ANALYTICAL STUDIES ON THE LOW-VELOCITY IMPACT RESPONSE OF TIMOSHENKO NANOBEAMS

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Abstract

In this paper, analytical solutions of low velocity transverse impact of a nanoparticle (mass) on a nanobeam are presented using the nonlocal theories to bring out the effect of the nonlocal behavior on dynamic deflections. Impact of a mass on simply supported and clamped nanobeams are investigated by using nonlocal Timoshenko beam theory. In order to obtain an analytical result for this problem, an approximate method has been developed wherein the applied impulse is replaced by a suitable boundary condition. The dynamic deflections predicted by the classical theory are always smaller than those predicted by the nonlocal theory due to the nonlocal effects. Furthermore, the mass and the velocity of the nanoparticle (striker) have significant effects on the dynamic behavior of nanobeams.

1 Introduction

Impact is a well known concern in composite structures and the effects of impact damage is a major issue in the design of aircraft made of composite materials. As it is well known, impact is a complex event involving several phenomena and many parameters, and capturing the essential dynamics experimentally and/or computationally is difficult.

There are many different approaches and methodologies in nanomechanics including analytical, numerical and experimental studies. Due to small length scale in micro and nano applications of beam nonlocal elasticity has been used in recent years. Basic difference between classical elasticity and nonlocal elasticity is definition of stress. Most classical continuum theories are based on hyperelastic constitutive relations which assume that the stress at a point is functions of strains at that point. However, in the nonlocal elasticity theory proposed by Eringen [1-3], the stress at a point is a function of strains at all points in the continuum.

In macro scale the impact of targets by mass involves highly complex processes which have been investigated analytically largely [4-8].

The motion of neutral atoms and nanoparticles in nanotubes has been of considerable interest in view of the rapid progress of nanotechnology, and carbon nanotubes are used as molecular channels for the transportation of nanoparticles, such as water and protons [9]. During these applications, nanotubes may be subjected to impact of mass, and this leads to transverse dynamic deflection of nanotubes (nanobeam). Due to this fact, it is very important to understand the impact behavior of nanobeam. In this study, the dynamic response of nanobeams with simply-supported and clamped under impact of mass using the nonlocal constitutive differential equations is presented. By using nonlocal Timoshenko beam theory the applied impulse is replaced by a suitable boundary condition. In this paper, new equilibrium conditions, domain governing differential equation and boundary conditions for the nanobeams under impact of nanoparticle(mass) are derived and analyzed.

2 Transverse impact of a nanoparticle (mass) on nanobeam

In order to obtain an analytical result for this problem, an approximate method has been developed wherein the applied impulse is replaced by a suitable boundary condition. Thus, only a solution of the equation of free vibration of the nanobeam is required [4-5]. By introducing the following nodimensional terms as[1]:

$$\bar{x} = \frac{x}{L} \tag{1a}$$

$$\bar{w} = \frac{w}{L}, \qquad \hat{\bar{w}} = \frac{\hat{w}}{L} \tag{1b}$$

$$\lambda^2 = \omega^2 \frac{\rho A L^4}{EI} : Frequency \quad parameter \tag{1c}$$

$$\Omega = \frac{EI}{K_s G A L^2} : Shear \quad deformation \quad parameter \tag{1d}$$

$$\alpha = \frac{e_0 a}{L} : Scaling \quad effect \quad parameter \tag{1e}$$

$$\xi = \frac{L\sqrt{A}}{\sqrt{I}} : Slenderness \quad ratio \tag{1f}$$

$$\bar{v}_2 = \frac{v_2}{L} \text{ : initial velocity of striker} \tag{1g}$$

The governing equation may be written as

$$\Omega\left(1 - \frac{\alpha^2 \lambda^2}{\xi^2}\right)\frac{d^2\phi}{d\bar{x}^2} + \left(\frac{\lambda^2 \Omega}{\xi^2} - 1\right)\phi - \left(\alpha^2 \lambda^2 \Omega + 1\right)\frac{d\bar{w}}{d\bar{x}} = 0 \tag{2}$$

$$\left(\frac{d\phi}{d\bar{x}} + \frac{d^2\bar{w}}{d\bar{x}^2}\right) + \lambda^2 \Omega \bar{w} = 0 \tag{3}$$

Equations (2) and (3) may be uncoupled to produce two fourth-order differential equations in terms $\overline{w}$ of and ϕ as shown below[1]:

$$(1-\frac{\alpha^2\lambda^2}{\xi^2})\frac{d^4\overline{w}}{d\overline{x}^4}+\lambda^2(\Omega+\frac{1-\Omega\alpha^2\lambda^2}{\xi^2}+\alpha^2)\frac{d^2\overline{w}}{d\overline{x}^2}+\lambda^2(\frac{\lambda^2\Omega}{\xi^2}-1)\overline{w}=0 \tag{4}$$

$$(1-\frac{\alpha^2\lambda^2}{\xi^2})\frac{d^4\phi}{d\overline{x}^4}+\lambda^2(\Omega+\frac{1-\Omega\alpha^2\lambda^2}{\xi^2}+\alpha^2)\frac{d^2\phi}{d\overline{x}^2}+\lambda^2(\frac{\lambda^2\Omega}{\xi^2}-1)\phi=0 \tag{5}$$

The general solutions for equations (2) and (3) are, respectively, given by [13]

$$\overline{w}=C_1\cosh(k_e x)+C_2\sinh(k_e x)+C_3\cos(k_f x)+C_4\sin(k_f x) \tag{6}$$

$$\phi=D_1\sinh(k_e x)+D_2\cosh(k_e x)+D_3\sin(k_f x)+D_4\cos(k_f x) \tag{7}$$

Where

$$\begin{pmatrix} k_e \\ k_f \end{pmatrix}=\left(\frac{\{\mp\lambda^2(\Omega+\frac{1-\Omega\alpha^2\lambda^2}{\xi^2}+\alpha^2)+\sqrt{\lambda^4(\Omega+\frac{1-\Omega\alpha^2\lambda^2}{\xi^2}+\alpha^2)^2-4\lambda^2(1-\frac{\alpha^2\lambda^2}{\xi^2})(\frac{\lambda^2\Omega}{\xi^2}-1)}\}}{2(1-\frac{\alpha^2\lambda^2}{\xi^2})}\right)^{1/2} \tag{8}$$

Through equation (3), the constants C_i and D_i are related as follows:

$$D_1=C_1\Psi_\beta,\quad D_2=C_2\Psi_\beta,\quad D_3=C_3\Psi_\gamma,\quad D_4=-C_4\Psi_\gamma \tag{9}$$

Where

$$\Psi_\beta=-\frac{\beta^2+\lambda^2\Omega}{\beta},\quad \Psi_\gamma=\frac{\gamma^2-\lambda^2\Omega}{\gamma} \tag{10}$$

2.1 Simply suported beam

For the case of central impact on a simply supported beam of lenght L, the boundary conditions are given by $w = 0$ and $M=0$. Where M is the nonlocal bending moment.

Therefore the boundary conditions of a simply supported beam can be expressed as [12]

$$\overline{M} = \frac{ML}{EI} = (1 - \frac{\alpha^2 \lambda^2}{\xi^2})\frac{d\phi}{d\overline{x}} - \alpha^2 \lambda^2 \overline{w} = 0 \to \frac{d\phi}{d\overline{x}} = 0, \qquad \overline{w} = 0, \qquad at \quad \overline{x} = 0,1 \tag{11}$$

Substitution of the two conditions of Eq. (11) (at $x=0$) in Eq. (6) shows that $C_1 = 0$ and $C_2 = 0$.

For simply supported beam due to symmetry, beam slope at center of the beam is expressed as

$$\frac{\partial \widehat{\overline{w}}(\frac{1}{2}, t)}{\partial \overline{x}} = 0 \Rightarrow \frac{\partial \overline{w}(\frac{1}{2})}{\partial \overline{x}} = 0 \tag{12}$$

Substitution of the condition shows that $C_4 = -C_2 \dfrac{k_e \cosh(\frac{k_e}{2})}{k_f \cos(\frac{k_f}{2})}$.

The discontinuity in the beam shear at the contact point must also equal the reversed effective force of the striker [4-5]. For central impact, due to symmetry, this is expressed as

$$Q = \frac{1}{2} m_2 \frac{\partial^2 \widehat{\overline{w}}}{\partial t^2} \qquad at \quad \overline{x} = 1/2 \tag{13}$$

Where Q is the shear force, therefore Eq. (13) can be written as

$$K_s GA(\varphi + \frac{\partial \widehat{\overline{w}}}{\partial \overline{x}}) = \frac{1}{2} m_2 \frac{\partial^2 \widehat{\overline{w}}}{\partial t^2} \Rightarrow K_s GA(\phi . e^{i\omega t} + \frac{d\overline{w}}{d\overline{x}} . e^{i\omega t}) = -\frac{1}{2} m_2 \overline{w} \omega^2 . e^{i\omega t} \quad at \qquad \overline{x} = 1/2 \tag{14}$$

Furthermore, the characteristic equation for ω as follow

$$\frac{\omega^2 \rho L}{K_s G(\Psi_\beta + \Psi_\gamma \frac{k_e}{k_f})}\left[\frac{k_e}{k_f}\tan(\frac{k_f}{2}) - \tanh(\frac{k_e}{2})\right] = 2m \tag{15}$$

Where $m = \frac{m_1}{m_2}$ is the ratio of the mass of the nanobeam to that of the nanoparticle (striker) and $m_1 = \rho AL$ is the mass of the nanobeam.

Note that Eq. (15) has several roots of ω (frequency). Therefore, the general solutioncan be written as

$$\widehat{w}(\overline{x},t) = \sum_{a=1}^{\infty} \overline{w}_a(\overline{x}).e^{i\omega_a t} = \sum_{a=1}^{\infty} \overline{w}_a(\overline{x}).(E_a \sin(\omega_a t) + iF_a \cos(\omega_a t)) \tag{16}$$

Furthermore, since $\widehat{w}(\overline{x},0) = 0, F_a = 0$. Upon applying these results to Eq. (16), the latter may be expressed as

$$w(x,t) = \sum_{a=1}^{\infty} G_a X_a \sin(\omega_a t) \tag{17}$$

Where $G_a = A_a E_a \cosh(\frac{k_{e_a}}{2}),$ $X_a = \frac{\sinh(k_{e_a}\overline{x})}{\cosh(\frac{k_{e_a}}{2})} - \frac{k_{e_a}}{k_{f_a}} \frac{\sin(k_{f_a}\overline{x})}{\cos(\frac{k_{f_a}}{2})}$

The evaluation of the constant G_a actually involves the replacement of the impulsive force by an appropriate initial velocity condition for the beam. If the striker is considered to impart a velocity indistinguishable from $\overline{v}_2$ (initial velocity of striker) to an infinitesimal section of the beam just under the contact point, then the momentum of this section and striker should equal the initial momentum $m_2\overline{v}_2$ as [4]

$$\int \frac{\partial \widehat{w}(\overline{x},0)}{\partial t} dS = m_2 \overline{v}_2 \tag{18}$$

Where integration dS is carried out with respect to the total mass of nanobeam and nanoparticle (striker). If, then, $\frac{\partial \hat{\overline{w}}(\overline{x},0)}{\partial t} = \sum_{a=1}^{\infty} \omega_a G_a X_a \equiv \varphi(\overline{x})$, and both sides are multiplied by X_b , there results

$$\sum_{a=1}^{\infty} \omega_a G_a X_a X_b \equiv \varphi(\overline{x}) X_b \tag{19}$$

If Eq. (19) multiplied by m_2 and evaluated at $\overline{x} = \frac{1}{2}$ is added to the integral of Eq. (19) with respect to beam mass $dm_1 = (m_1)d\overline{x}$, there is obtained, in view of symmetry,

$$\sum_{a=1}^{\infty} \omega_a G_a \left\{ 2m_1 \int_0^{\frac{1}{2}} X_a X_b dx + m_2 X_a(\frac{1}{2}) X_b(\frac{1}{2}) \right\} = 2m_1 \int_0^{\frac{1}{2}} X_b \varphi(x) dx + m_2 X_b(\frac{1}{2}) \varphi(\frac{1}{2}) \tag{20}$$

Which represents really an integration with respect to the total mass as suggested by Eq. (18). It can be shown with direct integration that the left-hand side of Eq. (20) vanishes when $a \neq b$, and thus G_a may be written in the form

$$G_a = \frac{1}{\omega_a} \left(\frac{2m_1 \int_0^{\frac{1}{2}} X_a \varphi(\overline{x}) d\overline{x} + m_2 X_a(\frac{1}{2}) \varphi(\frac{1}{2})}{2m_1 \int_0^{\frac{1}{2}} X_a{}^2 d\overline{x} + m_2 X_a{}^2(\frac{1}{2})} \right) \tag{21}$$

Now $\frac{\partial \hat{\overline{w}}}{\partial t} = 0$ for $t = 0, x \neq \frac{1}{2}$, and $\frac{\partial \hat{\overline{w}}}{\partial t} = \overline{v}_2$ for $t=0$ and $x = \frac{1}{2}$; hence G_a becomes

$$G_a = \frac{1}{\omega_a} \left(\frac{m_2 \overline{v}_2 X_a(\frac{1}{2})}{2m_1 \int_0^{\frac{1}{2}} X_a{}^2 d\overline{x} + m_2 X_a{}^2(\frac{1}{2})} \right) \tag{22}$$

And the displacement $\hat{\overline{w}}$ is obtained by combination of Eq. (22) and Eq. (17) as

$$\widehat{w}(\bar{x},t)=\sum_{a=1}^{\infty}\frac{1}{\omega_a}\left(\frac{m_2\bar{v}_2 X_a(\frac{1}{2})}{2m_1\int_0^{\frac{1}{2}}X_a{}^2 d\bar{x}+m_2 X_a{}^2(\frac{1}{2})}\right)\frac{\sinh(k_{e_a}\bar{x})}{\cosh(\frac{k_{e_a}}{2})}-\frac{k_{e_a}}{k_{f_a}}\frac{\sin(k_{f_a}\bar{x})}{\cos(\frac{k_{f_a}}{2})}\sin(\omega_a t) \tag{23}$$

In general, the effect of the nonlocal parameter μ is to increase the maximum dynamic deflections and reduce the frequencies, as can be seen from the results presented in Fig. 1.

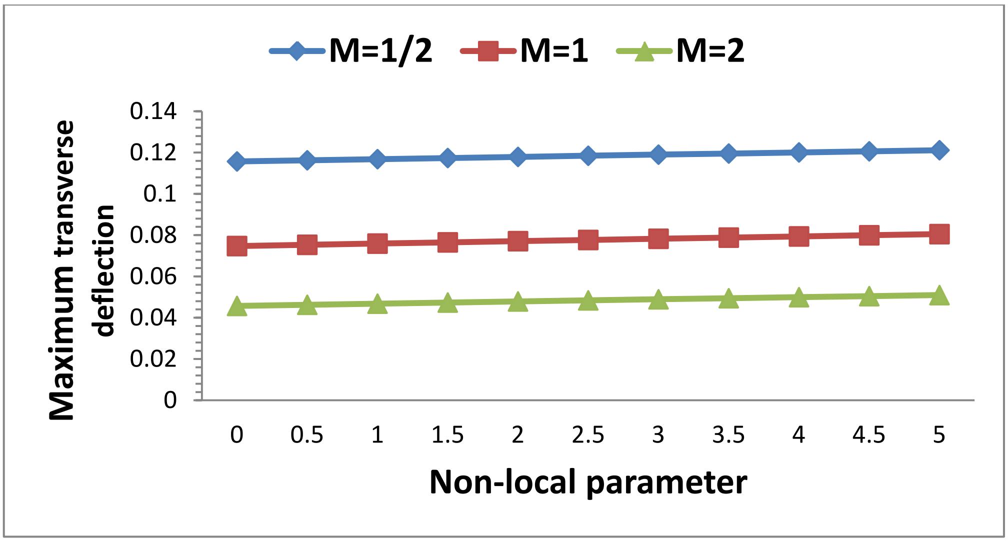

Figure 1. Comparison of maximum dynamic deflection versus nonlocal parameter for different values of *M* in simply supported nanobeams

3 Conclusion

In this article, dynamic analysis of low velocity transverse impact of a nanoparticle on nanobeams has been derived based on the nonlocal elasticity model by Eringen. The effects of the nonlocal parameter and the ratio of the mass of the beam to that of the striker on the dynamic responses of nanobeam are investigated.

The inclusion of the nonlocal effect increases the magnitudes of dynamic deflections and decreases frequencies. The maximum time parameter t_m, at which the maximum dynamic deflection occurs, is increased with increasing the nonlocal parameter.

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