NUMERICAL CHARACTERISATION OF RANDOM GLASS FIBRE COMPOSITE MATERIAL

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Abstract
The current work demonstrates a numerical approach to spatially random short fibre composite materials based on the concept of Representative Volume Elements (RVE) in order to obtain the mechanical properties of Random Short Fibre Composites (RaFC). Mechanical characterization of RaFC is of high importance for lightweight structural applications due to their relatively high performance and low cost of production. A two phase heterogeneous material was created and analysed through a RVE approach and Finite Element Analysis (FEA) was implemented. A homogenisation method was applied in order to obtain the micromechanical parameters of homogeneous-equivalent analyses in a bulk composite. RVE size was affirmed by a statistical analysis.

1. Introduction
Composite materials have recently come under intense investigation due to a variety of specific properties such as stiffness, strength and durability and that provide enhanced lightweight performance in comparison with other traditional engineering materials. Short fibre composite materials possess the additional advantage of relatively easy processing and the lower cost of manufacturing in contrast to advanced long fibre composites.

Lightweight structures made from random composite materials have a range of applications in engineering. Their favourable lifecycle and ease of manufacturing and processing make them particularly attractive for the construction industry [1]. The need for an improved understanding of these materials is becoming increasingly evident, as more additives are successfully included as their constituents, and traditional models that describe their behaviour may not adequately correspond to experimental observation. A multitude of micromechanical analytical models have been developed in order to predict the behaviour of such materials [2]. The most successful models rely on the mean field models, based on Eshelby's field solution. Eshelby [3] solved the transformed inclusion problem for the dilute case, by describing the stress, strain and displacement fields for a single ellipsoidal inclusion embedded in an infinite matrix. The work presented by Mori and Tanaka [4] belongs to the family of effective field methods and has registered greater interest after the clarification of Benveniste [5]. The main assumption of the MT theory is that each identical inclusion in a composite with many such constituents experiences a far field strain equal to the average matrix strain. The determination of the influential tensor, or Mori Tanaka concentration tensor, is a function of Eshelby's strain concentration tensor. Another model for the prediction of the mechanical properties of short fibre composite materials is the semi-empirical Halpin-Tsai model [6] which is based on the original work of Hill [7] for the self-consistent method. This model has been implemented by industry due to its simplicity and the accurate results, comparable with experimental data. The Cox shear lag model is another significant analytical model that takes into account fibre orientation and length distributions [8].

A relatively new method that has been adopted to characterise random fibre materials is the concept of a Representative Volume Element (RVE), based on the implementation of computational calculations and numerical methods**.** The definition of RVE is dependent on the purpose and the nature of the material [9]. Generally RVE is a statistical representation of the material [10] and according to Hashin [11] RVE is a model of the material that may be used to determine the corresponding effective properties for the homogenised macroscopic model. The RVE should be both large enough to contain sufficient information about the microstructure in order to be representative and much smaller than the macroscopic volume. As reported by Pan *et al.*[12] an RVE must be sufficiently large such that it contains a large number of inclusions in a heterogeneous material and that the effective properties derived from the RVE represent the true material properties on a macroscopic scale.

Gitman et al. [9] studied the existence and the size of the RVE for a 3 phase composite (matrix, inclusion and interface transition zone). Moreover, the work investigated the existence of a representative volume element for the elastic, hardening and softening regimes for inclusion volume fractions up to 60%. A two dimensional composite consisting of three phases with an interfacial transition zone was simulated. Inclusions were circular with randomly determined diameters and were randomly distributed in space. The study combined numerical and statistical analysis and concluded that localisation effects were present for quasi brittle material in the softening regime, thus the material lost its statistical homogeneity.

Pelegri et al [13] published a series of reports aimed at evaluating the overall elastic properties of 3D random fibre composites using a modified random sequential adsorption (RSA) to create the random 3D microstructure geometry. Fibres were modelled as cylinders with spherical edges and were able to kink in case of intersection. This feature allowed them to increase the fibre volume fraction (Vf) up to 35-40%, thereby simulating typical industrial productions. A finite element analysis (FEA) was performed and the results were compared with Halpin-Tsai models.

Taking into account the aforementioned investigation of the mechanical properties of short fibre composites, this work aims to study the effect of Aspect Ratio (AR) and RVE size on the effective stiffness of a two-phase composite material reinforced with randomly oriented fibres.

2. RVE Model

Fibres were depicted as ellipses in order to accurately simulate the two dimensional geometry on a microscopic level and to calculate the aspect ratio as the fraction between fibre dimensions on the major and the minor axes. The choice of elliptical inclusions instead of rectangles ensured the absence of stress concentration at the end of fibres and was physically consistent due to the geometrical similarities between a two dimensional fibre and a narrow ellipse. Each family of aspect ratio inclusions is identical, and oriented in the same direction at zero degrees. Five realisations were created for each size in order to provide a mean value for the different spatial position of the inclusions.

2.1 Generation of 2D random geometry

Intersection between the inclusions was avoided to prevent any influence on the final result of the simulation. An algorithm was created to solve the packing problem. A single inclusion was placed in each iteration, after checking for intersections between the given fibre and the previously analysed and accepted fibres, and was repeated until the criterion of a specified volume fraction was satisfied. The initial parameters were major and minor axes of the ellipses, radii of the circles and the volume fraction. In the case of circular inclusions, the centre and the radius of the circle were generated with information used to define the position of the circle. Non-intersection was ensured by controlling the Euclidean distance between them. The flowchart of the algorithm is shown in Figure 1.

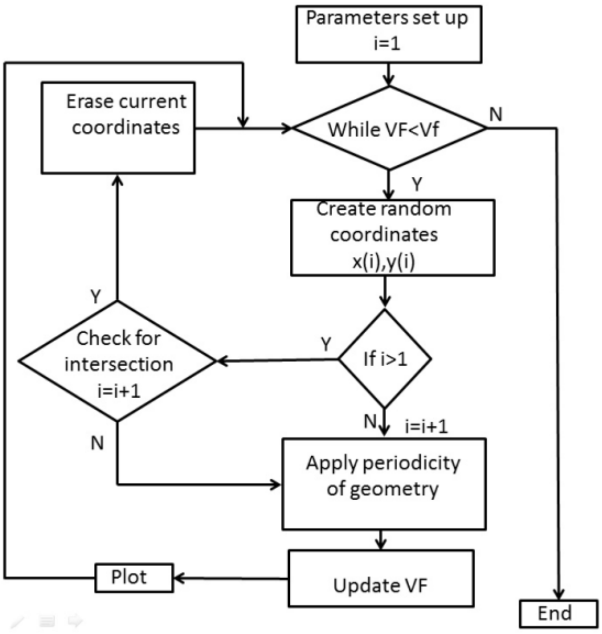

Figure 1.A flowchart of the algorithm implemented to create the random geometry.

2.2 *Periodicity of the Geometry*

As previously mentioned, RVE is a statistical representation of the material, or more simply a small volume of the structure that is large enough to contain sufficient information about the mechanical properties of the material. Under this definition it is necessary for the representative realisation to be periodically replicable along x and y directions. This is implemented by the algorithm for each acceptable inclusion by omitting all the inclusions that exceed the representative square, and shifting them to the opposite side. Figure 2 illustrates geometrical periodicity of RVE.

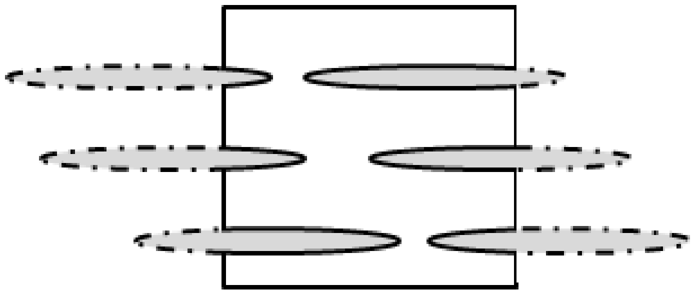

Figure 2. Geometrical periodicity of the RVE. Each inclusion that exceeds the boundary of RVE is transposed from the opposite side such that the whole inclusion is contained within the RVE.

3. Homogenisation process

Implementation of a homogenisation technique is required by the process of deriving homogeneous properties from heterogeneous media. Such a technique must be able to return accurate homogeneous properties for the macroscopic level while taking into account the response of the microscopically heterogeneous media. Macroscopic stresses and strains can be defined over the volume of the entire specimen and are abbreviated as $\overline{\sigma}$ and $\overline{\varepsilon}$:

$$\overline{\sigma_{ij}} = \frac{1}{V}\int_V \sigma_{ij}(x)dV \text{ i ,j=1,2 x} \in \text{V} \tag{3.1}$$

$$\overline{\varepsilon_{ij}} = \frac{1}{V}\int_V \varepsilon_{ij}(x)dV \text{ i ,j=1,2 x} \in \text{V} \tag{3.2}$$

The actual local stress and strain are denoted by σ_{ij} and ε_{ij} and were derived from each single element of the model and integrated though the volume of the element. The effective stiffness can be derived by the implementation of Hook's law with the known macroscopic stresses and strain:

$$\overline{\sigma_i} = \overline{C_{ij}}\,\overline{\varepsilon_j} \text{ i,j=1,2,3} \tag{3.3}$$

Equivalence between the random composite heterogeneous material and the homogenised equivalent one is ensured by the Hill condition:

$$\frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}dV = \frac{V}{2}\overline{\sigma_{ij}}\,\overline{\varepsilon_{ij}} \tag{3.4}$$

This indirectly verifies equivalence of strain energy for both media.

3.1 *Boundary conditions*

Three types of boundary conditions can be applied in order to satisfy Hills condition in RVE method

- Traction boundary conditions on the boundary of RVE,

$$t_i = \sigma_{ij} n_j \tag{3.5}$$

- Kinematic uniform boundary conditions,

$$u_i = \varepsilon_{ij} x_j \tag{3.6}$$

- Periodicity boundary conditions.

4. FEA Results

Finite Element Analysis of the RVE model was performed using Abaqus 6.10-2 software. The geometry of the model was created in Matlab and imported into Abaqus via Python script. Two linear elastic systems were modelled, where the properties of the fibre were E_f =73GPa, ν_f=0.21 and with the properties of matrix were E_m=1.2GPa, ν_m =0.335. Models were meshed with the free meshing algorithm built into Abaqus with triangular three note CPS3 elements. Local stresses and strains were measured at the integration point of each element. The procedure to derive the properties of the equivalent homogeneous material required three loading conditions accompanied by the analogous boundary conditions, in order to sequentially activate only one of the three independent elements of the strain tensor. Average stress and strain were calculated by analysing the response of the heterogeneous media. The effective stiffness tensor was derived by solving Hooke’s law for the elements at each interval. Results for longitudinal Young modulus, transverse Young modulus and shear modulus as well as predictions made by the Halpin-Tsai equation are presented in Figures 3-6.

Results in figure 3 show effective stiffness as a function of RVE size for circular inclusions. It can be observed that the effective stiffness in transverse and longitudinal directions is the same as predicted due to the symmetry of the inclusions. Figure 4 describes ellipsoidal inclusions with AR=2.2. The effective stiffness for the ellipsoidal inclusions is slightly higher than that of the circular inclusions in the longitudinal direction due to the orientation of the reinforcing inclusions. The same trend was observed for the ellipsoidal inclusions in figures 5 and 6. The effective shear modulus seems to decrease with an increase in aspect ratio, with a higher value recorded for circular inclusions.

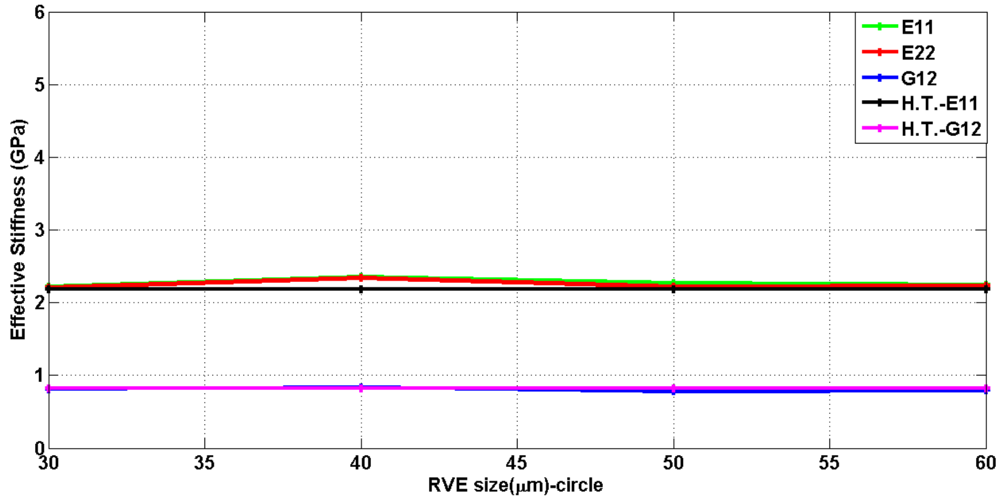

Figure 3. Effective stiffness of RVE in transverse and longitudinal directions and predictions of the Halpin-Tsai model for circular inclusions with AR=1 for four sizes.

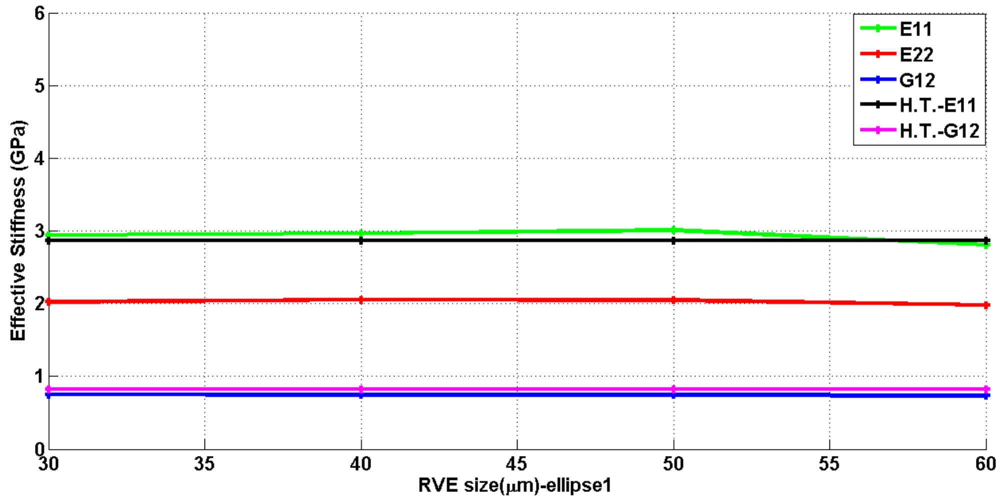

Figure 4. Effective stiffness of RVE in transverse and longitudinal directions and predictions of theHalpin-Tsai model for ellipsoidal inclusions with AR=2.2 for four sizes.

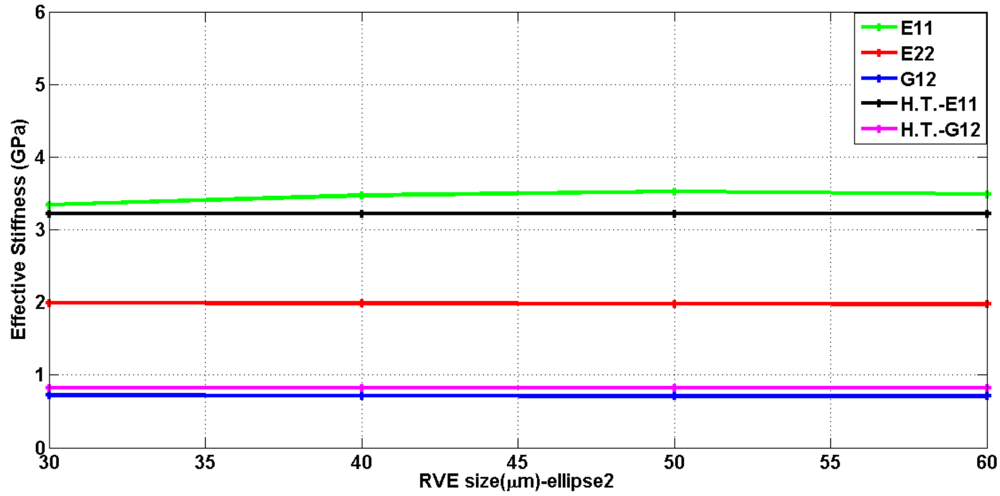

Figure 5. Effective stiffness of RVE in transverse and longitudinal directions and predictions of the Halpin-Tsai model for ellipsoidal inclusions with AR=3.3 for four sizes.

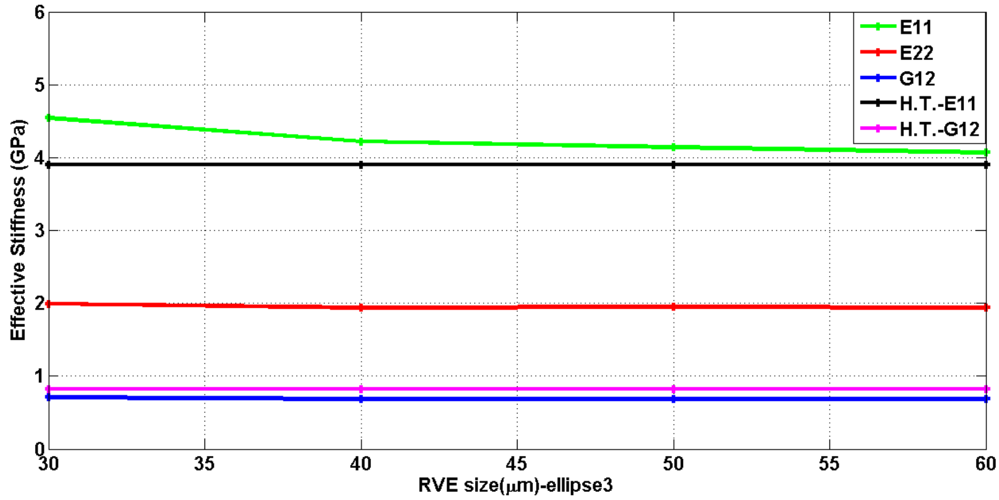

Figure 6. Effective stiffness of RVE in transverse and longitudinal directions and predictions of the Halpin-Tsai model for ellipsoidal inclusions with AR=5 for four sizes.

5. Conclusions

The effect of aspect ratio on the effective stiffness of unidirectional random fibrous composite materials with constant volume fraction was investigated. The solid mechanics problem was solved using the Finite Element Method and the elastic constants were derived by implementing a homogenisation technique, and compared with the semi-empirical model of Halpin-Tsai. Results show that with the increase in the aspect ratio the stiffness in the principle direction of the specimen increased with respect to the Halpin-Tsai predictions. Ellipses accurately simulate 2D fibres and the homogenisation technique is sufficiently accurate to determine effective stiffness.. Future work will include all the parameters of a stochastic material such as random orientation, random size and random spatial distribution, as well as a comparison between numerical and experimental data. Moreover an appraisal of three-dimensional modelling will be provided in order to summarise the effect of the two dimensional simplicity. In addition, future simulations will help to identify optimum fibre geometry, content and distribution with the aim of improving the thermo-mechanical behaviour of thermoplastic composites. Furthermore a comparison between numerical and statistical analysis will be performed on the mechanical behaviour of stochastic composite materials in order to determine the most efficient way of characterising short fibre composite materials.

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