NONLINEAR DYNAMIC BEHAVIOUR OF TAPERED SANDWICH PLATES SUBJECTED TO BLAST LOADING

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Abstract

The dynamic behaviour of simply supported tapered laminated sandwich plates subjected to the air blast loading is investigated numerically. The theory is based on the classical sandwich plate theory including von Kármán large deformation effects, in-plane stiffness and inertias, and shear deformation. The equations of motion for the plate are derived by the use of the virtual work principle. Approximate solution functions are assumed for the space domain and substituted into the equations. The Galerkin method is used to obtain the nonlinear differential equations in the time domain. The finite difference method is applied to solve the system of coupled nonlinear equations. The displacement-time and strain-time histories are obtained. The theoretical results are compared with ANSYS finite element results and are found to be in an agreement.

1 Introduction

The plate and shell structures are used in many engineering applications. A sandwich structure is a class of composite material which is manufactured by attaching two thin laminated stiff faces to a light and low strength thick core. Sandwich structures are especially used in the construction of advanced supersonic and hypersonic flight vehicles subjected to several dynamic loadings such as blast loads. For this reason, predicting the transient dynamic response of plates subjected to time dependent loads is very important to design more reliable structures for the aerospace and marine industry. The plates could be optimized to minimize their weight. A common way of the optimization of the plates is to give a taper ratio. In many cases, they are produced by using a proper taper ratio to obtain optimum structures.

Several analytical and numerical studies have been done about response of flat sandwich panels and plates subjected to time dependent loads [1-9]. Hause et al. investigated the dynamic response of doubly curved anisotropic sandwich panels with constant thickness subjected to blast loadings [10]. Studies on plates with variable thickness are generally restricted to the static and free vibration analysis of isotropic and orthotropic tapered plates [11-17]. In these studies, free vibration frequencies and mode shapes of tapered plates are determined using the numerical techniques. Thomsen et al. presented a developed high order sandwich theory which enables the analysis of sandwich beams and plates with linear variable core thickness and tested on bending loads [18]. However, there is not a study reported on dynamic response of the air blast loading on tapered sandwich composite plates.

In this study, the dynamic behavior of simply supported tapered laminated sandwich plates subjected to the air blast loading is investigated numerically. The theory is based on the classical sandwich plate theory including the large deformation effects, such as geometric nonlinearities, in-plane stiffness and inertias, and shear deformation. The geometric nonlinearity effects are taken into account by using the von Kármán large deflection theory of thin plates. The equations of motion for the plate are derived by the use of the virtual work principle. Approximate solution functions are assumed for the space domain and substituted into the equations of motion. The Galerkin method is used to obtain the nonlinear differential equations in the time domain. The finite difference method is applied to solve the system of coupled nonlinear equations. The displacement-time and strain-time histories are obtained on certain points through the tapered direction by concerning about the effect of taper ratio on dynamic response. The results obtained by using the present method are compared with the ones obtained by using a commercial finite element code ANSYS. The results are found to be in an agreement.

2 Formulation of the problem

The blast loaded tapered sandwich plate is modelled using both theoretical and numerical method. In the theoretical method, a closed form solution is considered by assuming the proper displacement functions for the plate. In the numerical method, ANSYS software is used to obtain the finite element model (FEM) of the plate.

2.1 Theoretical method

The tapered rectangular sandwich plate with the length a, the width b, and the blast load are depicted in Figure 1. The thickness denoted by $h(x)$ is varying only in the x direction for both core and face sheets and shown in Equation (1). The taper ratio is defined by β and the Cartesian axes are used in the derivation of the formulation.

$$h(x) = h_0(1+\beta \frac{x}{a}) \tag{1}$$

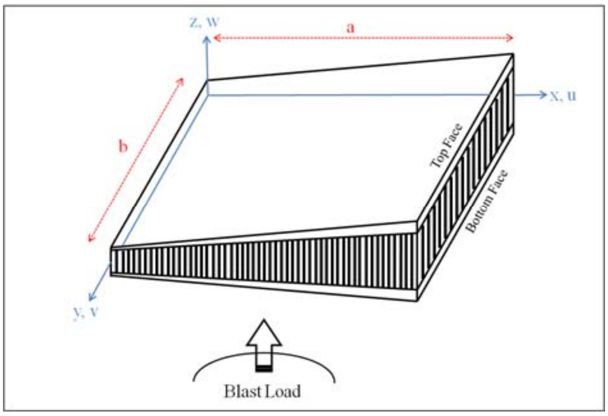

Figure 1. The schematic view of the tapered sandwich plate subjected to the blast load.

The mid-plane displacement in the z direction, w is separated into contributions due to bending and shear in Equation (2) [1]. The constitutive equations can be obtained as

Equations (3) and (4) by using the reduced stiffness coefficients for a layer of the laminated composite and stress components. Force and moment components of plate in closed form are shown in Equation (5).

$$w = w_b + w_s \tag{2}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^k \tag{3}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{44} & 0 \\ 0 & \bar{Q}_{55} \end{bmatrix}^k \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k \tag{4}$$

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{5}$$

A_{ij}, B_{ij} and D_{ij} are the extensional, coupling and bending matrices, respectively. Besides, the following expressions are obtained for stiffness matrices of the sandwich plate. The coefficients in the matrices in Equation (5) are calculated by using Equation (6) and (7).

$$A_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k - h_{k-1}) \qquad i,j = 1,2,6; \quad A_{ij}^{core} = Q_{ij}^{core} h_c \qquad i,j = 4,5 \tag{6}$$

$$\begin{aligned} A_{ij}^{san} &= A_{ij}^{face1} + A_{ij}^{face2} \qquad i = 1,2,6 \\ A_{ij}^{san} &= A_{ij}^{core} \qquad i = 4,5 \\ B_{ij}^{san} &= \frac{h_c}{2}\left(A_{ij}^{face1} - A_{ij}^{face2} \right) \qquad i = 1,2,6 \\ D_{ij}^{san} &= \left(\frac{h_c + h_{f1}}{2} \right)^2 A_{ij}^{face1} + \left(\frac{h_c + h_{f2}}{2} \right)^2 A_{ij}^{face2} \qquad i = 1,2,6 \end{aligned} \tag{7}$$

Using the constitutive equations and the strain-displacement relations for the von Kármán plate in the virtual work and applying the variational principles, nonlinear dynamic equations of a laminated composite sandwich plate can be expressed in terms of mid-plane displacements in Equation (8). The explicit expressions of the linear and nonlinear operators L_{ij} and N_i are given in Kazancı [1]. $\bar{m}$ is the mass of unit area of the mid-plane, q_x, q_y and q_z are the load vectors in the axes directions.

$$\begin{aligned} L_{11}u^0 + L_{12}v^0 + L_{13}w_b^0 + L_{14}w_s^0 + N_1(w^0) + \bar{m}\ddot{u}^0 - q_x &= 0 \\ L_{21}u^0 + L_{22}v^0 + L_{23}w_b^0 + L_{24}w_s^0 + N_2(w^0) + \bar{m}\ddot{v}^0 - q_y &= 0 \\ L_{31}u^0 + L_{32}v^0 + L_{33}w_b^0 + L_{34}w_s^0 + N_3(u^0, v^0, w^0) + \bar{m}\ddot{w}^0 - q_z &= 0 \\ L_{41}u^0 + L_{42}v^0 + L_{43}w_b^0 + L_{44}w_s^0 + N_4(u^0, v^0, w^0) + \bar{m}\ddot{w}^0 - q_z &= 0 \end{aligned} \tag{8}$$

The boundary conditions for a simply supported plate can be given in the following form. Initial conditions are given by Equation (10).

$$\begin{gathered}
u^0(0,y,t) = u^0(a,y,t) = u^0(x,0,t) = u^0(x,b,t) = 0 \\
v^0(0,y,t) = v^0(a,y,t) = v^0(x,0,t) = v^0(x,b,t) = 0 \\
w^0(0,y,t) = w^0(a,y,t) = w^0(x,0,t) = w^0(x,b,t) = 0 \\
M_x = 0 \text{ at x = 0, a} \\
M_y = 0 \text{ at y = 0, b}
\end{gathered} \tag{9}$$

$$\begin{aligned}
&u^0(x,y,0) = 0, \qquad v^0(x,y,0) = 0, \qquad w^0(x,y,0) = 0 \\
&\dot{u}^0(x,y,0) = 0, \qquad \dot{v}^0(x,y,0) = 0, \qquad \dot{w}^0(x,y,0) = 0
\end{aligned} \tag{10}$$

Approximate solution functions are chosen crucially by considering the results of static large deformation analysis of laminated composite and shown in Equation (11). Only the first term of the series for in-plane displacements and first two terms for out-of-plane displacements for the simply supported plate are accounted.

$$\begin{aligned}
u^0 &= U(t)\sin\frac{2\pi x}{a} y^2(y-b)^2 \\
v^0 &= V(t)x^2(x-a)^2 \sin\frac{2\pi y}{b} \\
w_b^0 &= W_{1b}(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + W_{2b}(t)\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b} \\
w_s^0 &= W_{1s}(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + W_{2s}(t)\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b}
\end{aligned} \tag{11}$$

The plate is subjected to the blast load only in the z direction. The load is expanded in Fourier series but only the first term is chosen. The blast load is assumed to be varying exponentially in time and Friedlander decay function is used to express the air blast load as shown in Equation (12). Here, p_m is the peak pressure, t_p is positive phase duration, and α is a waveform parameter.

$$p(x,y,t) = p_m(1 - t/t_p)e^{-\alpha t/t_p} \tag{12}$$

The time dependent nonlinear differential equations are obtained by applying the Galerkin method to the equations of motion given in Equation (8). The nonlinear-coupled equations of motion are then solved by using the finite difference method. Finally, the equations of motion are reduced into a form that can be easily solved by one of the methods for solution of linear equation systems such as LU decomposition.

2.2 The finite element model of the tapered sandwich plate

The blast loaded simply supported tapered sandwich plate is modelled using ANSYS finite element software. The plate is discretized using 14x14 eight nodded layered shell elements

(Shell281) which have the geometric nonlinearity capability. The number of the elements is chosen by a mesh sensitivity analysis. Transient response analysis is based on the Newmark method. The function, *h(x)* is used to describe the variation of the thickness. The finite element model is constructed for each taper ratio considered in this study and for the flat plate separately. The air blast loading, given in Equation (12), is applied on the plate.

3 Numerical results

The dynamic behaviour of simply supported tapered sandwich composite plates is obtained using the closed form solution by writing a FORTRAN program and using FEM. The pressure distribution because of the blast loading, p is assumed to be uniform on the plate, and the parameters shown in Equation (12) are p_m=29 kPa, t_p=0.0018 s and α=0.35. Only square shaped plates are considered in this paper and the dimensions are taken as a=b=225 mm.

	Honeycomb (Core)	Glass/Epoxy (0°/90°) Fabric (Face Sheet)
E_1 (GPa)	0.0296	10
E_2 (GPa)	0.0145	10
ν_{12}	0.3	0.18
G_{12} (GPa)	0.014	4
ρ (kg/m^3)	32	1800

Table 1. Material properties of the constituents [1].

A sandwich plate, that has a honeycomb core and face sheets of one layer fiber-glass fabric with (0°/90°) fiber orientation angle, is designed (Figure 1). Mechanical properties of constituents are given in Table 1. The taper ratios are chosen as β=0 (flat plate), β=0.2 and β=1.2 to understand the effect of taper ratio on the dynamic response. The thickness of the core is 5.08 mm and the thickness of the two face sheets is 0.23 mm for the flat plate. Thus, the plate thickness is taken as 5.54 mm at the center of the plate for all configurations so that the weight of the plate is kept constant. The minimum thickness, h_0 can be calculated by using Equation 1.

The results of present study are compared with the results of Kazancı [1] and FEM solutions. Figure 2 shows comparison of displacement-time histories for certain points and strain-time histories at the middle surface of the center for the flat plate. It is found that although there is a small discrepancy between the histories obtained by using the present method and the FEM, the present study suppresses the shift because of adding one term to each approximate displacement functions, while a shift occurred between Kazancı [1] and ANSYS because of using only one term. Increasing the number of terms of displacement functions increases the number of time dependent nonlinear differential equations too.

The displacement-time histories of the points x=3a/14, x=7a/14 and x=11a/14 through the center line of the plate (y=b/2) are obtained. Figure 3 illustrates displacement-time histories for β=0.2 while Figure 4 presents histories for β=1.2. The displacement-time histories obtained by using the present method are found to be in an agreement with the FEM results. The theory has better agreements with ANSYS for smaller displacement values and smaller β values as shown in from Figure 2 to Figure 4. Additionally, the strain-time histories are obtained at the middle (ε_{xm} - t), bottom (ε_{xb} - t) and top (ε_{xt} - t) surface of the center of the plate and shown for β=0.2 and β=1.2 in from Figure 5 to Figure 7. The strain-time histories obtained by using the present method are found to be in an agreement with the FEM results.

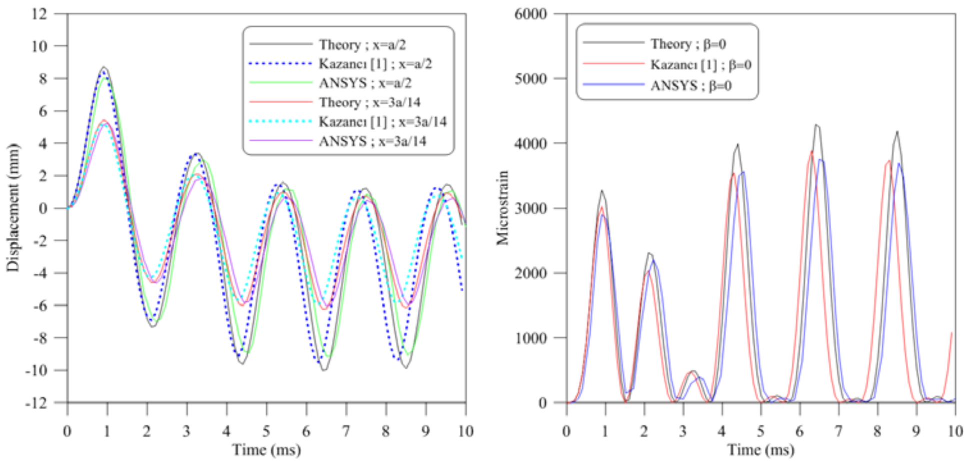

Figure 2. Comparison of displacement-time and strain-time histories

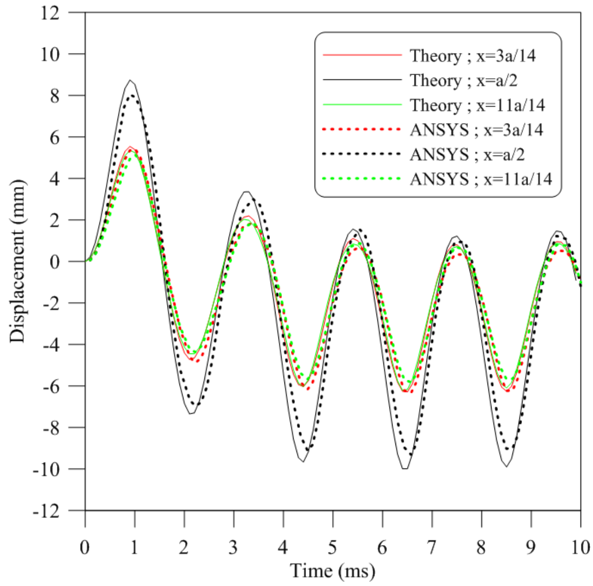

Figure 3. The displacement-time histories of certain points for β=0.2.

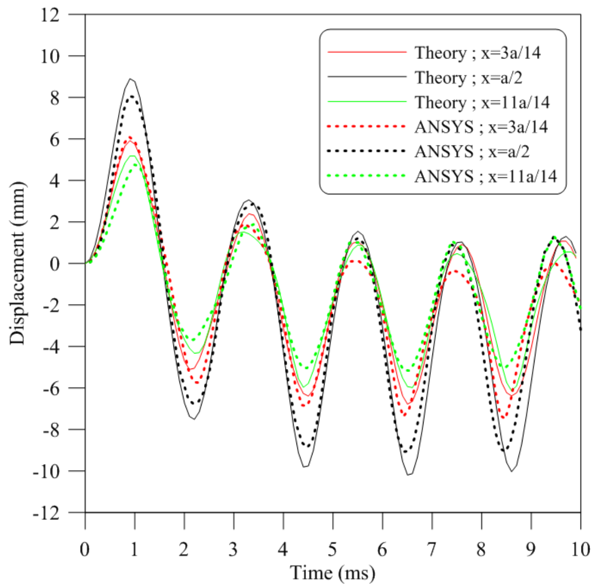

Figure 4. The displacement-time histories of certain points for β=1.2.

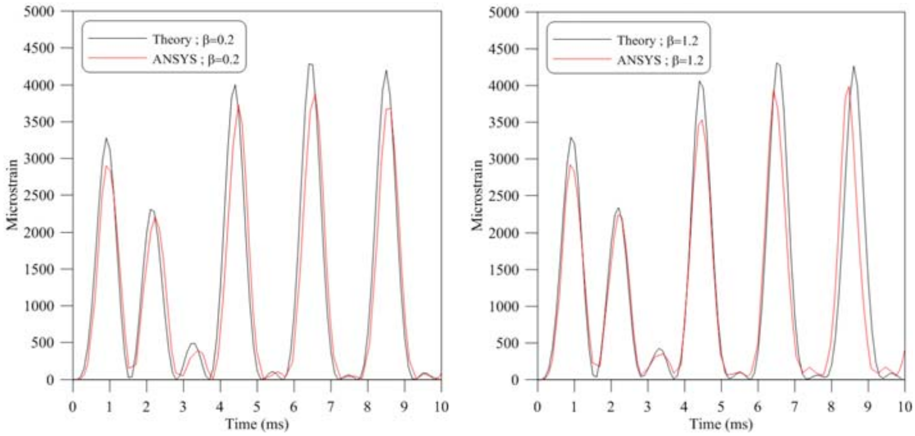

Figure 5. The strain-time histories (ε_{xm}-t) at the middle surface of point (a/2, b/2) for β=0.2 and β=1.2.

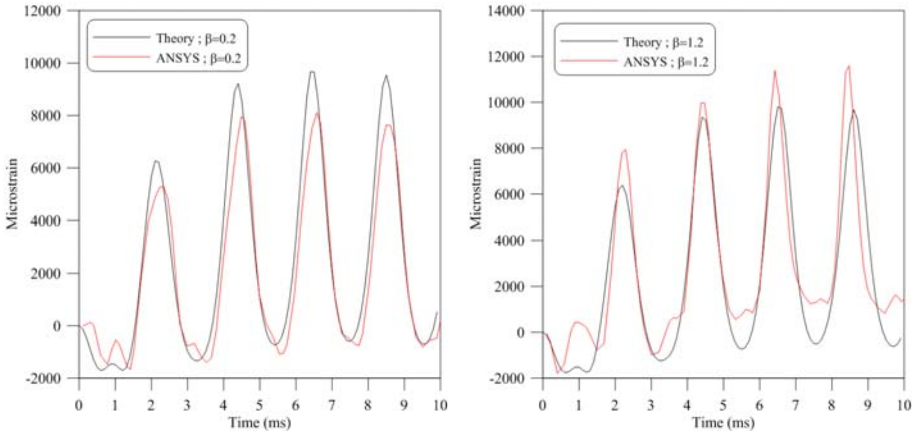

Figure 6. The strain-time histories (ε_{xb}-t) at the bottom surface of point (a/2, b/2) for β=0.2 and β=1.2.

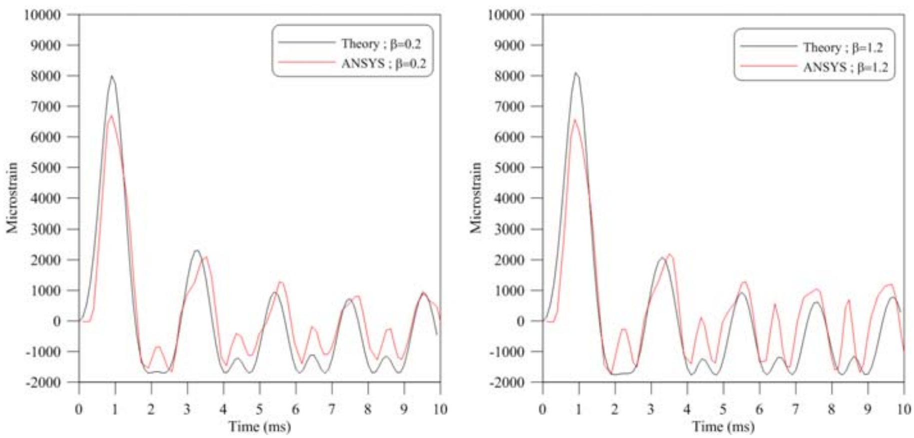

Figure 7. The strain-time histories (ε_{xt}-t) at the top surface of point (a/2, b/2) for β=0.2 and β=1.2.

4 Conclusion

In this study, the dynamic response of tapered sandwich composite plates subjected to the air blast loading is investigated theoretically and numerically. The theoretical results are compared with ANSYS finite element results. It is shown that there is an agreement between the displacement-time and strain-time histories predicted using both methods. This study can be extended to the analysis of the sandwich plates with parabolically changing taper ratios subjected to different type of dynamic loads. Number of layers on both top and bottom faces could be increased, so that the effects of number of laminates, fiber orientation angle and stacking sequence on the response of plate apart from taper ratio can be analyzed. The effect of the blast pressure character on the dynamic behavior can also be investigated.

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