THE EFFECT OF TRANSVERSE LOAD BIAXIALITY ON THE FIBRE-MATRIX DEBOND INITIATION IN FRC. APPLICATION OF A COUPLED STRESS AND ENERGY CRITERION AT THE MICROMECHANICAL LEVEL

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Keywords: matrix failure, crack initiation, failure criterion, finite fracture mechanics

Abstract
Crack initiation at the fibre-matrix interface under transverse loading is studied by applying a coupled stress and energy criterion in the framework of the Finite Fracture Mechanics. This criterion assumes that a debond onset of a finite length occurs when a stress condition is fulfilled along the assumed path of the new crack and the crack onset is energetically allowed. The effect of the load biaxiality on this crack initiation is studied, showing only a moderate influence of the secondary load with respect to a primary tension load. This effect appears to be larger for brittle than for tough fibre-matrix configurations .

1. Introduction

In spite of the responsibility of fibre-reinforced composite materials in industrial structures, failure in these materials is not sufficiently understood yet. In particular, matrix failure, associated to predominant load perpendicular to the fibre-axis (called transverse load), shows a high complexity. The stages of this failure mechanism are well-known [1]: first, microdebonds appear at the fibre-matrix interface (or very close to it) which propagate subsequently along this interface. Eventually these microdebonds kink out the interface to the matrix coalescing with other microcracks which can produce the failure of the ply. The present work focuses on the first step: the crack initiation at the fibre-matrix interface. The problem of a debond along the fibre-matrix interface under a remote transverse loads $(\sigma_x^\infty, \sigma_y^\infty)$ has been largely studied during decades. However, the debond initiation was not intensively studied until recently when computational methods as Cohesive Zone Model [2, 3] or Linear Elastic Brittle Interface Model [4] were used. Additionally, a theoretical model was developed in [5] to predict a debond onset of a finite length at the fibre-matrix interface under uniaxial transverse load σ_x^∞ by applying the coupled criterion [6] of the Finite Fracture Mechanics (FFM). This criterion assumes that a crack onset of a finite length occurs when tractions along the assumed crack path exceed a critical value (stress criterion) and the crack onset is energetically allowed (energy criterion).

Bimaterial	E_1(GPa)	ν_1	E_2(GPa)	ν_2	α	β	ε	E^* (GPa)	k	m
glass/epoxy	70.8	0.22	2.79	0.33	0.919	0.229	-0.074	6.01	1.44	1.56

Table 1. Isotropic bimaterial constants for the glass/epoxy studied (1, fibre; 2, matrix)

The influence of a secondary transverse load σ_y^∞, perpendicular to the primary one σ_x^∞, see Figure 1, has been remarked in [7]. In spite of this, this influence is not taken into account by the majority of failure criteria found in the literature for these materials. The aim of this work is generalize the theoretical model developed in [5] to take into account the secondary transverse load in order to quantify the influence of this secondary transverse load on the critical value of the primary remote load leading to the debond onset. The initial problem, before the debond onset, is modelled as a single fibre perfectly bonded to an infinite matrix considering both materials as linear elastic isotropic at the plane perpendicular to the fibre-axis, see Figure 1(a). In this work, a glass/epoxy is studied as example, see Table 1. A presupposed debond after its onset is represented in Figure 1(b). Note that, it is symmetric with respect to the x-axis (primary load direction). As a consequence, only its upper half part is studied, therefore, angles at the interface are defined as $\theta, \Delta\theta \geq 0$. On the contrary the presupposed debond is not symmetric with respect to the y-axis in accordance with the analysis carried out in [8].

First, both stress and energy criteria are applied to this problem in Sections 2 and 3, respectively. Then, conditions imposed by both criteria are combined in order to obtain the minimum remote load which fulfills both criteria in Section 4, where main results are discussed as well.

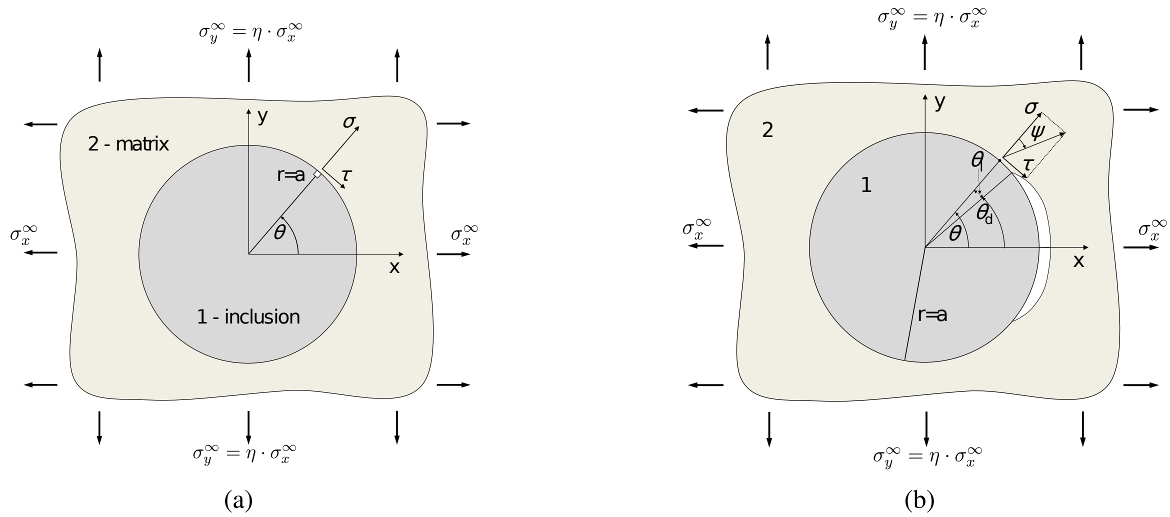
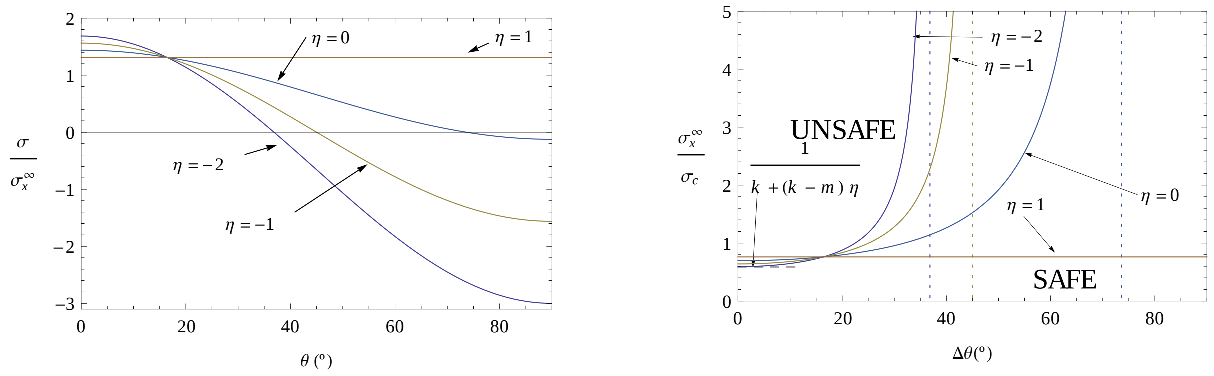


Figure 1. Schema of the problem (a) before and (b) after the debond onset.

2. Stress criterion

Stress criterion used here imposes that a debond onset is possible if normal tractions σ along the entire presupposed debond path before the onset exceed a critical value σ_c. In the present case the presupposed debond path is the interface, so σ and $\sigma_c > 0$ are, respectively, the normal tractions along the fibre-matrix interface and the tensile strength of this interface. This condition



(a) (b)

Figure 2. (a) Dimensionless normal and tangential tractions along the fibre-matrix interface as functions of the angle θ for glass/epoxy. (b) Plot of the stress criterion for the debond semiangle $\Delta\theta$ and the critical remote load σ^{∞}_{cx} for glass/epoxy.

is expressed as,

$$\sigma(\theta) \geqslant \sigma_c, \qquad \text{for } 0 \leq \theta \leq \Delta\theta, \tag{1}$$

where θ is the polar angle of each point at the interface and $\Delta\theta$ is the debond semiangle after the onset, see Figure 1.

Normal tractions along the fibre-matrix interface before the debond onset can be calculated from Goodier's solution [9] as a function of the primary and secondary loads, σ^{∞}_x and σ^{∞}_y, the bimaterial properties and the polar angle θ of an interface point,

$$\frac{\sigma(\theta)}{\sigma^{\infty}_x} = k + (k - m)\eta - (1 - \eta)m \sin^2(\theta), \tag{2}$$

where η, named biaxiality parameter in the following, is the ratio of the secondary to primary remote loads,

$$\eta = \frac{\sigma^{\infty}_y}{\sigma^{\infty}_x}, \tag{3}$$

and k y m are bimaterial elastic constants defined in [5] as a function of the Dundurs parameters,

$$k(\alpha,\beta) = \frac{1}{2}\frac{1+\alpha}{1+\beta}\frac{2+\alpha-\beta}{1+\alpha-2\beta}, \tag{4a}$$

$$m(\alpha,\beta) = \frac{1+\alpha}{1+\beta}. \tag{4b}$$

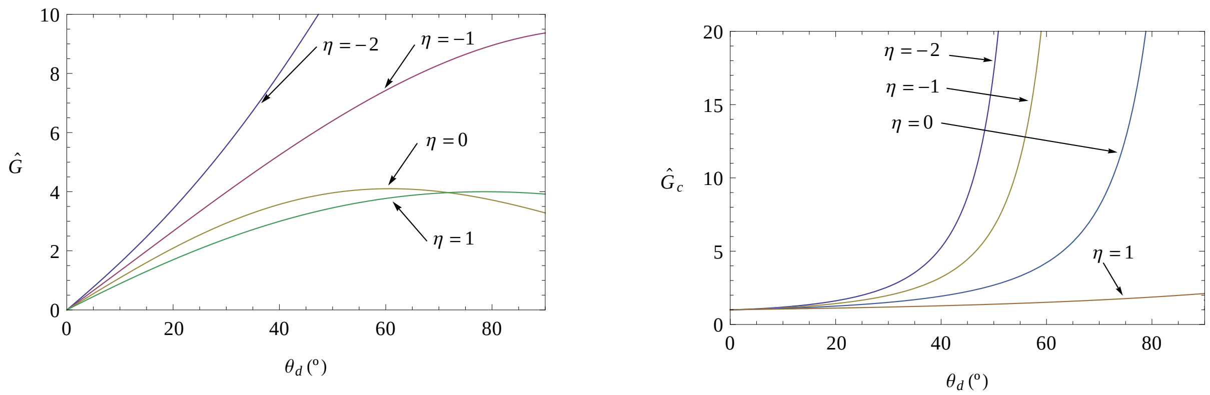
Figure 2(a) shows the normal tractions along the fibre-matrix interface given by Goodier's solution as a function of θ and the biaxiality parameter η. In the following, it will be assumed that $\sigma^{\infty}_x > 0$ and $\sigma^{\infty}_y \leq \sigma^{\infty}_x$, this implies $\eta \leq 1$. It is interesting to remark that normal tractions around the point $\theta = 0°$ increase when a secondary remote compression $\sigma_y < 0$ is applied. However for points far from $\theta = 0°$, the effect of a secondary remote compression is reversed and normal tractions therein decrease with a secondary compression. For some values of η, a point θ_0 exists at the interface where normal tractions vanish and for points $\theta > \theta_0$ compressions at the interface are observed, therefore, according to (1), $\Delta\theta \leq \theta_0$.

Then, taking into account that σ is decreasing for $0^\circ \leq \theta \leq 90^\circ$ as demonstrated in [10] and introducing the analytic solution for normal tractions (2) in the definition of the stress criterion in (1), the final expression of this criterion writes as

$$\frac{\sigma_x^\infty}{\sigma_c} \geqslant \frac{1}{k + (k-m)\eta - (1-\eta)m\sin^2\Delta\theta}. \tag{5}$$

This condition is plotted in Figure 2(b), where it is seen that this criterion defines a minimum remote load σ_x^∞ originating the debond onset as a function of the debond semiangle $\Delta\theta$ and the biaxiality parameter η. This figure shows that for small debond semiangles $\Delta\theta$ a remote compression makes easier the debond onset, whereas for the large $\Delta\theta$ its effect is opposite. A deeper analysis of the implications of this criterion in this problem can be found in [10].

3. Energy criterion



(a) (b)

Figure 3. (a) Dimensionless Energy Release Rate $\hat{G}$ (ERR) and (b) dimensionless interface fracture toughness $\hat{G}_c$ as functions of the debond semiangle θ_d for glass/epoxy and several values of the biaxiality parameter η.

Energy criterion is based on applying the first law of thermodynamics to the energetic balance between the state before, Figure 1(a), and after, Figure 1(b), the debond onset,

$$\Delta\Pi + \Delta E_k + \Delta\Gamma = 0, \tag{6}$$

where $\Delta\Pi$ and ΔE_k are, respectively, the changes of the potential elastic and kinetic energy stored in the body and $\Delta\Gamma$ is the energy dissipated associated to the irreversible processes during the debond onset. The change in the potential elastic energy can be related to the Energy Release Rate (ERR) G by means of the classical relation $G = -\frac{\mathrm{d}\Pi}{\mathrm{d}(2a\theta_d)}$ where θ_d is an "instantaneous" debond semiangle. Assuming quasistatic initial state: $\Delta E_k \geq 0$. The energy dissipated $\Delta\Gamma$ can be estimated by integrating the interface fracture toughness G_c. Hence, the energy balance in (6) leads to

$$\int_0^{\Delta\theta} G(\theta_d, \eta)\mathrm{d}\theta_d \geqslant \int_0^{\Delta\theta} G_c(\psi(\theta_d, \eta))\mathrm{d}\theta_d. \tag{7}$$

ERR G is given from Toya's analytic solution [10, 11], assuming the open model of interface cracks, in the form

$$G(\theta_d; \sigma_x^\infty, \eta; a; E^*, \alpha, \beta) = \frac{(\sigma_x^\infty)^2 a}{E^*} \hat{G}(\theta_d; \eta; \alpha, \beta), \tag{8}$$

where $E^* = 2/\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)$. Plots of dimensionless ERR $\hat{G}$ as a function of the "instantaneous" debond semiangle for glass/epoxy and several values of η are shown in Figure 3(a). Note that, in general, $\hat{G}$ increases when the secondary remote load σ_y^∞ decreases. G_c can be approximated by the phenomenological law of Hutchinson and Suo [12]

$$G_c(\psi, G_{1c}, \lambda) = G_{1c}\hat{G}_c(\psi, \lambda) = G_{1c}\left(1 + \tan^2 [(1 - \lambda)\psi]\right), \tag{9}$$

where ψ is the stress-based fracture mode mixity at the crack tip[1] extracted from Toya's analytic solution for stresses [10, 11] for an "instantaneous" debond semiangle θ_d which would correspond to a slow growing of the debond initiated at $\theta = 0°$. ψ can be defined in an alternative manner avoiding the assumption of a presupposed debond growing sequence [14]. λ is a mode sensitivity parameter and G_{1c} is the interface fracture toughness in pure mode I.

By introducing the expressions of G and G_c into (7), the final expression of the energy criterion is achieved after a and rearrangement,

$$\frac{\sigma_x^\infty}{\sigma_c} \geqslant \gamma \sqrt{g\,(\Delta\theta, \eta)}, \tag{10}$$

where

$$g(\Delta\theta, \eta; \alpha, \beta, \lambda, \theta_l) = \frac{\int_0^{\Delta\theta} \hat{G}_c(\psi(\theta_d, \eta))\mathrm{d}\theta_d}{\int_0^{\Delta\theta} \hat{G}(\theta_d, \eta)\mathrm{d}\theta_d} > 0.$$

Note that g can be interpreted as a ratio between a dimensionless dissipated to a released energy. γ is a structural dimensionless parameter introduced in [5]–a measure of the brittleness-ductility of the interface,

$$\gamma = \frac{1}{\sigma_c}\sqrt{\frac{G_c E^*}{a}}. \tag{11}$$

The energy criterion, as can be observed in Figure 4 imposes a minimum value for $\Delta\theta$ for a given remote load σ_x^∞, σ_y^∞ up to $\theta_{\min}^E$, where the function g has a minimum [5, 10].

4. Coupled criterion and results

Leguillon's hypothesis [6] assumes that whereas each of the above studied criteria gives a necessary condition for the debond onset, the combination of both criteria leads to a sufficient condition. According to this idea, for a fixed value of the biaxiality parameter η, the debond

[1] It can be demonstrated, see e.g. [13], that in the open model of interface cracks it is not possible to define a value of ψ strictly for the crack tip. As a consequence, in the present work, ψ is actually evaluated at a small distance ahead of the crack tip $\theta_l = 0.1°$ following [5]

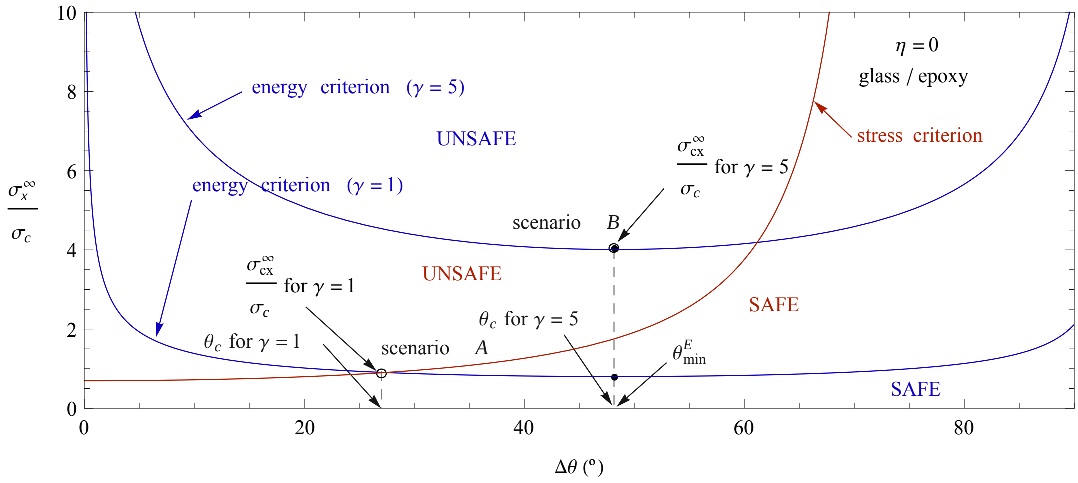


Figure 4. Example of coupling both stress and energy criteria for $\eta = 0$ and glass/epoxy for two values of γ corresponding to two different scenarios.

onset is predicted for the minimum value of σ_x^∞, denoted σ_{cx}^∞ in the following, which fulfills both criteria simultaneously.

As have been demonstrated in [5, 10], two different scenarios depending on γ value are found:

- Scenario A: Curves given by both criteria have an intersection point in the decreasing part of the energy criterion. In this case the minimum value of σ_x^∞ and $\Delta\theta$ for which both criteria are fulfilled is determined by this intersection point, see Figure 4.
- Scenario B: Both curves have no common point or this point corresponds to a debond semiangle $\Delta\theta > \theta_{\min}^E$. In this scenario, see Figure 4, the minimum load is given by the energy criterion as $\frac{\sigma_{cx}^\infty}{\sigma_c} = \gamma\sqrt{g(\theta_{\min}^E(\eta))}$ and $\Delta\theta = \theta_{\min}^E$.

In view of this analysis, a simple algorithm is necessary to determine the critical primary remote load σ_{cx}^∞ for different values of η and γ as detailed in [10].

From this model, several additional results can easily be extracted as, for example, variations of the debond semiangle $\Delta\theta$ at the debond onset as a function of the different problem parameters and a size effect prediction. Also an indirect experimental procedure for measurement of the fracture and strength properties of the interface can be proposed. All this is excluded from the present paper for the sake of brevity, see [10] for details.

We focus here on the influence of the secondary transverse load on σ_{cx}^∞ because this is essential for three-dimensional failure criteria of unidirectional composites. Using the algorithm presented in [10], a map of critical biaxial transverse load is plotted in Figure 5 for glass/epoxy and several values of γ. This map shows that, in general if a remote secondary transverse compression σ_y^∞ is applied the value of σ_{cx}^∞ decreases which could lead to a premature failure if the secondary compression is neglected. This effect is relevant for small values of γ which correspond to brittle configurations. For small values of γ, this effect depends strongly on the bimaterial elastic constants k and m, see [10] for a detailed analysis. The effect of the secondary transverse load agrees with the preliminary experimental evidences presented in [7, 15] for a material with similar values for k and m.

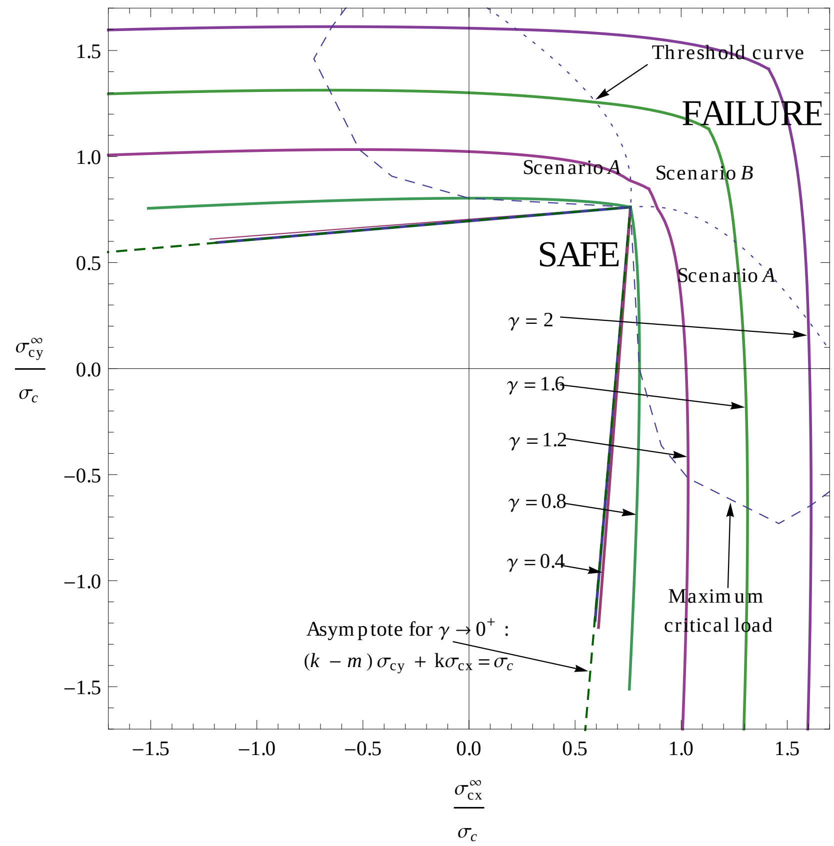


Figure 5. Biaxial critical loads σ_{cx}^{∞} and σ_{cy}^{∞} for glass/epoxy and several values of γ.

5. Concluding remarks

A theoretical model for the debond initiation at the fibre-matrix interface under a biaxial remote load perpendicular to the fibre-axis has been developed by generalizing the model presented in [5] applying the coupled criterion of the Finite Fracture Mechanics.

The influence of the secondary remote transverse load on the critical primary transverse load leading to the debond onset has been studied and quantified. The analysis carried out has shown a moderate influence on the critical primary load predicted for brittle configurations. In particular this difference is important when the secondary load is compressive, as found experimentally in [7, 15], because of a premature failure predicted when this effect is neglected. For the sake of brevity, only results which quantify the influence of the biaxiality have been presented, but this model allows studying easily the influence of any parameter on the debond onset as shown in [10].

Acknowledgements

This work was supported by the Junta de Andalucía and the Spanish Ministry of Science and Innovation, through the Projects TEP4051 and MAT2009-14022, respectively, and a FPU Grant of the Spanish Ministry of Education corresponding to I.G. García.

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