OPTIMAL DESIGN OF DAMPING PROPERTIES OF HYBRID ELASTOMER-COMPOSITE PLATES

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Abstract
A global optimisation technique for the design of damping properties of hybrid elastomer/composite laminates is presented. The goal of the procedure is to maximise the first N modal loss factors of the laminate subject to constraints on the in-plane and out-of-plane stiffness along with a constraint on the weight of the plate. The problem is considered in the most general case: no simplifying hypotheses are made on the behaviour of the hybrid laminate, thus allowing us to consider as design variables the number of layers (both of the elastic and viscoelastic layers), their thickness and orientations as well as the position of the viscoelastic plies within the stacking sequence. As an example, the method is applied to a rectangular plate and the results demonstrate the effectiveness of the proposed strategy.

1 Introduction
Several works have been carried out on the study of damping properties of hybrid plates, shells and beams. Rather complete, but not exhaustive reviews on this subject can be found in [1-4]. Several numerical studies have been conducted on the effect of adding viscoelastic layers to vibrating beams and plates [5-7]. As it can be resumed from the state of the art, until now, the problem of designing the damping characteristics of the hybrid laminates has been stated considering as design variables only the thickness and orientations of the elastic plies along with the thickness and/or the material properties (shear modulus, material loss factor, density) of the viscoelastic layers. The main objective of the present work consists in determining also which are the best number of the constitutive layers of the hybrid laminate and the best positions of the elastomeric layers within the stacking sequence (along with the values of orientation and thickness for each ply) in order to maximise the damping properties of the structure. Moreover, constraints on the in- and out-of-plane stiffness along with a constraint on the total mass of the hybrid plate are considered in order to avoid the degradation of the mechanical properties and the increase of the weight of the structure.
The problem is formulated in the most general case: no simplifying hypotheses are made on the behaviour of the hybrid laminate and on the position of the viscoelastic plies within the stack, differently from which is usually done in literature where it is a-priori assumed that the positions of elastomeric layers within the stack are always located between two consecutive stiffer plies. In addition, since the material properties of the elastomeric plies depends on the frequency, the evaluation of the undamped eigenfrequencies and of the structural loss factors

leads us to consider a non-linear modal analysis, thus the Iterative Modal Strain Energy (IMSE) method is employed to overcome this difficulty.
In order to obtain a configuration that represents a global optimum and also to include the number and position of layers among the design variables we use, as optimisation tool, the genetic algorithm (GA) BIANCA (see [8, 9]) with crossover on species. The main difficulty, when dealing with the optimisation of modular structures, is how to take into account the variable number of modules, even in the case wherein the modules are non identical, as the case of hybrid laminates with variable number of plies made of different materials. As explained in Sec. 3, in the framework of GAs, this problem corresponds to the search of solutions in a design space made up of individuals with variable number of chromosomes and, hence, belonging to different species. For this purpose, we developed new genetic operators that perform the crossover and mutation operations among individuals of different species, see [9]. In this way the number of layers is directly related to the number of the individual's chromosomes and, hence, the optimal number of layers is an outcome of the genetic process, which automatically issues the best species. Moreover, during the optimisation process, the GA is coupled with the FE code ANSYS in order to evaluate the objective and constraint functions.

2 Problem description

2.1 Geometry and materials

The optimisation strategy presented in this work allows to find a solution for the problem of designing the damping properties of hybrid laminates and it is applied to a rectangular hybrid plate, whose dimensions are depicted in Fig. 1.

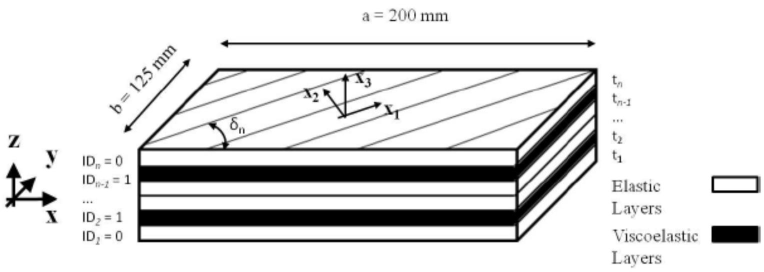

Figure 1. Geometry of the hybrid plate.

Concerning the typical dimensions of the plate, the thickness of each layer is constrained to remain sufficiently small compared to both width and length of the plate, in order to keep valid the assumptions of the thin plate model. Moreover, we assume that the fiber-reinforced plies have linear elastic orthotropic behaviour. The material used for the viscoelastic layers is a rubber-like material having linear isotropic behaviour. In addition, the properties of that material are considered dependent upon the loading frequency *f*. Introducing the fourth-order complex viscoelasticity stiffness tensor $\mathbf{D}^v$ the constitutive law is:

$$\sigma = D^v(f)\cdot\varepsilon \text{ with } \mathrm{D}^v(f) = D_r^v(f) + iD_i^v(f) \tag{1}$$

where $D_r^v(f)$ and $D_i^v(f)$ are the fourth-order tensors which characterise the energy storage and the dissipative response of the material, respectively. Moreover, the Young's modulus, the Poisson's ratio and the material loss factor are frequency-dependent. The variation of the Young's modulus with the frequency is expressed as:

$$E(f) = E_s + E_d \log\left(\frac{f}{\tilde{f}}\right) \tag{2}$$

Concerning the numerical value of all material properties, see [10].

2.2 Loading Conditions

The design of the hybrid laminate represents a compromise between its damping capability and the ability of keeping good mechanical properties in terms of stiffness, without increasing too much the weight. The dynamic response of the structure is evaluated through a classical free vibration analysis. Only the first $N = 5$ non-rigid modes are calculated considering free displacement boundary conditions on the edges of the plate. Since the material properties of the viscoelastic layers depend upon the frequency, the calculation of the eigenfrequencies, as well as the modal loss factors, is iterative for each eigenfrequency. To this purpose the IMSE method [4, 11], which is an extension of the MSE method originally introduced by Ungar and Kerwin [12], is employed to overcome this difficulty. The material properties are updated according to the adopted material law, in our case the law of eq. (2), at the value of frequency of the current iteration, within the vicinity of the considered natural mode. Once the convergence on the *i-th* undamped natural frequency is reached, the corresponding modal loss factor η_i is evaluated as:

$$\eta_i = \eta_v(f_i)\frac{W_v(f_i)}{W_{tot}(f_i)} \tag{3}$$

where $\eta_v(f_i)$ is the material loss factor at the current frequency, while $W_v(f_i)$ and $W_{tot}(f_i)$ are the strain energy of the viscoleastic layers and the total strain energy of the structure for the ith mode, respectively. The logical flow of the IMSE approach that we have implemented within the ansys environment is shown in Fig. 2. For more details, see [10].

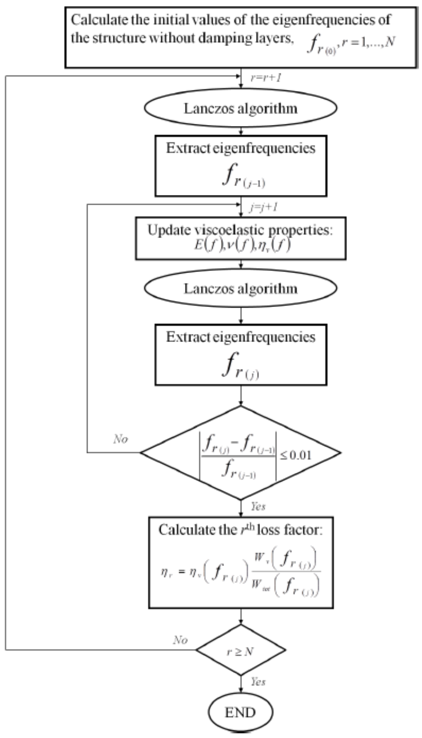

Figure 2. Flow of the IMSE strategy for the prediction of the loss factors of the structure.

2.3 Finite element model of the hybrid plate

Two different mechanisms of dissipating the vibratory energy can essentially be observed in viscoelastically damped structures (see [1]): the first dissipation phenomenon is linked to the shear strains, which are predominant in the constrained viscoelastic materials, while the second one is related to the direct normal strains, in the case of unconstrained viscoelastic materials. In order to predict these phenomena, 3D brick elements have been considered to model the rubber layers: we need to build a mathematical model able to describe (with a good level of accuracy and reliability) the mechanical response of the physical system. To this purpose the FE model of the hybrid plate has to be able to catch those aspects which normally, even with higher-order 2D theories, are not well described, e.g. the damping response associated to the shear strains through-the-thickness. Since the model is built in ANSYS environment, we use SOLID185 elements, which are solid elements with 8 nodes and 3 degrees of freedom (DOFs) per node. Moreover, this type of element is also employed for the elastic plies.

3 Formulation of the optimisation problem

In this section, the problem of designing the damping properties of a hybrid plate is stated as a constrained optimisation problem. The goal of our strategy consists in maximising the first N modal loss factors of the structure, without degrading the stiffness properties of the plate and increasing too much its weight. The problem is stated in the most general case, thus the design variables are:

- the total number of layers (both elastic and viscoelastic), n;
- the position and the number of the viscoelastic layers within the stack, which are directly linked to the variable ID_k, (k = 1, ..., n), that identifies the nature of the *k-th* ply, i.e. $ID_k = 1$ if the *k-th* ply is viscoelastic, $ID_k = 0$ otherwise;
- the thickness of each layer, t_k (k = 1, ..., n);
- the fiber orientation of the elastic plies, δ_k (k = 1, ..., n).

It is worth noting that, since the number of layers is included among the optimization variables, the total number of design variables of the whole optimisation process can change for each possible point-solution in the design space, or, in other words, the procedure determines by itself the optimal number of design variables.

3.1 Mathematical statement

The optimisation problem can now be established. The maximisation of the N first modal factors can be expressed as the minimisation of the following objective function:

$$\Phi = -\sum_{i=1}^{N} \eta_i \quad (4)$$

that represents the opposite of the sum of the first N modal loss factors. Moreover, the constraints on the maximum decrease of the stiffness properties and on the maximum increase of the mass of the plate have to be considered. Therefore, the constrained minimization problem can be stated as a classical Non-Linear Programming Problem (NLPP) as follows:

$$\min \Phi(n, ID_k, t_k, \delta_k) \quad (\text{with } k = 1,\dots,n),$$
$$\text{subject to}$$
$$\begin{cases} g_1(n, ID_k, t_k, \delta_k) = \dfrac{R_x^{ref} - R_x(n, ID_k, t_k, \delta_k)}{R_x^{ref}} - \varepsilon_x \leq 0\,, \\ g_2(n, ID_k, t_k, \delta_k) = \dfrac{R_y^{ref} - R_y(n, ID_k, t_k, \delta_k)}{R_y^{ref}} - \varepsilon_y \leq 0\,, \\ g_3(n, ID_k, t_k, \delta_k) = \dfrac{R_z^{ref} - R_z(n, ID_k, t_k, \delta_k)}{R_z^{ref}} - \varepsilon_z \leq 0\,, \\ g_4(n, ID_k, t_k) = \dfrac{M(n, ID_k, t_k, \delta_k) - M^{ref}}{M^{ref}} - \varepsilon_M \leq 0\,. \end{cases} \tag{5}$$

In eq. (5) R_x, R_y and R_z are the reactions of the plate which represent a measure of the stiffness of the structure, while M is the mass of the plate. The apex *ref* stands for reference value. The reference values of the reactions and mass are calculated, before the optimisation process, on a reference undamped structure, i.e. a plate without elastomeric layers. The quantities ε_x, ε_y, ε_z and ε_M are the user-defined tolerances on each constraint. The meaning of the constraints on the reaction forces and the mass of the hybrid plate are the following: the maximum loss in stiffness and the maximum increase in mass of the optimised structure are superiorly bounded by the value of the corresponding tolerances. It can be noticed that the NLPP of eq. (5) is highly non-linear and non-convex in the space of design variables.

3.3 Numerical strategy

The previous considerations on the nature and on the varying number of design variables involved into the optimisation process oriented our choice on GAs, as numerical tool, in order to search solutions of the problem (5). According to the metaphor adopted by GAs, each point in the design space corresponds to an individual whose genetic structure is composed of chromosomes and genes [13, 14]. When the object of the optimisation problem is a modular system, each constitutive module can be represented by a chromosome, each chromosome is composed of genes, and each gene represents a design variable related to the module. In agreement with the paradigms of natural sciences, individuals characterised by a different number of chromosomes, i.e. modular structures composed of different number of modules, belong to different species. In this work, we use the new version of the GA BIANCA, see [9], able to cross individuals belonging to different species. Specific operators of cross-over and mutation have been developed in order to perform reproduction over individuals having variable number of chromosomes: more details about the main features of this improved GA along with a description of the new genetic operators for the evolution of species can be found in [8, 9]. It is worth noting that, the constrained minimisation problem, formulated in Eq. (5), is transformed into an unconstrained one defining the penalised objective function. For more details, see [10].

4 Numerical results

In order to demonstrate the capabilities of our strategy we study the optimisation of the damping properties of a rectangular hybrid plate whose in-plane dimensions are those shown in Fig 1. In particular we performed the optimisation process in the most general case wherein no simplifying assumptions on the stacking sequence of the hybrid plate are made (thus, the total number of design variables depends upon the number of layers). The user-defined tolerances on the constraints of the problem (5) are set equal to 0.05, i.e. the maximum loss in stiffness and the maximum increase in mass between the optimised structure and the reference one are limited to 5%. The design variables, their nature and bounds are detailed in [10]. Due to the greater complexity of the optimisation process, the population size is fixed to $N_{ind} = 60$ and the maximum number of generations is assumed equal to $N_{gen} = 80$. The crossover and mutation probability are $p_{cross} = 0.85$ and $p_{mut} = 1/N_{ind}$, respectively. Selection is performed by roulette-wheel operator and elitism is active. The ADP method is used for handling constraints. The best solution found by BIANCA is shown in Table 1.

	Reference	**Best Solution**
n	6	6
ID_k		[1/1/0/0/0/0]
δ_k	[90/45/0] $_S$	[V/V/0/90/90/0]
t_k	[0.3/0.3/0.3] $_S$	[0.32/0.31/0.43/0.42/0.31/0.30]
η_1		0.01756
η_2		0.00483
η_3		0.01228
η_4		0.01066
η_5		0.01298
$f_1[Hz]$		70.09
$f_2[Hz]$		164.87
$f_3[Hz]$		217.90
$f_4[Hz]$		317.45
$f_5[Hz]$		346.97
$R_x[N]$	-17352	-17065 (-1.6%)
$R_y[N]$	-44480	-43688 (-1.7%)
$R_z[N]$	-23.06	-28.31 (+22.7%)
$M[Kg]$	0.0675	0.07 (+3.7%)
Φ		-0.05831
g_1		-0.03349
g_2		-0.03218
g_3		-0.27763
g_4		-0.013

Table 1. Best solution found by BIANCA for the optimization problem (5), V denotes the position of the viscoelastic ply.

The optimal number of plies is 6. Fig. 3 shows the variation of the best solution and of the best species along the generations: the global constrained minimum has been found after 62 generations, whilst the optimal number of plies is found after only 7 generations. The optimal configuration of the hybrid plate shows 2 viscoelastic plies at the top of the structure. Indeed, this is a non-conventional configuration: for this configuration the damping phenomenon, depending on the considered eigenfrequency, involves all the strain components. As conclusive remark, it can be noticed that such a solution is equivalent to a 5 layers solution with the following stack and thickness: [V/0/90/90/0] and [0.63/0.43/0.42/0.31/0.30], respectively.

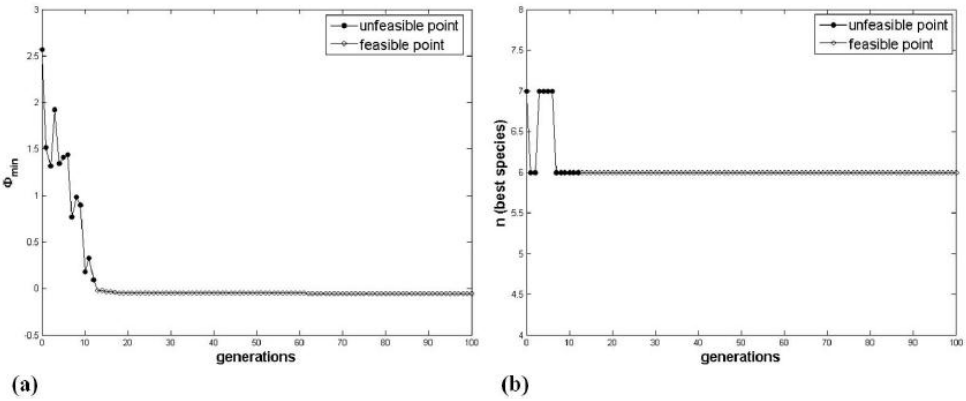

Figure 3. (a) Best values of the objective function and (b) number of layers along generations for problem (5).

4 Conclusions

In this work, an optimisation procedure for the design of damping properties of hybrid elastomer/composite laminates is presented. The goal of the procedure is to maximize the first N modal loss factors of the laminate subject to constraints on the in-plane and out-of-plane stiffness along with a constraint on the weight of the plate. The main key points of our strategy consist in determining which are: a) the best number of layers of the hybrid plate, and b) the best number and positions of the elastomeric layers within the stacking sequence. The main difficulty, when dealing with this kind of problems, is how to take into account the variable number of layers among the optimisation variables. In order to deal with such a problem we used our improved GA which presents new genetic operators that perform the crossover and mutation operations among individuals of different species. The use of an evolutionary strategy along with the fact that the problem is stated in the most general case, lead us to find some non-conventional configurations, i.e. non-constrained layer configurations, which show better damping properties when compared to the classical constrained layer treatments. The proposed approach appears to be very flexible and applicable to various engineering problems wherein the results are given by complex and expensive models and a high number of analyses is necessary to reach a suitable optimum. Moreover, the procedure has a high level of versatility: more constraints could be easily added to the optimisation problem, e.g. constraints on the strength, elastic symmetries, yielding or de-lamination of the hybrid plate, without reducing the power and the robustness of the proposed approach.

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