STRUCTURAL OPTIMISATION OF 3D COMPONENTS MANUFACTURED BY THE DIRECTED CARBON FIBRE PREFORMING PROCESS

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Abstract
This paper presents a model to intelligently optimise the fibre architecture of components manufactured from discontinuous fibre meso-scale composites. A stiffness optimisation algorithm is adopted to derive distributions of section thickness and constituent properties concurrently, whilst minimising material cost. A spare wheel well geometry is used to demonstrate the optimisation algorithm and the quality of the optimisation is assessed with the convergence of the structural performance. Use of structural optimisation facilitates properties closer to that of a textile fibre benchmark with little cost impact to the final component.
Keywords: DCFP, optimisation, FEA

1 Introduction

Directed Carbon Fibre Preforming (DCFP) is an automated process for producing complex 3D discontinuous carbon fibre preforms for liquid moulding. Fibre deposition is robotically controlled and net-shape, offering excellent repeatability and low wastage. DCFP offers greater design flexibility compared with conventional laminated composites, as the fibre orientation distribution, fibre volume fraction, and fibre length can all be continuously varied locally in the component according to the robot and chopping apparatus parameters. In addition, recent studies have shown that significant gains in mechanical performance can be achieved by introducing fibre alignment [1].

Current DCFP components however, are often manufactured as direct substitutes for existing metallic counterparts using the same geometry, assuming a homogeneous isotropic fibre architecture. The lack of suitable design tools prevents the full potential of these versatile materials from being exploited. Traditional optimisation routines are primarily concerned with structural issues, such as the overall weight and stiffness of the component. Topology, shape and size are the three main categories of structural optimisation, and a number of methods are well established for designing with isotropic materials. Non-linear programming algorithms, such as sequential programming [2] and the ‘method of moving asymptotes’ [3] are iterative mathematical approaches commonly utilised in optimisation routines. However, the application of these methods involves constant re-evaluation of the design objectives and constraints for each iteration, which makes the process very computationally expensive and consequently unsuitable for large structures.

Metaheuristic optimisation methods are alternative approaches which imitate natural phenomena. Genetic algorithms (GA) are the most widely used type of metaheuristic method in structural optimisation [4], but the computational time is strongly dependent on the size of the solution, which can be a problem for larger structures with continuous design variables. There are several other methods such as cellular automata [5] and optimality criteria approaches [6], which use simple local rules to update each design variable. These methods provide very similar solutions to the mathematical programming approaches, but with lower computational cost, and are therefore more practical for problems with complex or large-scale geometries.

With the continual development of composite materials, research into structural optimisation has been extended to suit composites, where material properties can be easily varied across a component. GAs are generally the preferred method for laminated composites [7]. The variation of the laminate lay-up sequence is greatly limited by the manufacturing route, and optimisation of the local thickness is difficult because of the ply-based lay-up. This simplifies somewhat the application of GAs to discrete problems with a very small population of searching points. Consequently the computational cost becomes insignificant for laminate based composites.

Structural optimisation of meso-scale discontinuous fibre architecture composites involves a combination of continuous and discrete design variables. The local thickness can be continuously varied across the component and is independent of the fibre architecture, whereas a continuous change of fibre length or tow size is unrealistic and therefore can only be varied in discrete regions. It is worth noting that the section stiffness of a structure is determined by its cross sectional dimensions and effective material properties, where the effective material properties for discontinuous fibre composites are a function of the fibre architecture. Therefore, the optimisation problem for DCFP can be solved in two stages: The first stage is to evaluate the optimised thickness contour of the component and the corresponding local material properties, using structural optimisation algorithms. The second stage is to determine the optimised fibre deposition strategy based on the structural optimisation results. This can be implemented using decision making methods such as weighted score and TOPSIS [8], providing the material properties database and the design preference. The spray strategy can be calculated following the optimisation and the results can be compared back to the ideal case.

This paper aims to establish a structural optimisation algorithm to evaluate the thickness contour and local assignment of material properties, utilising the design flexibility of DCFPs. A stiffness optimality criteria is derived and the method of Lagrangian multipliers is adopted to determine the local recurrent criteria. The local section thickness and stiffness values are updated concurrently through an iterative process, with a material cost model employed to determine the contribution of thickness and modulus increments during each iteration. The algorithm is validated using a semi-structural automotive component for a niche vehicle. The optimisation quality is assessed by comparing the component stiffness, deflection and final mass for the optimised design against a baseline textile solution.

2 Structural Optimisation Methodology

2.1 Problem definition

The stiffness is often the primary issue in most structural design problems. Stiffness optimisation aims to achieve the stiffest structure whilst fulfilling all of the design constraints, so that the overall deflection of the part is minimised under the prescribed load case. The objective of maximising the structural stiffness is equivalent to minimising the total strain energy within the structure [9]. It has been proposed in [10] that, for a single load case evaluation subjected to a constant volume constraint, the total strain energy is minimised when the strain energy density distribution through the part is uniform. However, this average strain energy density criteria was derived assuming the material properties are constant. When designing a shell-like structure the local thickness is normally the only variable to be updated during a size optimisation process. The use of DCFP has introduced the local effective modulus of the material as an additional design variable, therefore a new stiffness optimality criteria has been determined to optimise the thickness and modulus values concurrently.

With the additional stiffness design variable, a second constraint is required to determine the limits when updating the local modulus values. Restricting the material cost is a reasonable approach, since increasing the section thickness requires a larger quantity of material to be used, while increasing the modulus potentially requires increased fibre volume fraction or a smaller fibre tow size, further increasing the unit cost of the material. The mechanical performance for meso-scale discontinuous fibre architecture composites is linked to the homogeneity of the bundle ends and the number of fibre to fibre contacts [11] and this therefore means that smaller, more expensive tows make stronger and stiffer components. The optimisation problem can be constructed as

$$\begin{aligned} &\text{min} && U(E, t) \\ &\text{subject to} && V(t) = V_0,\ C(E, t) = C_0 \\ &\text{and} && E \geq E_{min},\ t \geq t_{min} \end{aligned} \tag{1}$$

where E and t denote the modulus and thickness design variables respectively. U denotes the total strain energy in the structure. V and C denote the overall volume and material cost of the structure, and V_0 and C_0 are the target volume and cost. E_{min} is the lower bound of the modulus, which has been taken from the literature [11], and t_{min} is the lower bound of thickness value to prevent local buckling of the structure. The minimum thickness is influenced by the lower modulus bound, since the stiffness and strength of the component changes with thickness due to the homogeneity effects.

2.2 Stiffness optimality criteria and the Lagrangian multiplier approach

The optimisation process is performed based on the results from finite element analyses of the structure. The overall strain energy, component volume and material cost can be individually expressed as a summation of the corresponding value from each element in the part. The optimality criterion is derived by solving the Karush–Kuhn–Tucker (KKT) conditions of the Lagrangian expression. The Lagrangian expression from equation (1) is

$$L = U + \lambda_1(V - V_0) + \lambda_2(C - C_0) + \lambda_3(E_{min} - E_i) + \lambda_4(t_{min} - t_i) \tag{2}$$

where λ_1, λ_2, λ_3 and λ_1 are the Lagrange multipliers corresponding to each constraint. The subscript i denotes the element number. The stationary of the Lagrangian leads to the following KKT conditions:

$$\frac{\partial L}{\partial E} = \sum \frac{\partial U_i}{\partial E_i} + \lambda_2 \sum \frac{\partial C_i}{\partial E_i} + \lambda_3 = 0 \tag{3}$$

$$\frac{\partial L}{\partial t} = \sum \frac{\partial U_i}{\partial t_i} + \lambda_1 \sum \frac{\partial V_i}{\partial t_i} + \lambda_2 \sum \frac{\partial C_i}{\partial t_i} + \lambda_4 = 0 \tag{4}$$

If the lower bounds for modulus and thickness are inactive, then λ_3 and λ_1 are both equal to zero. It was stated in [12] that for an optimal design problem with the number of variables equal to the number of active constraints, the solution yields a fully utilised design. The stationary conditions in this case produce *n* constraint equations with *n* unknowns, and each can be solved independently of the Lagrangian multipliers. (i.e. each constraint equation is sufficient to determine one design variable). Therefore, Equation (3) and (4) can be rearranged as:

$$\frac{-\sum \frac{\partial U_i}{\partial E_i}}{\lambda_2 \sum \frac{\partial C_i}{\partial E_i}} = 1 \quad , \quad \frac{-\sum \frac{\partial U_i}{\partial t_i}}{\lambda_1 \sum \frac{\partial V_i}{\partial t_{i_2}}} = 1 \qquad (5), (6)$$

The optimality criteria can be employed based on the expressions derived in Equation (5) and (6) with an iterative scheme. The recurrence relations for modulus and thickness may be written as:

$$E_i^{k+1} = \left(\frac{-\frac{\partial U_i}{\partial E_i}}{\lambda_2 \frac{\partial C_i}{\partial E_i}} \right)^{\frac{1}{r}} E_i^k \quad , \quad t_i^{k+1} = \left(\frac{-\frac{\partial U_i}{\partial t_i}}{\lambda_1 \frac{\partial V_i}{\partial t_i}} \right)^{\frac{1}{n}} t_i^k \qquad (7), (8)$$

where the superscript *k* denotes the iteration number, and *r* and *n* are the moving limit parameters to define the step size of each iteration.

The algorithms described in Equation (7) and (8) require the partial derivatives of *U*, *V* and *C* to be calculated for each element at each iteration. Assuming the section forces and moments applied to each element remain constant, the elemental strain energy can be written as:

$$U_i = \frac{1}{2}[d_i]^T[K_i][d_i] = \frac{1}{2}[F_i]^T[K_i]^{-1}[F_i] \tag{9}$$

where $[d_i]$ and $[F_i]$ are the element displacement and force vectors, and $[K_i]$ is the section stiffness matrix of the *i*th element. Therefore, the partial derivatives of U_i are:

$$\frac{\partial U_i}{\partial E_i} = \frac{1}{2}[F_i]^T \frac{\partial [K_i]^{-1}}{\partial E_i}[F_i] \quad , \quad \frac{\partial U_i}{\partial t_i} = \frac{1}{2}[F_i]^T \frac{\partial [K_i]^{-1}}{\partial t_i}[F_i] \qquad (10), (11)$$

The elemental volume is simply a function of thickness, thus

$$\frac{\partial V_i}{\partial E_i} = 0 \quad , \quad \frac{\partial V_i}{\partial t_i} = A_i \qquad (12), (13)$$

where A_i is the area of the *i*th element.

The material cost model employed here assumes a basic linear relationship between the material cost and the modulus value, such that:

$$C_i = \alpha E_i A_i t_i \tag{14}$$

where the factor α can be estimated by calculating the material cost for a range of DCFP laminates with known properties. The current optimisation method uses a constant cost as an equality constraint, thus the evaluation of the actual cost value is unnecessary. In a more refined solution, a more extensive database of material properties and costs would be used to provide greater fidelity. The value of α is taken as unity in the current study, which denotes the ratio of the current cost to the constraint value. Therefore;

$$\frac{\partial C_i}{\partial E_i} = A_i t_i \quad , \quad \frac{\partial C_i}{\partial t_i} = E_i A_i \qquad (15), (16)$$

The values of the Lagrangian multipliers λ_1 and λ_2 need to be correctly determined such that the overall volume and cost constraints are satisfied at every iteration, i.e.

$$\sum \left(\frac{-\dfrac{\partial U_i}{\partial t_i}}{\lambda_1 \dfrac{\partial V_i}{\partial t_i}} \right)^{\frac{1}{r}} t_i^k A_i = V_0 \quad , \quad \sum \left(\frac{\dfrac{\partial U_i}{\partial t_i} \dfrac{\partial U_i}{\partial E_i}}{\lambda_1 \lambda_2 \dfrac{\partial C_i}{\partial E_i} \dfrac{\partial V_i}{\partial t_i}} \right)^{\frac{1}{n}} t_i^k A_i E_i^k = C_0 \qquad (17), (18)$$

Substituting Equation (10), (11), (12), (13), (15) and (16) into Equation (17) and (18) yields a system of two non-linear equations with two unknowns λ_1 and λ_2, where λ_1 can be determined by Equation (17), and the value of λ_2, E_i^{k+1} and t_i^{k+1} can be calculated subsequently.

The design variables are also subjected to bound constraints, E_{min} and t_{min}. A reduced step size method is adopted here to prevent the updated value of each design variable exceeding its limit. If the output value at the current step falls below the lower bound when updating E_i and t_i at the *k*th step (using Equations (7) and (8)), the step size parameter (*r* or *n*) is adjusted globally until the bound limits are satisfied.

Solving the Lagrangian multipliers is the classical approach to solving optimisation problems with equality constraints. Other optimality criteria methods do exist [13], but they don't calculate the value of the Lagrangian multipliers and every iteration is performed in two steps. The first step solves the unconstrained problem by estimating a value for the objective function, and the second step is to rescale the design variables so that the constraints are satisfied. The rescaling method can often slow down the convergence, and in some cases convergence may not be achieved.

3 Case Study

3.1 Application description

An automotive component has been selected to demonstrate how the model can be successfully used to optimise the fibre architecture of discontinuous fibre components. The component represents the floor structure at the rear of the vehicle within the boot (trunk) area, which primarily houses the spare wheel, but also a pair of 12V batteries. The 'spare wheel well' (see Figure 1) is required to be sufficiently stiff to avoid NVH issues within the passenger cell, but also needs to be durable, as the exterior surface is exposed to impact from road debris. The production component comprises an inner and outer skin with local areas of foam core to create a sandwich construction. The layup has been simplified into a monolithic component (ignoring cores) for the purpose of this study and the assumed layup is presented in Figure 1. A single ply of 650gsm 2×2 twill weave carbon fabric is sandwiched about 3 plies of 600gsm 3×1 twill weave glass, in order to prevent a conductive path between the battery terminals and the body in white.

A simple replacement for the glass/carbon woven composite has been proposed using an all-carbon fibre layup. The ply count has been reduced to just three 650gsm 2×2 twill weave carbon fibre plies, in order to provide a comparable maximum deflection to the glass/carbon hybrid. However the anticipated reduction in component mass cannot be justified in this case because of the added material cost, which is typically greater than 50%. An optimised DCFP replacement has therefore been designed using the current algorithm, to demonstrate that the performance of DCFP can compete against continuous fibre solutions, but at a fraction of the cost. The material cost for DCFP is approximately 50% lower than a textile solution because no intermediate processing is required and there is no fibre wastage. The process is also fully automated, which eliminates all touch labour to create further cost savings.

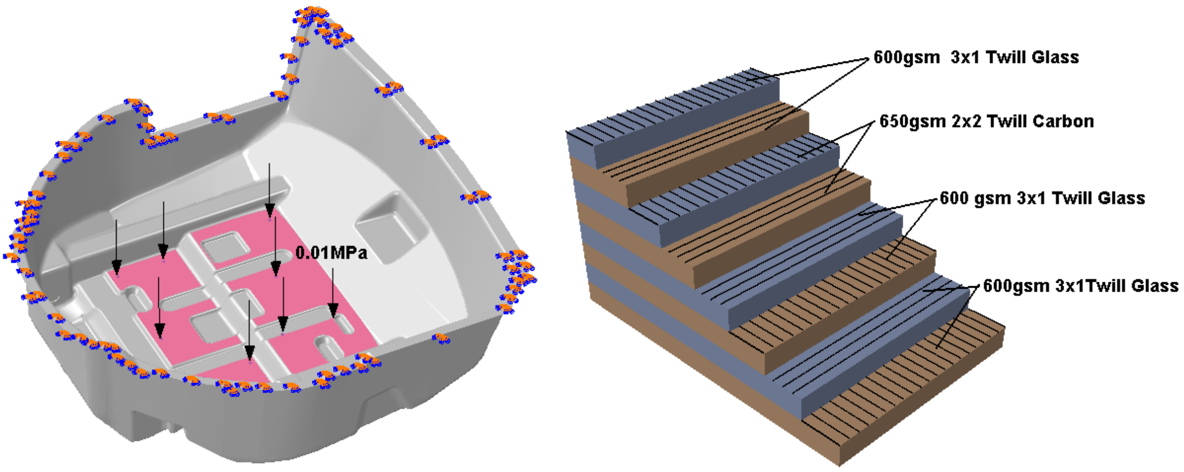

Figure 1 Left: Un-deformed shape of the spare wheel well. (0.01MPa uniform pressure applied on the pink surface) Right: Ply layup arrangement of the current composite laminate.

Table 1 Material properties for composites layup model and initial values for DCFP model.

	E_1 (MPa)	E_2 (MPa)	Nu_{12}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)	Ply thick (mm)	Density Kg/m^3
3x1 Twill Glass Fibre	43400	9770	0.256	4090	4090	3560	0.44 *	1946
2x2 Twill Carbon Fibre	132500	10530	0.256	4444	4444	3787	0.66**	1534
DCFP (6K, 60mm, 36% vf)	30000	30000	0.3	11540	11540	11540	2.00	1421

* Idealised laminate contains 2×0.22mm UD plies.
** Idealised laminate contains 2×0.33mm UD plies.

3.2 Finite Element modelling

Structural optimisation is performed using FEA of the component at the macro level. The CAD geometry of the wheel well is defined as a 3D conventional shell, and exported into ABAQUS CAE for mesh generation and analysis. Conventional shell elements in ABAQUS represent the geometry of a reference surface, and the thickness and material properties can be defined though the shell section definition. The optimisation strategy can be adopted for both 6-node (STRI65) and 8-node (S8R5) quadratic elements.

The model is subjected to a static load case using appropriate boundary conditions and is analysed in ABAQUS/Standard. A pressure of 0.01MPa has been applied to the flat regions of the wheel well and an encastre constraint has been applied to the top rim, as indicated in Figure 1. The mid surface of the wheel well is defined as the reference surface. The section stiffness for the benchmark model is defined by creating and assigning a composite layup in ABAQUS CAE, simplifying the twill weave plies into individual unidirectional plies at 0º and 90º. The section stiffness for the DCFP model is defined by specifying the shell thickness and the effective composite properties using a *DISTRIBUTION tab, where the effective composite properties are defined by isotropic material definitions with linear elastic stress-strain relationships. The thickness and modulus values are updated on an element by element basis during the optimisation process. In addition, the lower bounds for the design variables are t_{min}=1mm and E_{min}=15,000MPa. The material properties for both models are summarised in Table 1.

3.3 Results

A summary of the FE results is presented in Table 2 for both the continuous fibre benchmark model and the optimised DCFP model. The performance is compared in terms of the total strain energy, maximum deflection, total mass and specific stiffness. It is worth noting that at this stage, since the fibre architecture has not been determined, the variation of the effective composite density is ignored. Therefore, the constant volume constraint is effectively a constant mass constraint in this example (the mass of the two DCFP models is the same).

Rapid convergence is observed for both the strain energy and the deflection; with both values converging approximately after the third iteration. Figure 2 includes the von Mises stress plot and the deflection plot for the benchmark model (glass/carbon) and the optimised DCFP model. The optimised structure exhibits lower peak and mean values for the von Mises stress and the magnitude of the deflection is reduced.

According to Table 2, moving towards an all-carbon fibre woven solution over the glass/carbon hybrid reduces the maximum deflection from 3.85mm to 2.62mm, whilst also reducing the component mass by 15%, resulting in an increase in specific stiffness of 70%. Whilst this is an attractive solution, high material costs prohibit this from being commercially viable. Table 2 indicates that an optimised DCFP model can compete very well against a woven carbon solution at a fraction of the cost. The deflection of the un-optimised DCFP model is 3.66mm, which is similar to that of the continuous glass/carbon hybrid (specific stiffness of 1.3). However, the specific stiffness increases to 1.6 following the optimisation, which compares very well against the value of 1.7 for the continuous carbon fibre model. The current optimisation method reduces the total strain energy for the DCFP model by 16.8% and the maximum deflection by 17.8% during the optimisation process. Optimisation yields a 24% reduction in mass compared with the glass/carbon benchmark.

The thickness and modulus distributions for the optimised DCFP model are presented in Figure 3. Local changes in thickness and modulus appear in the regions where local stress concentrations exist (as shown in Figure 2(a) and (b)). These continuously variable distributions are currently unrealistic from a manufacturing perspective, since the fibre deposition process is difficult to control to this level of precision. Discrete regions of high stiffness material need to be applied over much larger areas. This can be addressed with future developments, by introducing a zoning algorithm to group and then smooth areas of similar moduli. Conversely, a continuously varying thickness can be achieved by varying the cavity height of matched tooling.

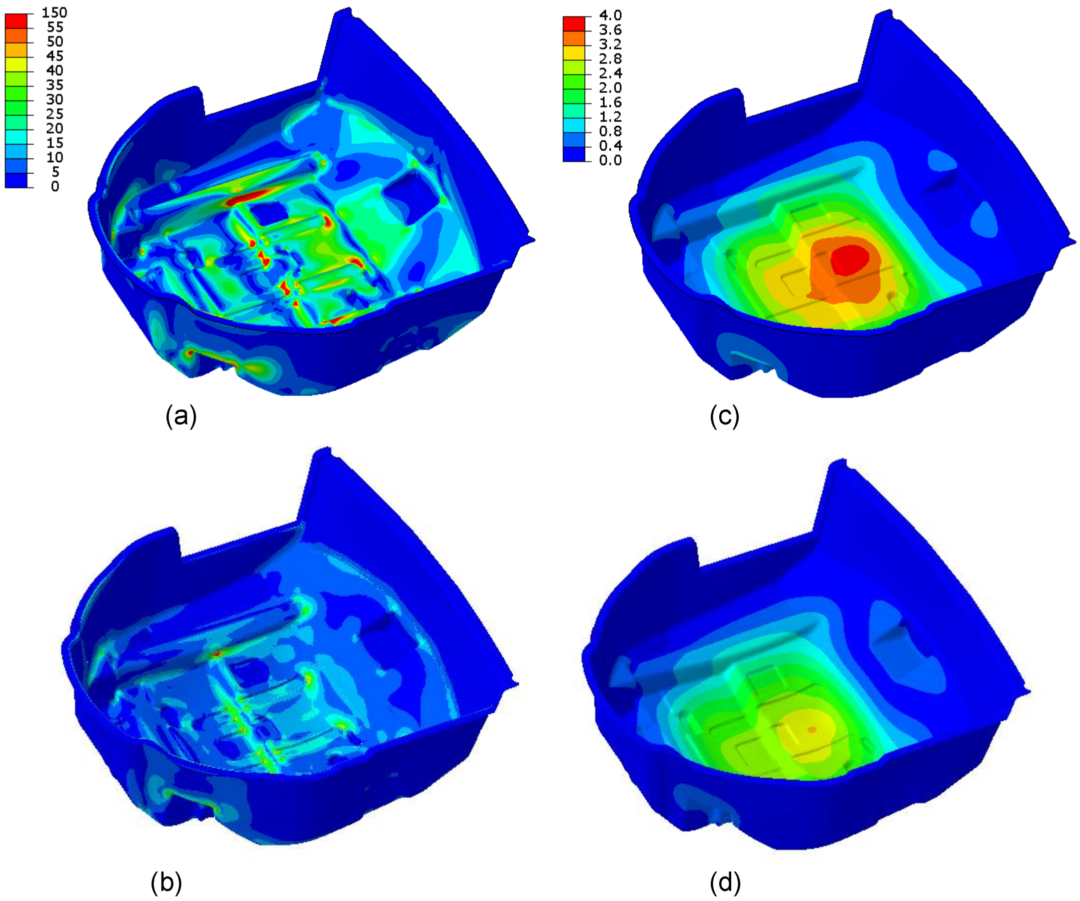

Figure 2 The von Mises stress (MPa) contour for (a) continuous glass/carbon fibre benchmark, (b) optimised DCFP. The magnitude of deflection (mm) contour for (c) benchmark and (d) optimised DCFP.

It should be noted that the method proposed in this paper is only concerned with optimising the stiffness of the component at this stage. In the case of any other design objective, such as strength or impact requirements, new optimisation criteria must be derived for the new problem. It is anticipated that the mass saving will be affected when optimising for these additional design objectives, as the section thickness will locally increase.

Table 2 FEA results for the benchmark model and the optimised DCFP.

	Total Strain Energy (kN.mm)	Max Deflection (mm)	Total Mass (kg)	Specific Stiffness* (unity)
Continuous Glass/carbon	1215	3.85	4.18	1
Continuous Carbon	837	2.62	3.56	1.7
DCFP (un-optimised)	1230	3.66	3.16	1.3
DCFP (optimised)	1023	3.01	3.16	1.6

*Values normalised to the benchmark case of continuous glass/carbon

4 Conclusions

A stiffness optimisation method has been proposed to concurrently optimise the thickness and modulus distribution for a DCFP component. The stiffness of the component is maximised under the constraints of constant volume and material cost. The optimisation algorithm is derived using the stiffness optimality criteria, and the Lagrangian multipliers are solved for each optimisation constraint.

The optimisation method has been demonstrated using a spare wheel well model. The component was originally designed with laminated carbon and glass fibre composites. Results have suggested that the current method can effectively improve the structural stiffness of the component and hence reduce the strain energy and deflection when the component is subjected to a prescribed load case.

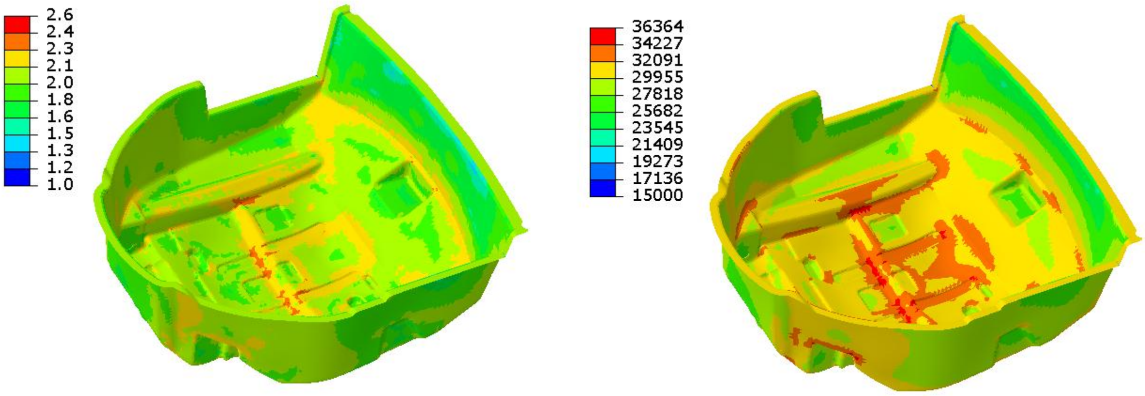

Figure 3 Thickness map (left, unit: mm) and modulus map (right, unit: MPa) for the optimised DCFP model.

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