E^{lam} is the equivalent Young modulus of the laminate in the loading direction.

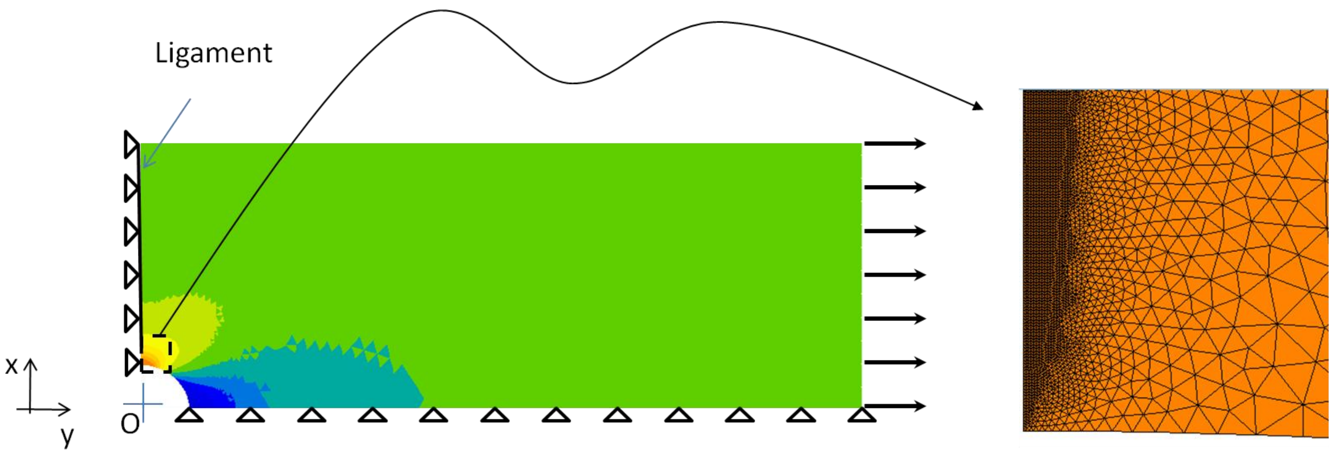

Figure 1. Geometry and boundary condition of an open-hole specimen and detail of the mesh near the hole

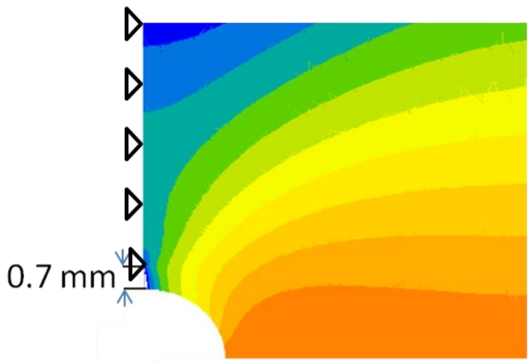

Figure 2. Result of the finite element calculation with a crack length of 0.7mm. The symmetry boundary conditions are released on the length of the crack (BC applied for x>R+0.7mm)

All the material properties (i.e. the homogenized elastic properties and the fracture properties of the laminate) involved on L^c could be determined thanks to the properties of the ply. Indeed, thanks the elastic properties of the ply it is possible using the classical laminate theory to determine the homogenized elastic properties of the ply. The tensile strength in the direction y is determined using the progressive failure approach proposed in [9]. Finally, the toughness of the laminate is calculated from the toughness of the 0° ply thanks to the approach described in [10]. The elastic properties of the ply and the strength could be measured using standardized tests. The fracture toughness of the UD 0° ply could be determined using Compact Tension Test as proposed in [11]. Only 6 tests performed on the ply could be used to identify all the material parameters necessary to apply the model.

3 Results and discussion

3.1. Strength prediction of a central open-hole specimen

This section is devoted to the prediction of the strength of notched specimen with a central hole. The experimental results are taken from [12]. The specimens are manufactured using IM7-8552 UD plies with a stacking sequence of $[90/0/\pm45]_{3S}$. The properties of the UD ply are given table 1. Different diameter of holes are investigated (d=2,4,8,10) with a constant ratio W/d=6 (where D is the width of the specimen).

As explain in section 2.2, the coupled criterion necessitates to determine the fracture properties of the laminate (strength and toughness). The progressive failure analysis proposed in [9] predicts the strength of the un-notched specimen in the longitudinal direction equal to 844MPa which is close to that measured experimentally. The toughness of the quasi-isotropic laminate is determined using [10] and is estimated to be equal to 20kJ/m^2.

Elastic properties	
E_1 (GPa)	171.42
E_2 (GPa)	9.08
G_{12} (GPa)	5.29
ν_{12}	0.32

Strength	
X_t (MPa)	2326.2
X_c (MPa)	1200.1
Y_t (MPa)	62.3
Y_c (MPa)	199.8
S_c (MPa)	92.3

Toughness	
G_1^c (kJ/m^2)	81.5

Table 1. Elastic properties and strength of the UD ply IM7/8552 (taken from [12])

The crack nucleation length and the facture loading have been calculated using the procedure given in section 2.2. The comparison between the experimental results and the results obtained with the coupled criterion are reported in Figure 3.

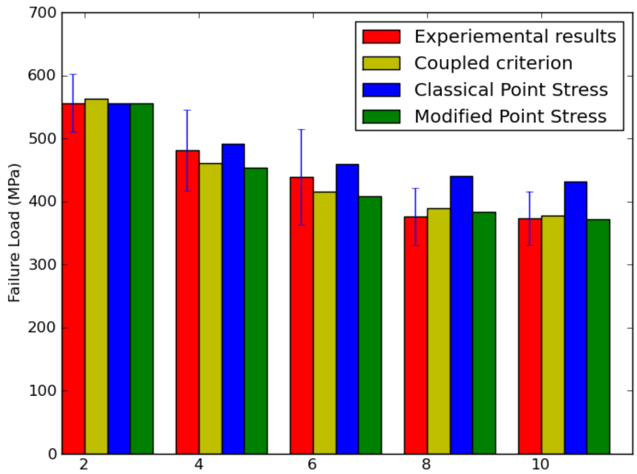

Figure 3. Comparison between the experimental failure load and the numerical prediction as a function of the diameter of the hole for the IM7/8552 central hole specimen

A quite good agreement could be observed between the experimental results and the prediction provided by the coupled criterion. It is worth mentioning that all the parameters involved in the model have been identified thanks to tests on UD plies (no test on open-hole specimen has been used to identify the model). As a comparison, the point stress method has also been applied. For this method, it is necessary to identify the internal length l_0 and its evolution as a function of the diameter of the hole [1]

$$l_0 = \rho\sqrt{d} \tag{8}$$

The test with the diameter d=2mm has been chosen to identify the parameter ρ. The evolution of l_0 as a function of d is presented in Figure 4 (referenced as "Classical Point Stress", CPS). The length l_0 could be compared with the crack length nucleation (a^*) provided by the Coupled Criterion (CC). In Figure 4, it can be seen that internal length l_0 is greater than a^*. As the stress in the ligament is a decreasing function of x, the failure load predicted by the CPS is greater than that provides by the CC (Figure 3).

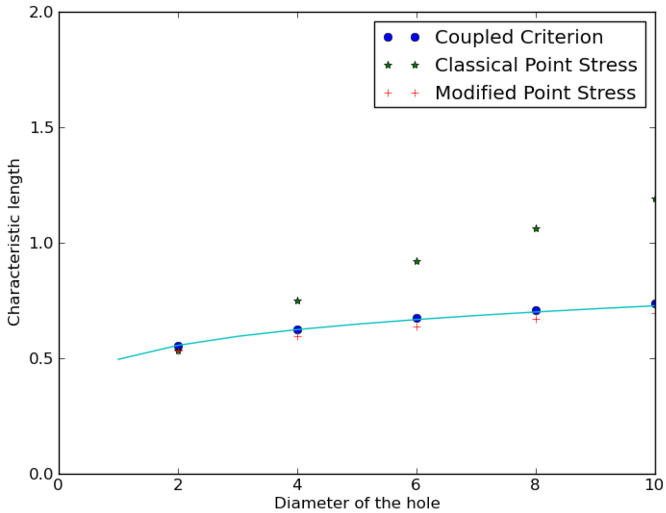

Figure 4. Evolution of the characteristic length of the different model as a function of the diameter of the hole

Thanks to the results provide by the Coupled Criterion, it is possible to propose a modification of the point stress method and more particularly of the evolution of the internal length as a function of the diameter of the hole

$$l_o^{mps} = \rho^{mps} d^{1/6} \tag{8}$$

The test with the diameter d=2mm has been chosen to identify the parameter ρ^{psm}. The evolution of l_0^{mps} as a function of d is presented in Figure 4 (referenced as "Modified Point Stress", MPS). The predictions of the MPS are very close to that of the CC and are in quite good agreement with the experimental results. However, it is worth mentioning that the final result is very sensitive to the test chosen to identify ρ^{psm}. Moreover, the internal length used in the Point Stress methods has no physical meaning, this is the reason why, the coupled criterion seems more suitable to predict the failure of notched specimens.

3.2 Strength of composite laminates with multiple holes

This section is devoted to the prediction of the strength of notched specimen with a multiple holes. The experimental results are taken from [13]. The specimens are manufactured using T700/M21 UD plies with two different stacking sequences $[0_2/\pm60_2]_s$ and $[0_2,90,0_2]$. The properties of the UD ply are given in table 2. Different geometries are investigated (see table 3).

Elastic properties		**Strength**	
E_1 (GPa)	130	X_t (MPa)	2000
E_2 (GPa)	8.3	X_c (MPa)	1180
G_{12} (GPa)	4.4	Y_t (MPa)	76
ν_{12}	0.31	Y_c (MPa)	200
		S_c (MPa)	80

Table 2. Elastic properties and strength of the UD ply IM7/T700/M21 (taken from [13])

	(0,0) d=8mm
	(-6,0) d=8 mm (6,0.) d=8 mm
	(0,-13.) d=4 mm (0, -5.) d=4 mm (0. ,5.) d=4 mm (0.,13.) d=4 mm

Table 3. Geometry of the multiple holes specimens (the position of the hole are indicated in brackets). The width is 38mm for all the specimens

As in section 3.1 the strengths of the two laminates have been calculated using the progressive failure analysis. No experimental data is available for the toughness of the T700/M21 material. This is the reason why the CC has been used to perform an inverse identification of the G_1^c of the ply using the result obtained on the $[0_2/\pm60_2]_s$ specimen with one hole. Using this data, the comparison between the experimental results and the results obtained with the coupled criterion are reported in Figure 4. A quite good agreement could be observed between the experimental results and the prediction provided by the coupled criterion (excepted for the central hole specimen with $[0_2,90,0_2]$ on which an experimental problem occurs).

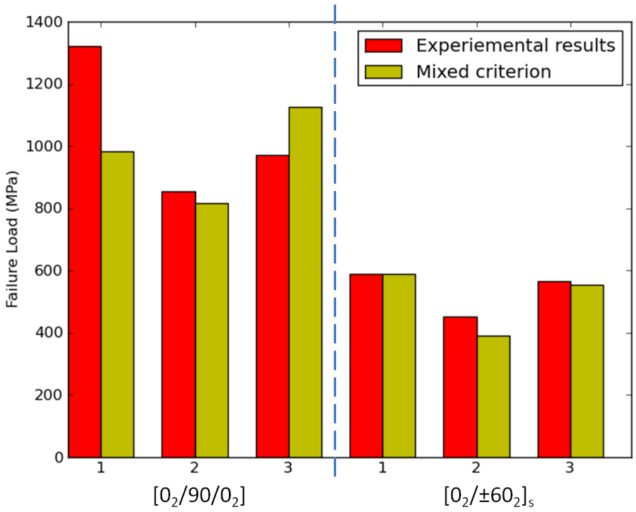

Figure 5. Comparison between the experimental failure load and the numerical prediction for the T700/M21 multiple holes specimen as a function of the laminate and of the geometry

4 Results and discussion

A method based on a coupled criterion that involved an energetic condition and a stress criterion have been developed to predict the failure of notched specimen. This criterion necessitates the calculation of dimensionless parameters that has been performed using elastic finite element simulations. The application of this coupled criterion necessitates material parameters that can be identified using simple test on UD ply. It is shown that this approach (i) describes the hole size effect whereby the strength decreases with increasing the hole radius, (ii) permits to predict that failure for different laminates.

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