DESIGN OPTIMIZATION OF LAMINATED COMPOSITE STRUCTURES WITH MANY LOCAL STRENGTH CRITERIA

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Keywords: Structural optimization, strength design, failure criteria, large scale problems

Abstract

This paper presents different strategies for handling very many local strength criteria in structural optimization of laminated composites. Global strength measures using Kreisselmeier-Steinhauser or p-norm functions are introduced for patch-wise parameterizations, and the efficiency of the approach is illustrated for multi-material topology optimization of laminated composite structures including failure criteria.

1 Introduction

The objective of this work is to investigate different strategies for design optimization of laminated composite structures where very many local strength criteria are taken into account together with other global structural performance measures. The use of gradient based structural optimization techniques makes it is possible to tailor the laminated composite structure such that the desired structural performance is obtained. Typical criteria functions for the design optimization can be global measures such as mass, stiffness, cost, eigenfrequencies, and buckling load factors, and local measures such as point-wise strength criteria. Gradient based structural optimization techniques of finite element discretized models are well established for such problems, but the inclusion of local strength criteria introduces some challenges for large scale engineering design problems such as structural optimization of wind turbine blades. The failure indices of the laminated composite structure are normally determined using refined finite element models, and this introduces a very large number of local strength criteria that must be handled efficiently in the design optimization approach.

One way of obtaining an efficient way of handling very many strength measures for design optimization of laminated composite structures is by introducing global strength measures such as Kreisselmeier-Steinhauser (KS) functions [1] or p-norm functions. In this paper the methods will in particular be applied for including failure criteria in multi-material topology optimization of laminated composite structures, where the amount of design variables is very large [2, 3, 4, 5], but the approach can as well be applied when the design variables considered are fiber angles and thicknesses of monolithic laminates. A simple benchmark example is studied for different discretizations in order to test the algorithms developed, and the use of the approach is also illustrated for multi-criteria design of a generic main spar from a wind turbine blade. Focus is on design problems where the maximum failure index is minimized while global constraints such as compliance and mass constraints are taken into account, but the approach might as well be applied for design problems where strength criteria are specified as constraints of the optimization problem.

2 Design optimization using strength criteria

The design optimization approach taken in this work is based on the ideas introduced by Schmit in 1960 [6] of minimizing cost by means of Mathematical Programming techniques. The optimization problem can be written as:

$$\begin{aligned} \text{Objective}: \quad & \min_{\mathbf{x}} \; f(\mathbf{x}) \\ \text{Subject to}: \quad & g_j(\mathbf{x}) \leq G_j, \quad j = 1,\ldots,J \\ & \underline{x_i} \leq x_i \leq \overline{x_i}, \; i = 1,\ldots,n \end{aligned} \tag{1}$$

where $f(\mathbf{x})$ is the objective function, $g_j(\mathbf{x})$ the constraint functions with upper bounds $G_j, j = 1,\ldots,J$, and x_i, $i = 1,\ldots,n$, are the design variables with lower and upper bounds $\underline{x_i}$ and $\overline{x_i}$, respectively.

Structural optimization of laminated fiber composites using gradient based techniques has been applied for decades with the earliest works in the seventies by [7]. In this work focus is on optimization problems where the maximum failure index in the structure is minimized, and the failure indices are typically evaluated at the bottom and top of all layers in all finite elements used for the discretized model. Thus, the optimization problem is a min-max problem where the objective function $f(\mathbf{x})$ is given as a max over a set of functions $f_k(\mathbf{x})$:

$$f(\mathbf{x}) \;=\; \max_{k=1,\ldots,n_0} \; f_k(\mathbf{x}) \tag{2}$$

When the objective function $f(\mathbf{x})$ contains more than one function value, i.e. $n_0 > 1$, the non-differentiable min-max optimization problem can be reformulated into a differentiable problem using the so-called bound formulation [8, 9, 10], where an additional scalar design variable β is introduced:

$$\begin{aligned} \text{Objective}: \quad & \min_{\mathbf{x},\beta} \; \beta \\ \text{Subject to}: \quad & f_k(\mathbf{x}) \;\leq\; \beta \qquad k = 1,\ldots,n_0 \\ & g_j(\mathbf{x}) \;\leq\; G_j \qquad j = 1,\ldots,J \\ & \underline{x_i} \;\leq\; x_i \;\leq\; \overline{x_i}, \quad i = 1,\ldots,I \end{aligned} \tag{3}$$

This is a simple, robust and very efficient method for multi-criteria optimization where the min-max problem is transformed into the problem of minimizing a bound β subject to the constraints $f_k(\mathbf{x}) \leq \beta$, $k = 1,\ldots,n_0$, where each of the functions $f_k(\mathbf{x})$ are assumed to be differentiable.

The bound formulation (and similar approaches) has been used for many years in structural optimization with great success for such min-max problems, but the resulting optimization problem may become very large for laminate design problems due to the many design variables x_i and many highly nonlinear local strength criteria $f_k(\mathbf{x})$ for engineering design problems such as wind turbine blades. Active set strategies may be used for reducing the number n_0 of failure indices $f_k(\mathbf{x})$ to include, but it is important to include all relevant failure indices in order to obtain a stable convergence of the problem. For example, values exceeding 70% of the maximum value in a given design iteration can be included in the optimization problem that typically is solved using Sequential Linear Programming (SLP), Sequential Quadratic Programming (SQP), etc., see for example [11,12].

3 Global strength measures for structural optimization

One way of making the optimization problem less computationally challenging is by introducing global strength measures. Initial work on stress based single-material topology optimization was based on the transformation of stresses into one global stress measure for the structure, see [13, 14], but it is difficult to find a general and robust function suitable for all cases of stress reduction. Recently it has been demonstrated for single-material topology optimization problems with stress constraints that it is advantageous to introduce a number of global stress measures instead of having only one global measure. This is because the efficiency decreases when a large number of values are lumped into a single global value [15]. [16] grouped the finite elements into blocks and used a so-called block aggregation approach by computing a global stress measure for each of the blocks. In [17] a so-called regional stress measure approach was used and different strategies for grouping the stresses before computing the global stress measures were studied.

For laminate design problems a patch parameterization is normally used, such that larger patches of elements are associated with the same parameterization. This can be reused for the computation of global strength measures by evaluating a global strength measure for each patch in the model, and this is the approach taken in this work. Failure indices (FI) are computed at the bottom and top of each layer of each finite element, and the number of FI values, n_{FI}^j, to include for each patch *j* may be defined by the user. Global strength measures are obtained using modified Kreisselmeier-Steinhauser (KS) functions [1, 18] or *p*-norm functions, see the discussion on these strength measures in [19]. For each patch $j,\ j = 1,\dots,n^P$, a KS function f_{KS}^j can be computed as:

$$f_{KS}^j = \frac{1}{\rho}\ln\left[\sum_{k=1}^{n_{FI}^j} e^{\rho f_k^j}\right] \quad \text{or} \quad f_{KS}^j = f^{j,max} + \frac{1}{\rho}\ln\left[\sum_{k=1}^{n_{FI}^j} e^{\rho (f_k^j - f^{j,max})}\right] \tag{4}$$

where $\ln = \log_e$, the scalar ρ typically is between 2 and 200 (20 is used for the examples in this paper), and $f^{j,max}$ is the largest FI value among $f_k^j(\mathbf{x}),\ k = 1,\dots,n_{FI}^j$ in patch *j*. In general the latter KS definition has numerical advantages and is the one used in this work. However, the FI values typically are in the order of 1 and thus both KS functions perform well. The parameter ρ determines the difference between the original function and its approximation. The maximum value $f^{j,max}$ of the values $f_k^j(\mathbf{x}),\ k = 1,\dots,n_{FI}^j$, is weighted more heavily when a high value of ρ is used, but this also causes oscillation or even divergence of the optimization problem due to ill-conditioning.

Similarly, a *p*-norm function f_{PN}^j can be computed for patch *j* as:

$$f_{PN}^j = \left(\sum_{k=1}^{n_{FI}^j} (f_k^j)^p\right)^{1/p} \tag{5}$$

The parameter p controls the level of smoothness, and the p-norm approaches the original max function as $p \rightarrow \infty$. However, a high value of p causes numerical problems, and in this work a value of 6 is used for p.

Thus, the optimization problem in Eq. (1) is reduced to the following mathematical programming problem:

$$\begin{aligned}
\text{Objective :}\quad & \min_{\mathbf{x},\beta} \ \beta \\
\text{Subject to:}\quad & f(\mathbf{x})^j \leq \beta \qquad j = 1,\dots,n^P \\
& g_j(\mathbf{x}) \leq G_j \qquad j = 1,\dots,J \\
& \underline{x_i} \leq x_i \leq \overline{x_i}, \quad i = 1,\dots,I
\end{aligned} \tag{6}$$

where the global strength functions $f^j,\ j = 1,\dots,n^P$, are computed as f^j_{KS} or f^j_{PN}.

4 Design parameterization

The so-called Discrete Material Optimization (DMO) approach [2, 3, 4, 5] has been applied for parameterization of the examples considered in this work. This parameterization approach makes it possible to solve the combinatorial problem of proper choice of material and fiber orientation for multi-material topology optimization of laminated composites, and the DMO approach relaxes the discrete material selection problem to a continuous formulation using interpolation schemes with penalization. Thus, the material properties are computed as weighted sums of properties of candidate materials, which may be different kinds of fiber reinforced materials associated with given fiber angles, possibly together with core materials if sandwich structures are allowed for the design. In this paper the "generalized SIMP" interpolation scheme presented in [5] for compliance problems is used. Here the material property of interest, for example the constitutive matrix $\mathbf{C}^{eff}$, is computed in the following way, when there are n_c candidate materials to choose between, each characterized by its constitutive matrix $\mathbf{C}_i$:

$$\mathbf{C}^{eff} = \sum_{i=1}^{n^c} x_i^q \mathbf{C}_i, \quad 0 \leq \underline{x}_i \leq x_i \leq \overline{x}_i \leq 1, \quad \sum_{i=1}^{n^c} x_i = 1 \tag{7}$$

Thus, design variables x_i are directly associated with candidate material i, and the penalization power q is used to enforce a unique choice of material at the end of the optimization. The design variables can be considered as volume fractions x_i of each of the candidate materials with this parameterization. A large number of sparse linear constraints ($\sum_{i=1}^{n^c} x_i = 1$) to enforce the selection of at most one material in each design domain is introduced, but these constraints are handled effectively using the SNOPT optimization package [20] using either SLP or SQP.

The failure criteria used for fiber reinforced polymer (FRP) materials are normally defined in the material coordinate system 1-2-3, and the procedure applied for computing effective failure indices ($\mathbf{FI}_{eff}$) with the above DMO parameterization is the following:

1) Assemble the element stiffness matrices using $\mathbf{C}^{eff} = \sum_{i=1}^{n^c} x_i^p \mathbf{C}_i$

2) Solve the linear elastic static problem ($\mathbf{KD} = \mathbf{F}$) for displacements $\mathbf{D}$

3) Failure analysis postprocessing:

   a) For each element (ElemNo) extract the element displacement vector $\mathbf{d}$ from $\mathbf{D}$:

      i) For each layer (LayerNo):

         (1) Compute strain vector $\boldsymbol{\epsilon}$ in structural coordinate system

         (2) For each candidate material *i*:

            - Transform $\boldsymbol{\epsilon}$ to material coordinate system 1-2-3 of the candidate material $\Rightarrow$ $\boldsymbol{\epsilon}_i^{1-2-3}$ and evaluate failure index $\mathrm{FI}_i(\boldsymbol{\epsilon}_i^{1-2-3})$

         (3) Compute $\mathrm{FI}_{eff}\,(\text{ElemNo, LayerNo, top/bottom}) = \sum_{i=1}^{n^c} x_i^{\,r}\ \mathrm{FI}_i(\boldsymbol{\epsilon}_i^{1-2-3})$

A SIMP-type interpolation is also used for computing the failure index FI_{eff}, but here the power *r* is ≤ 1 in order to make it unfavorable to have intermediate volume fractions x_i. The maximum strain criterion is used for the examples in this paper, but many other failure criteria used for laminated composites have also been implemented and can be applied.

4 Examples

4.1 Single-layered clamped plate subjected to uniform pressure

The first numerical example considered is a clamped single-layered monolithic plate subjected to uniform pressure. The plate is divided into 8x8 patches with the same fiber angle within each patch, and the problem has been solved for a number of different discretizations. The four candidate materials to choose between are glass/epoxy material (GFRP) oriented at 0^0, -45^0, 45^0, and 90^0. Material parameters for all examples are taken from the software program ESACOMP ver. 4.1. The minimum compliance problem using 24 x 24 9-node shell elements divided into 8 x 8 patches yields the design shown in Fig. 1.

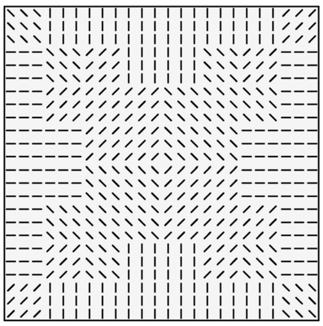

Figure 1. Stiffness optimal design of single-layered plate divided into 8x8 patches. The fiber angle of the chosen candidate material is shown for each finite element in the 24 x 24 mesh used.

The plate example has been solved for a number of different discretizations with the objective of minimizing the maximum failure index in the structure, and Fig. 2 shows the result obtained when minimizing the failure index (computed by the maximum strain criteria) for a 96x96 mesh, again with 8x8 patches for the parameterization. For each patch the 20 highest FI values have been extracted and converted to a global strength measure by using a KS function f_{KS}^{j}. The strength optimized design differs from the stiffness optimal design in several of the patches.

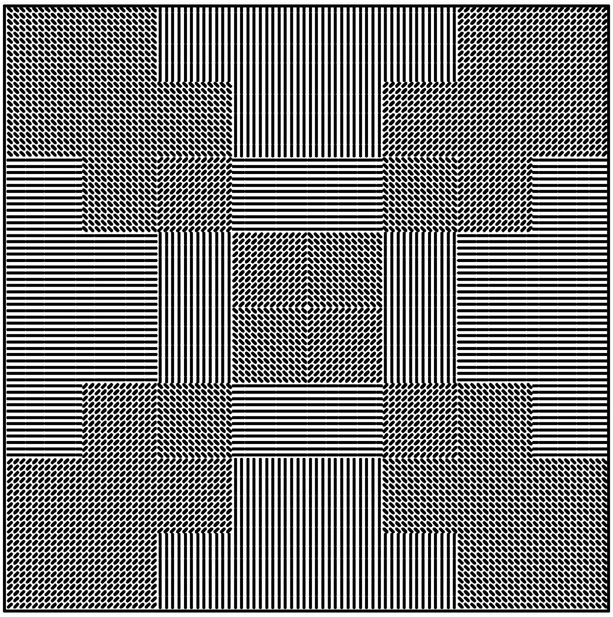

Figure 2. Optimal design for strength of single-layered plate divided into 8x8 patches and discretized by 96x96 elements.

The design problem has also been solved using *p*-norm functions, but in general the use of KS functions seems to yield optimization problems that are easier to solve. This is also the general observation of other studies, see, e.g., the derivations and discussions of these two global strength measures for optimization purposes in [10].

4.2 Multi-criteria optimization of generic main spar from wind turbine blade

The approach is also shortly demonstrated in the following for multi-material design of a generic main spar from a wind turbine blade subjected to the maximum flapwise bending load case. The midsection of the main spar is divided into 16 patches, each consisting of 10 layers of equal thickness. The candidate materials are GFRP oriented at 0^0, -45^0, 45^0, and 90^0, GFRP $\pm 45^0$ biax non-crimp fabric mats, and foam material. The objective is to minimize the failure index in the main spar while fulfilling compliance and mass constraints. 1/5 of the design domain should be filled with foam material. The FE model and the parameterization are seen on Figure 3, and the results are listed in Table 4. The model consists of 1652 9-node shell elements, and thus the number of computed effective FI values in each iteration is 1652 x 16 x 2 = 52864. These are reduced to 16 global strength measures by using a KS function f_{KS}^{j} for each of the 16 patches where the 20 highest values in each patch are taken into account. Both constraints are fulfilled for the final design.

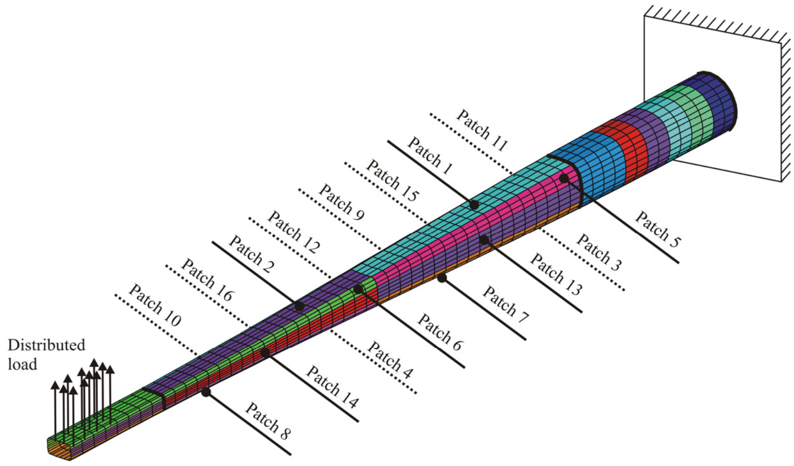

Figure 3. Generic main spar from wind turbine blade.

Patch \ Layer	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
P1	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°
P2	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°
P3	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°
P4	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°
P5	0°	foam	0°	foam	0°	0°	0°	0°	0°	0°
P6	-45°	45°	45°	90°	0°	90°	0°	0°	0°	45°
P7	0°	foam	foam	foam	0°	0°	foam	0°	foam	0°
P8	-45°	0°	0°	0°	0°	0°	0°	0°	0°	0°
P9	0°	0°	0°	foam	0°	foam	foam	0°	0°	0°
P10	45°	0°	0°	0°	0°	0°	0°	0°	0°	0°
P11	0°	foam	0°	foam	foam	0°	foam	0°	0°	0°
P12	45°	-45°	0°	0°	foam	90°	0°	90°	0°	-45°
P13	0°	foam	foam	foam	foam	foam	foam	foam	foam	0°
P14	0°	0°	0°	0°	0°	0°	0°	0°	-45°	45°
P15	0°	foam	foam	foam	foam	foam	foam	foam	foam	0°
P16	0°	foam	0°	0°	0°	0°	0°	0°	0°	0°

Table 1. Results obtained for main spar example.

6 Summary and conclusions

The paper has illustrated a novel method of including strength criteria in multi-material topology optimization of laminated composites. Focus has been on possible strategies for handling a very large number of strength criteria together with many design variables, and the paper has illustrated the use of applying global strength measures associated with patches of the finite element model used for the discretization. Both Kreisselmeier-Steinhauser functions and *p*-norm functions have been successfully applied, and two examples have illustrated the type of results that can be obtained by extending multi-material topology optimization with the proposed methodology for handling strength criteria.

Acknowledgement
This research is part of the project "Optimal Design of Composite Structures under Manufacturing Constraints" sponsored by the Danish Research Council for Technology and Production Sciences (FTP), Grant no. 10-082695. This support is gratefully acknowledged.

References

[1] Kreisselmeier G., Steinhauser R. Systematic control design by optimizing a vector performance index. In Proceedings *International federation of active controls symposium on computer-aided design of control systems*, Zurich, Switzerland, pp. 113–117 (1979).
[2] Stegmann J., Lund E. Discrete material optimization of general composite shell structures. *International Journal for Numerical Methods in Engineering*, Vol. 62, pp. 2009–2027 (2005).
[3] Lund E., Stegmann J. On structural optimization of composite shell structures using a discrete constitutive parametrization. *Wind Energy*, Vol. 8, pp. 109–124 (2005).
[4] Lund E. Buckling topology optimization of laminated multi-material composite shell structures. *Composite Structures*, Vol. 91, pp. 158–167 (2009).
[5] Hvejsel C.F., Lund E. Material interpolation schemes for unified topology and multi-material optimization. *Structural and Multidisciplinary Optimization*, Vol. 43, pp. 811–825 (2011).
[6] Schmit L.A. Structural design by systematic synthesis. In Proceedings of *2nd Conf. on Electronic Computation, ASCE*, New York, pp. 105–122 (1960).
[7] Schmit L.A., Farshi B. Optimum laminate design for strength and stiffness. *International Journal for Numerical Methods in Engineering*, Vol. 7, pp. 519–536 (1973).
[8] Bendsøe M.P., Olhoff N., Taylor J.E. A variational formulation for multicriteria structural optimization. *Journal of Structural Mechanics*, Vol. 11, pp. 523–544 (1983).
[9] Bendsøe M.P., Olhoff, N. A method of design against vibration resonance of beams and shafts. Optimal Control Applications & Methods, Vol. 6, pp. 191-200 (1985).
[10] Olhoff N. Multicriterion structural optimization via bound formulation and mathematical programming. *Structural Optimization*, Vol. 1, pp. 11–17 (1989).
[11] Johansen L.S., Lund E., Kleist J. Failure optimization of geometrically linear/ nonlinear laminated composite structures using a two-step hierarchical model adaptivity. *Computer Methods in Applied Mechanics and Engineering*, Vol. 198, pp. 2421–2438 (2009).
[12] Johansen L.S., Lund E. Optimization of laminated composite structures using delamination criteria and hierarchical models. *Structural and Multidisciplinary Optimization*, Vol. 38, pp. 357-375 (2009).
[13] Yang R.J, Chen C.J. Stress based topology optimization. *Structural Optimization*, Vol. 12, pp. 98-105 (1996).
[14] Duysinx P., Sigmund O. New development in handling stress constraints in optimal material distribution. In Proceedings of *7th AIAA/USAF/NASA/ISSMO symposium on multidisciplinary analysis and optimization*, A collection of technical papers, St. Louis, Missouri, Volume 3, AIAA Paper 98-4906, pp. 1501–1509 (1998).
[15] París J., Navarrina F., Colominas I., Casteleiro M. Topology optimization of continuum structures with local and global stress constraints. *Structural and Multidisciplinary Optimization*, Vol. 39, pp. 419–437 (2009).
[16] París J., Navarrina F., Colominas I., Casteleiro M. Block aggregation of stress constraints in topology optimization of structures. *Advances in Engineering Software*, Vol. 41, pp. 433–441 (2010).
[17] Le C., Norato J., Bruns T., Ha C., Tortorelli D. Stress-based topology optimization for continua. *Structural and Multidisciplinary Optimization*, Vol. 41, pp. 605–620 (2010).
[18] Martins J.R.R.A., Poon N.M.K. On structural optimization using constraint aggregation. In *Proc. 6th World Congress on Structural and Multidisciplinary Optimization, Rio de Janeiro, Brazil*, 8 pages (2005).
[19] Qiu G.Y., Li X.S. A note on the derivation of global stress constraints. *Structural and Multidisciplinary Optimization*, Vol. 40, pp. 625–628 (2010).
[20] Gill P.E., Murray W., Saunders M.A. SNOPT: An SQP algorithm for large-scale constrained optimization. SIAM Review, Vol. 47, pp. 99-131 (2005).