VISCOELASTIC BEHAVIOR OF COMPOSITE SANDWICH BEAM IN BENDING

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An analysis of viscoelastic bending of sandwich beam, consisting of elastic bearing layers and viscoelastic core, based on Volterra integral equations with exponential kernels and Volterra resolvent operators. The space and time operations are separated and Fourier series are applied. Theory is verified by experimental data from three- point bending.

1 Introduction

Polymer foam-core sandwich panels are increasingly being considered for load-bearing components in buildings. Sandwich panels are offering a high stiffness per unit weight and excellent thermal insulation and may be easily mass-produced [1]. But such sandwiches from polymers creep at room temperature, what is limiting their use in structural applications. In this paper we model the creep of sandwich beams with linear viscoelastic polymer foam cores. A previous study indicated that the creep of a polymer foam can be described knowing the creep response of the solid from which it is made and the relative density of the foam [2],[3]. In this study, we combine the viscoelastic model for foam creep with the standard analysis of deflection of a sandwich beam to develop an expression for the creep of a sandwich beam with a polymer foam core. The results are compared with data from a series of tests on sandwich beams with polymer based composite and rigid foam core.

2 Formulation and solution of the problem

Paper deals with integral equations Volterra and kernels mostly of exponential type.

Further, for simplicity, a symmetrical structure of the simply supported beam is considered.

The load $q(x,t)$ is acting in direction of beam thickness z according to relation

$q(x,t) = g(x)\,\Phi(t)$ where $\Phi(t)$ is Heaviside function.

Below, for the simplicity, the symmetric structure of the simply supported beam is considered.

Let us treat by beam core, where the shear stress τ is of main importance and define $\tau = \varphi_1(x,t)$, $\sigma_z = \varphi_2(x,t) - z\,\varphi_{1,x}$ where $\varphi_2 = 0$ can be considered. The relations satisfy the equilibrium condition in z direction z. Further

$u_{z,z} = \frac{\hat{C}}{E_c}\left(-z\,\varphi_{1,x}\right)$, $\quad u_{x,z} + u_{z,x} = \frac{\tilde{C}}{G_c}\,\varphi_1$, where time integral operators are of form $C = 1 + \lambda \;\; C^*(\alpha)$, and parameters λ, α are changing in dependence on adopted mechanical model. After integration we obtain

$$u_x = z\left(\frac{\tilde{C}}{G_c}\,\varphi_1 - w_{,x}\right) + \frac{z^3}{6}\frac{\hat{C}}{E_c}\,\varphi_{1,xx}$$

On this basis the similar relations for bearing layers are derived. Here we will define the integral operators S, S^{-1} for a model with structural equation $E - (E \,/\, K)$.

Decisive component of stress in bearing layer is σ_x . Relation $\sigma_x = E\,S\,\varepsilon_x$ is valid and after some arrangement

$$\sigma_x = E\,S\left[\pm s\left(\frac{\tilde{C}}{G_c}\,\varphi_{1,x} - \varphi_{3,xx}\right) \mp \frac{s^3}{6}\frac{\hat{C}}{E_c}\,\varphi_{1,xxx} - z\,w_{,xx}\right]$$

kde $\quad S = I - \frac{E}{K}\;E^*\left(-\frac{2E}{K}\right) \qquad S^{-1} = I + \frac{E}{K}\;E^*\left(-\frac{E}{K}\right)$

More advantageous formulation we can get on basis of generalized function $\omega(x,t)$, where we consider

$$\varphi_1 = s_0\,S\,\omega_{,xxx} \;, \qquad w = -\frac{\omega}{E\,r} + \frac{s\,S\,C_2}{G_c}\,\omega_{,xx} - \frac{s^3\,S\,C_1}{3E_c}\,\omega_{,xxxx}$$

Now one basic equation is satisfied identically and the second is transformed into form

$L\,\omega = -q$, where operator L is expressed by relation

$$L = \frac{1}{6}\,S\,\frac{d^4}{dx^4}\left[12 s_0^{\,2} + r^2 - \frac{r^3\,s\,E\,S}{G_c}\,\frac{d^2}{dx^2}\left(C_2 - \frac{s^2\,G_c\,C_1}{3E_c}\,\frac{d^2}{dx^2}\right)\right]$$

Boundary conditions for simple support are given by relations

$$\frac{d^{2k}}{dx^{2k}}\,\omega = 0 \qquad k = 0,1,2,3$$

At solving the problem for load $q(x,t) = g(x)\;\Phi(t)$ we will apply Fourier expansion of functions $g(x,t)$ a $\omega(x,t)$.

$$g(x) = \sum_m g_m \sin\frac{m\pi x}{l} \;, \qquad \omega(x,t) = \sum_m \omega_m\,\Phi(t)\,\sin\frac{m\pi x}{l} \qquad (m = 1,2,3,\ldots)$$

By substitution into the basic equation we obtain

$$\omega_m = \frac{6\frac{l^4}{m^4\pi^4}\,g_m}{12\,s_0^{\,2} + r^2 + \frac{r^3\,s}{G_c}\,E\,S\;\frac{m^2\pi^2}{l^2}\left(C_2 + \frac{s^2}{3}\frac{G_c}{E_c}\frac{m^2\pi^2}{l^2}\;C_1\right)}\;\frac{1}{S\,\Phi(t)}$$

As illustration we give expression for vertical displacement for the middle of the beam ($x = \frac{l}{2}$).

$$w = -\sum_m \left[\frac{1}{E\,r} + \frac{s}{G_c}\; m^2 \frac{\pi^2}{l^2} S \left(C_2 + m^2 \frac{\pi^2}{l^2} \frac{s^2}{3} \frac{G_c}{E_c} C_1 \right) \right] \Phi(t)\; \omega_m \; sin \frac{m\pi}{2}$$

$$m = 1,2,3,....$$

After substitution we obtain the relation

$$w = -\frac{6l}{\pi^4 E} \sum_m g_m \frac{1}{m^4} \frac{\frac{l}{r} + \frac{s}{l}\frac{E}{G_c} m^2\pi^2 S \left(C_2 + \frac{s^2}{3}\frac{G_c}{E_c}\frac{m^2\pi^2}{l^2} C_1 \right)}{\frac{12{s_0}^2 + r^2}{l^2} + \frac{r^3 s}{l^4}\frac{E}{G_c} m^2\pi^2 S \left(C_2 + \frac{s^2}{3}\frac{G_c}{E_c}\frac{m^2\pi^2}{l^2} C_1 \right)} \frac{\Phi(t)}{S\,\Phi(t)} \sin\frac{m\pi}{2}$$

For concentrated load we can introduce a generalized function $\omega(x,t)$ instead of functions $\varphi_1(x,t),\; w(x,t)$ by substitution $\omega_{,xxx} = -2 s_0\, \varphi_1$ and further

$$w = \frac{\omega}{2E\,r\,{s_0}^2} - \frac{\tilde{C}}{2G_c\, s_0}\,\omega_{,xx} - \frac{s_0 \hat{C}}{12E_c}\,\omega_{,xxxx}$$

By applying the boundary conditions $\omega = \omega'' = 0$ we receive for the middle of the beam

$$w\left(\frac{l}{2},t\right) = \frac{\overline{P}\, l^3}{48E.2r{s_0}^2} + \frac{\tilde{C}\,\overline{P}\, l}{8G_c\, s_0}$$

Operator $\tilde{C}$ is of the form $\tilde{C} = 1 + \lambda \;\; C^*(\alpha)$. Parameters λ, α are changed according to adopted mechanical model. For the Poynting-Thomsonův model (linear solid) and arrangement $G_c - G_x / K_2$ it will be valid

$$\tilde{C}\,\Phi(t) = \Phi(t) + \frac{G_c}{K_2} \int_0^t e^{-\frac{G_x}{K_2}(t-\tau)} \Phi(\tau)\, d\tau = 1 + \frac{G_c}{G_x}\left(1 - e^{-\frac{G_x}{K_2} t} \right)$$

Relation $\tilde{C}\,\Phi(t)$ will be substituted into the above equation:

$$w\left(\frac{l}{2},t\right) = \frac{\overline{P}\, l^3}{48E.2r{s_0}^2} + \frac{\tilde{C}\,\overline{P}\, l}{8G_c\, s_0} = \frac{P\,l}{8b\,s_0}\left(\frac{l^2}{12E\,r\,s_0} + 1 + \frac{G_c}{G_x}\,(1 - e^{-\frac{G_x}{K_2}t}) \right)$$

2 Experimental evaluating of creep for the core

For mechanical testing has been delivered a sandwich plate with bearing layers from composite, reinforced by glass fibres and polyester matrix. Cube testing sample was loaded by constant load in a set-up developed and manufactured in Klokner Institute. Transversal displacements were measured by LVDTs and strain gauges. Testing samples were loaded and unloaded at least 100 h. The course of strain for 24h loading and 24 h unloading is shown on Fig.1.

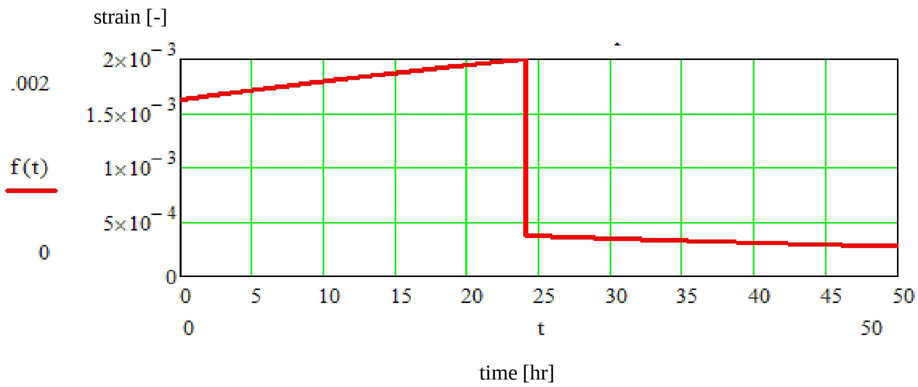

Figure 1 Strain vs time in compression of sandwich core

Theoretical course of strain vs time according to Poynting- Thomson model has relationship

$$\varepsilon = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{E_2}\left(1 - e^{-\frac{E_2}{K}t}\right)$$

and strain is depending on three material parameters E, which can be evaluated by collocation method. For the material in Fig.1 the following values have been found:
E1=24.615 N/m2, E2=45.714 N/m2, K=1961.0 N/m2.hr

3 Experimental evaluating of vertical deflection at tree-point bending

Testing samples were put into a set-up for three point testing and loaded by a constant load 300, 600 and 800 N. Distance between supports was 200 mm, length of sample 300 mm, width 50 mm. Vertical deflection has been measured by LVDT sensors and data acquisition system National Instruments NI 1052 in program environment LabWindows. Measured values have been evaluated by MS Excel and compared with theoretical values. The comparison of results shows good agreement (< 10%). Results of long-term testing of sandwich beam are shown in Fig.2.

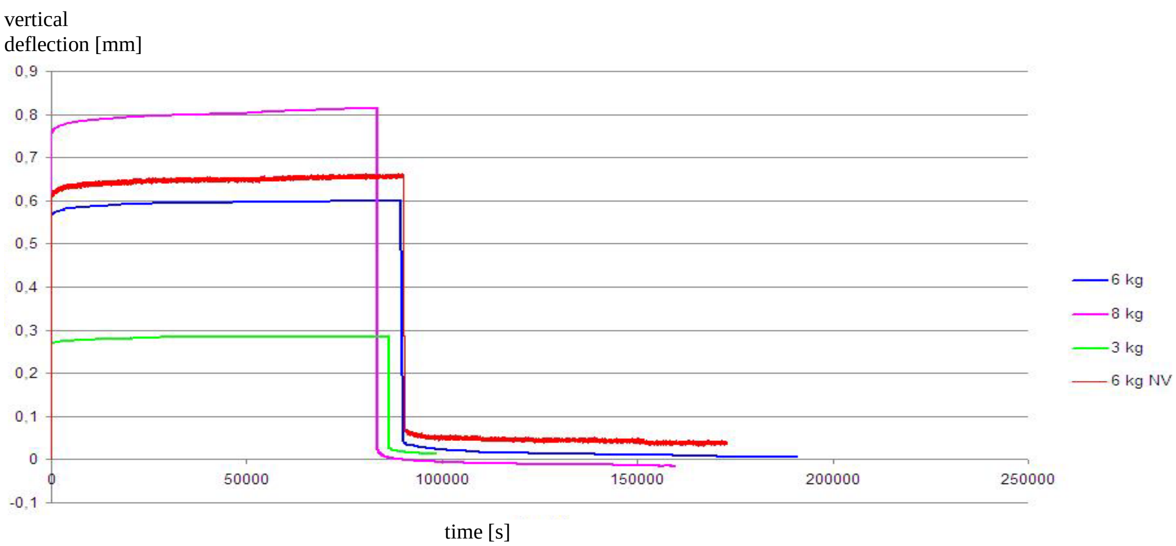

Figure 2. Results of long-term loading of sandwich beams

4 Conclusion

A theory of viscoelastic bending for sandwich beams subjected to arbitrary loading has been given. Sandwich beam has symmetrical structure with elastic bearing layers and viscoelastic core. Theory is based on Volterra integral equations with exponential kernels and Volterra resolvent operators. The space and time operations are separated and Fourier series are applied. Theory is verified by experimental data from three- point bending.

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