SIZE EFFECT ON TRANSVERSE CRACK INITIATION IN CROSS-PLY LAMINATES. APPLICATION OF A COUPLED STRESS AND ENERGY CRITERION

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Abstract

The classical problem of tranverse crack initiation in the inner-ply of symmetric [0,90]s laminates under tension is studied by a new approach: the coupled criterion of the Finite Fracture Mechanics, proposed by D. Leguillon in 2002. This criterion allows us to obtain semianalytic expressions for the critical strain originating the first tranverse crack by assuming that a crack of a finite extension appears when a stress condition is fulfilled and the crack onset is energetically allowed. A strong size effect of the inner-ply 90 thickness on the critical strain leading to the first crack onset is predicted. This prediction agrees with experimental results found in the literature.

1. Introduction

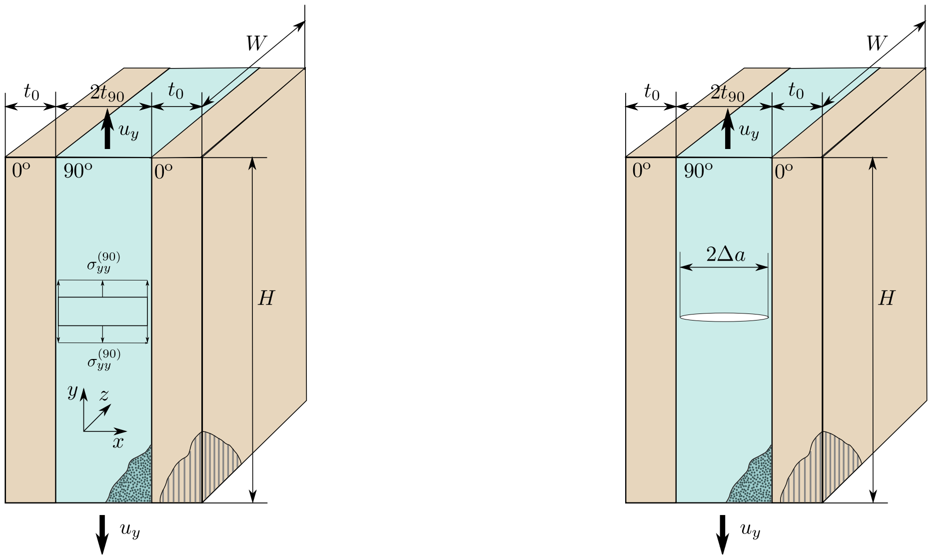
The increase of responsability and size of composite structures in high-technology industries requires a better understanding of damage and failure in composites. In particular, the effect of the size of composite structures is a key aspect to be studied in order to estimate the representativity of the laboratory-scale experimental results for the failure in full-scale composite structures. This will improve our knowledge about the failure in composites because "Any theory is not understood if the size effect is not explained" as told by Bažant in [1].

Cracks appearing and growing in the transverse inner-ply of a cross-ply laminate $[0_m, 90_n]_s$ represents a classical problem which has been studied for a long time. Basic steps of this damage mechanism are well known [2, 3, 4, 5]. First, some cracks appear perpendicular to the load in the inner-ply. Next, some cracks initiate at the interface between the inner and outer ply when transverse cracks reach this interface (or just before reaching it) [6, 7]. Finally, coalescense of several interface cracks occurs leading to macroscopic delaminations.

Experimental results by Parvizi et al. [8] showed a strong size effect of the 90° inner-ply thickness on the critical longitudinal strain leading to the first crack onset. This size effect has been explained from different points of view (see [9] for a review). The aim of this work is to apply the coupled criterion [10] of the Finite Fracture Mechanics (FFM) [11] to this problem in order to evaluate this size effect predicted and compare the theoretical predictions to the experimental results. In fact, the coupled criterion proposed in [10] was inspired in the experimental results by Parvizi et al. [8]. This criterion was further developed and generalized in subsequent works [11, 12, 13, 14]. This criterion is based on the assumption that it is necessary to fulfill two conditions simultaneously to allow the onset of a finite length crack: a stress condition which imposes stresses along the future crack path have to exceed a critical value and an energy condition which requires the onset is energetically allowed.

The specimens under study, see Figure 1, are $[0_m, 90_n]_s$ symmetric laminates as used by Parvizi in [8]. These are composed by glass/epoxy unidirectional plies, more specifically by two outer 0° plies with thickness $t_0 = 0.1$ mm and one inner 90° ply with variable thickness $t_{90} = 0.113 - 4.01$ mm. Material properties of the lamina used by [8] are shown in Table 1. The uncracked laminate, see Figure 1(a), is tested by imposing a displacement u_y at the extreme in the outer-plies direction y. Assuming the hypothesis of laminate theory, this u_y implies an homogenized longitudinal strain ε_{yy} in the whole laminate.

The critical value of ε_{yy} compared to the nominal critical tranverse strain $Y_{\varepsilon t} = Y_t/E_{22}$ leading to the first crack onset will be obtained as a function of t_{90} in Section 4 by combining the stress and energy criteria introduced previously in Sections 2 and 3, respectively.



(a) (b)

Figure 1. Schema of the problem (a) before and (b) after the first transverse crack onset.

2. Stress criterion

Stress criterion requires that normal traction along the future crack path exceed its critical value for the material considered. In the present case, the symmetric transverse crack is supposed to appear in the 90° inner ply, i.e. the crack is perpendicular to the y direction and is centered with

Composite	E_{11} (GPa)	E_{22} (GPa)	ν_{12}	ν_{23}	G_{12} (GPa)	G_{23} (GPa)	G_{ct} (N/m)	Y_t (MPa)
glass/epoxy in [8]	42	14	0.278	0.4	5.83	5	240	80

Table 1. Properties of the lamina used in [8]: longitudinal E_{11} and transverse E_{22} elastic moduli, shear moduli G_{12} and G_{23}, transverse fracture toughness G_{ct} and tranverse strenght Y_t.

respect to the inner ply thickness. Therefore, this criterion imposes that in this ply tractions $\sigma_{yy}^{(90)}$ have to exceed before the crack onset the transverse strength of the material Y_t along all the points x where a crack of a finite length $2\Delta a$ appears,

$$\sigma_{yy}^{(90)}(x) \geq Y_t, \quad \forall x \in [-\Delta a, \Delta a] \tag{1}$$

Normal tractions $\sigma_{yy}^{(90)}$ can be estimated by assuming the hypothesis of the laminate theory. This implies that $\sigma_{yy}^{(90)}$ does not depend on x and is homogeneous along the inner-ply. The analytic expression of $\sigma_{yy}^{(90)}$ as a function of ε_{yy} and the material properties has been obtained in [15]. Introducing the expression of ε_{yy} on (1), we arrive to the following condition imposed by the stress criterion expressed in terms of the normalized longitudinal strain $\frac{\varepsilon_{yy}}{Y_{\varepsilon t}}$,

$$\frac{\varepsilon_{yy}}{Y_{\varepsilon t}} \geqslant \frac{1-\nu_{12}\nu_{21}}{1-\frac{\nu_{12}\nu_{12}\left(1+\frac{t_0}{t_{90}}\right)}{\frac{t_0}{t_{90}}+\frac{E_{11}}{E_{22}}}}. \tag{2}$$

Note that this condition gives a minimum for $\frac{\varepsilon_{yy}}{Y_{\varepsilon t}}$ leading to the crack onset. This value depends on the material properties and the laminate configuration. However, this minimum is independent of the length of the crack Δa after the onset.

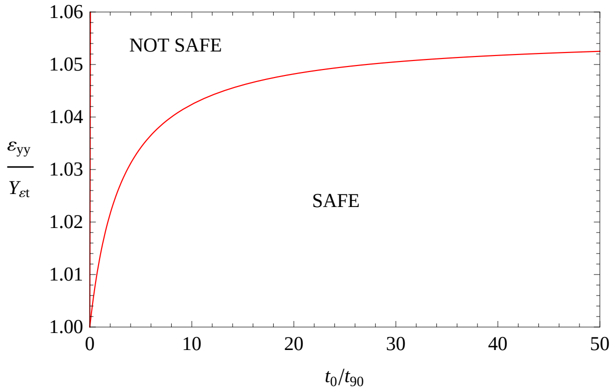


Figure 2. Stress condition for the mimimum critical longitudinal strain ε_{yy} leading to the first transverse crack onset as a function of the ratio of the ply thicknesses t_0/t_{90} for glass/epoxy.

Figure 2 shows the value of this condition for the glass/epoxy used in [8] as a function of t_0/t_{90}. It is interesting to remark that this value is about the unity for moderate values of t_0/t_{90}. A similar result can be obtained for carbon/epoxy as is demonstrated in [15].

3. Energy criterion

Energy condition is based on an incremental Griffith type criterion by comparing the energetic state of the specimen before and after the crack onset. The energetic balance between both states should fulfill the first law of thermodynamics,

$$\Delta\Pi + \Delta E_k + \Gamma = 0, \tag{3}$$

where $\Delta\Pi$ and ΔE_k are, respectively, the change on potential elastic and kinetic energy and Γ is the energy dissipated during the irreversible processes associated to the crack onset. Assuming a quasistatic initial state: $\Delta E_k \geq 0$. $\Delta\Pi$ can be related to the Energy Release Rate (ERR) G by a basic of Linear Elastic Fracture Mechanics (LEFM): $G = -\frac{\mathrm{d}\Pi}{\mathrm{d}a}$ where a is an "instantaneous" crack length. Γ can be approximated by $G_{ct} \cdot \Delta a$ where G_{ct} is the transverse fracture toughness. Then, the energy balance (3) leads to,

$$\int_0^{\Delta a} G(a)\mathrm{d}a \geq G_{ct}\Delta a \tag{4}$$

The ERR G is obtained by using a Boundary Element Method (BEM) code as explained in [4, 16]. A dimensional analysis is carried out in order to reduce the number of elastic solutions to calculate. This analysis, detailed in [15], allows defining a dimensionless ERR $\hat{G}$ as,

$$G = E_{22} \cdot \varepsilon_{yy}^2 \cdot t_{90} \cdot \hat{G}\left(\frac{a}{t_{90}}, \frac{t_0}{t_{90}}, \text{Mat. Prop.}\right). \tag{5}$$

Eventually, $\hat{G}$ is computed as a function of the "instantaneous" crack length a for several values of t_0/t_{90}. Figure 3 shows the results of this computation. Note that $\hat{G}(a = 0) = 0$ as is predicted by the LEFM for a vanishing crack length and $\hat{G}(a = t_{90}) = 0$ because of the weakness of the corner singularity when the crack reaches the interface between both plies, the second one being stiffer [4, 16]. In general, as can be seen in Figure 3, $\hat{G}$ increases when t_0/t_{90} decreases due to the lower influence of the stiffer outer-ply.

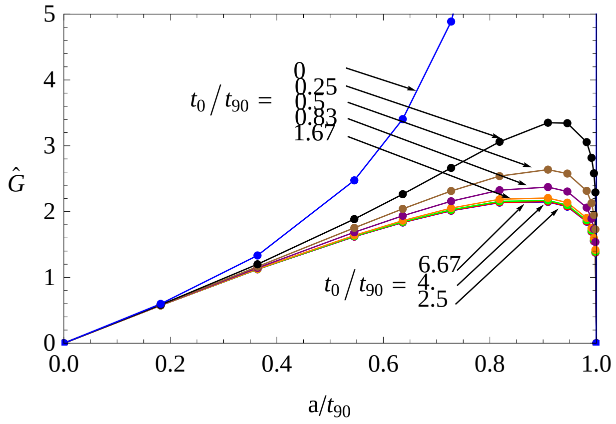


Figure 3. Dimensionless energy release rate (ERR) $\hat{G}$ as a function of the length a of the transverse crack.

By introducing (5) in (4) the expression of the energy criterion is obtained,

$$\frac{\varepsilon_{yy}}{Y_{\varepsilon t}} \geq \frac{1}{Y_t}\sqrt{\frac{G_{ct}E_{22}}{t_{90}} g\left(\Delta\hat{a}, t_0/t_{90}\right)}, \tag{6}$$

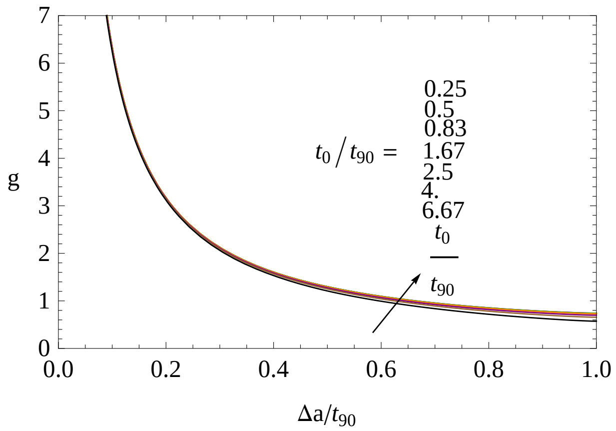


Figure 4. Dimensionless ratio g of the dimensionless dissipated to the released energy as a function of the length of the crack onset Δa.

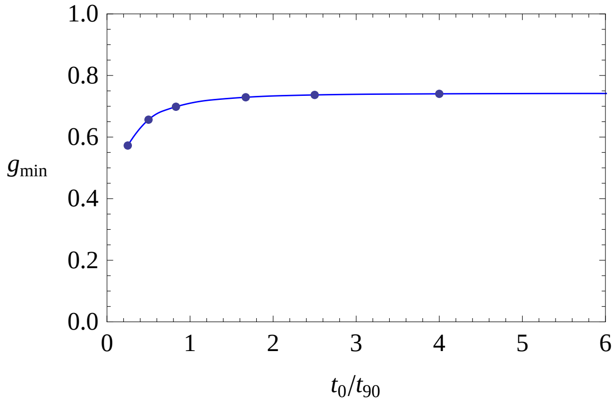


Figure 5. Value of the minimum of function g as a function of the ratio of thicknesses t_0/t_{90}.

where $g(\Delta\hat{a}, t_0/t_{90})$ is a ratio of the dimensionless dissipated to the released energy for a transverse crack onset of a dimensionless length $\Delta\hat{a} = \Delta a/t_{90}$,

$$g(\Delta\hat{a}, t_0/t_{90}) = \frac{\Delta\hat{a}}{\int_0^{\Delta\hat{a}} \hat{G}(a, t_0/t_{90})\mathrm{d}\hat{a}}. \tag{7}$$

The function g is plotted in Figure 4 and is apparently a decreasing function of $\Delta\hat{a}$. However, it can be demonstrated independently of the computational results that this function achieves a minimum value for $\Delta\hat{a} < 1$ denoted as $g_{\min}$. This implies, in view of the energy condition (6), that the only dependence of this on $\Delta\hat{a}$ is involved in the function g. Hence, the energy condition has a minimum value for $\frac{\varepsilon^c_{yy}}{Y_{\varepsilon t}}$ given for the corresponding crack length of onset. This value corresponds to the minimum value of g, $g_{\min}$ applied the expression (6). The dependence of $g_{\min}$ on the ratio t_0/t_{90} is plotted in Figure 5 showing that the lower value of the critical strain is required for smaller thicknesses of the outer-ply due to the corresponding larger values of $\hat{G}$ as explained above.

From an analysis of (6) and the plots for $\hat{G}$, g and $g_{\min}$ it follows that the energy criterion provides a minimum value required for the normalized strain $\frac{\varepsilon_{yy}^c}{Y_{\varepsilon t}}$, which depends on the crack length at the onset, material properties and laminate configuration.

4. Coupled criterion and results

According to Leguillon's hypothesis [10] the onset of a new crack of a finite length occurs when both criteria studied above are fulfilled simultaneously. The combination of both criteria gives the critical load (or strain in the present case) and the crack length Δa at the onset. The present analysis is focused on the critical strain predicted as a function of the 90° inner-ply thickness in order to study the size effect. Further results can be obtained with respecto to crack length at the onset and the subsequent unstable crack growth, see [15].

The combination of both criteria is very simple and the normalized critical strain can be obtained by maximizing the minimum values given by both criteria,

$$\frac{\varepsilon_{yy}^c}{Y_{\varepsilon t}} = \mathrm{Max}\left[\frac{1-\nu_{12}\nu_{21}}{1-\frac{\nu_{12}\nu_{12}\left(1+\frac{t_0}{t_{90}}\right)}{\frac{t_0}{t_{90}}+\frac{E_{11}}{E_{22}}}}, \frac{1}{Y_t}\sqrt{\frac{G_{ct}E_{22}}{t_{90}} g_{\min}\left(t_0/t_{90}\right)}\right]. \tag{8}$$

The results are computed for the values of G_{ct} and Y_t obtained experimentally by Parvizi et al. in [8], the elastic material properties and the outer-ply thickness t_0 and the range of inner-ply thicknesses t_{90} used in [8]. The dependence of the critical strain predicted is plotted in Figure 6 with a solid line. To compare, the experimental results obtained by [8] are showed in the same plot. The agreement found is very good taking into account the typical dispersion of the experimental results and of the strength and fracture properties estimations by Parvizi et al. [8], see e.g. the dispersion noticed in [17] for the transverse fracture toughness in unidirectional plies.

For small values of t_{90}, the stress criterion is fulfilled for $\frac{\varepsilon_{yy}}{Y_{\varepsilon t}} \sim 1$ whereas the energy condition requires a large value of $\frac{\varepsilon_{yy}}{Y_{\varepsilon t}}$ to allow the onset. The reason is that for smaller t_{90}, the energy released decreases as $\Delta\Pi \sim \varepsilon_{yy}^2 t_{90}^2$ whereas the energy dissipated as $\Gamma \sim t_{90}$. The different powers of the dependence on t_{90} implies that it is necessary an increase of the term ε_{yy} to equilibrate the balance when t_{90} decreases.

It is interesting to remark that the general expression in (8) depends only on the computational results through a constant $g_{\min}$ which depends on the ratio t_0/t_{90} and the elastic material properties. It means that different laminates with the same ratio of t_0/t_{90} can be studied without carrying out additional computations. From a theoretical point of view, this leads to a very simple expression of the size effect as is demonstrated in [15].

5. Concluding remarks

The coupled stress and energy criterion of the Finite Fracture Mechanics has been applied to predict the onset of a transverse crack at the inner-ply of a symmetric laminate $[0_m, 90_n]_s$. A semianalytic expression, depending on the computational results just thorugh a constant value

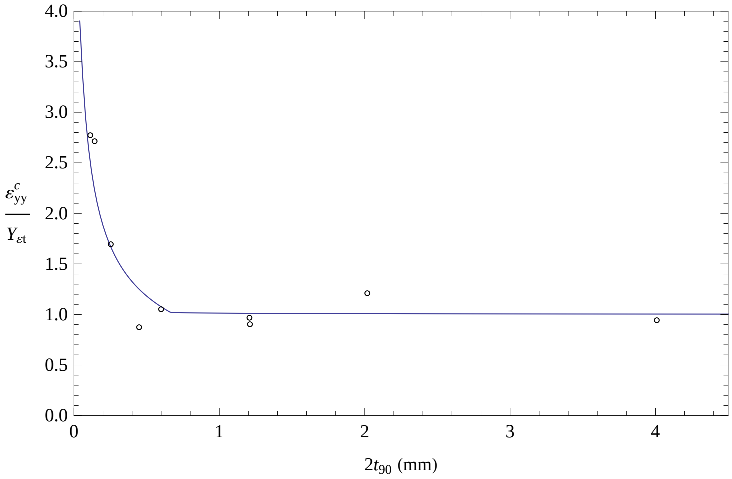


Figure 6. Comparison of the critical longitudinal strain ε_{yy} as a function of the inner-ply thickness $2t_{90}$ for a fixed value of $t_0 = 0.1$ mm predicted by the coupled criterion (solid line) and the experimental results (dots) obtained in [8].

$g_{\min}$, has been obtained for the normalized critical strain as a function of the strength, fracture and elastic properties of the material and the outer and inner ply thicknesses.

The analysis of this expression shows a strong size effect on the strenght of the 90° inner ply. The "apparent" strength of the inner-ply increases strongly when the thickness of this ply t_{90} decreases. A physical interpretation for this onservation has been based on energetic reasons: this is caused by the different geometrical dimensions of released and dissipated energy.

When the theory developed is applied to the material and laminate used in experiments by Parvizi et al. [8], the size effect predicted agrees with that observed in experiments.

Acknowledgements

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