# FAILURE BEHAVIOR OF COMPOSITE LAMINATES UNDER OUT-OF-PLANE LOADS

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**Abstract**
*In this study, failure behavior of fiber-reinforced composites under out-of-plane loads is investigated. For this purpose, four-point bending tests are simulated using both Classical Lamination Theory (CLT) and Finite Element Method (FEM) with brick elements. Unidirectional $[\theta_6]_s$ and balanced symmetric $[\theta_3/-\theta_3]_s$ composite laminates are considered and maximum allowable moment resultants as a function of fiber orientation angle, $\theta$, are obtained using different failure criteria. The differences between the model predictions are discussed.*

## 1 Introduction

Fiber - reinforced composite materials are widely used because of their high stiffness-to-weight and strength-to-weight ratios. For their safe use, one should use reliable failure theories that can correctly predict whether the plate will fail or not under given loading conditions for a chosen laminate configuration. Researchers [1-4] have extensively studied their failure behavior under in-plane loads both theoretically and experimentally and examined the validity of the failure theories. On the other hand, laminated composite plates under out-of-plane loads have not been studied adequately. Only some chosen configurations were studied under out-of-plane loads [5-6]; but the failure behavior as a function of fiber orientation angle, θ, was not investigated.

Composite failure criteria are categorized in several ways: the ones with or without stress interaction, failure mode dependent or independent, linear or quadratic, physically based, etc. In this study, some of the most widely recognized criteria in each category are studied and their predictions are compared by simulating four - point bending test, where the middle regions of the laminate are subjected to pure bending moment. Among the chosen failure criteria, Tsai-Wu and Quadric Surfaces are both nonlinear; they account for stress interaction; but Quadric Surfaces is also failure mode dependent; Maximum Stress is linear and failure mode dependent; Hashin is physically based, nonlinear, failure mode dependent and it accounts for stress interaction.

## 2 Analytical model of the problem

### *2.1 Stress-Moment Resultant Relationship*

The thickness of the plate is small compared to its width and length which turns the problem into a plane stress problem. Classical Lamination Theory (CLT) is utilized to relate loading to the resulting stress state. Bending-extension coupling matrix [B] reduces to zero thanks to symmetry conditions. Considering that only $M_{xx}$ is applied to the laminate, stresses in layer $k$ are as follows:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k = z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}_k^{-1} \begin{bmatrix} M_{\max} \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where $\sigma_{xx}$, $\sigma_{yy}$, and $\sigma_{xy}$ are the stresses in global coordinates, [$\overline{Q}$] is the reduced stiffness matrix of the $k^{\text{th}}$ lamina, [D] is the bending-twisting coupling matrix, and $M_{max}$ is the maximum allowable moment resultant, which makes the failure index of the laminate for the respective failure criterion equal to 1.0. The stresses are transformed into native coordinates. Residual stresses in multidirectional plates are calculated according to the method given by Hyer [7] and added to the stresses. Then, the maximum allowable moment is obtained for each lamina by implementing the failure criteria using the stresses in the principal material directions. The minimum value is then assigned to $M_{max}$.

## 3 Finite element model of the problem

Finite element analysis of the plate was performed using finite element software ANSYS v13 with Solid185 layered 3-D structural solid elements [8]. Convergence analysis was carried out to find a balance between computational cost and accuracy of results. Results of the analysis showed that 1×1×0.276 [$mm^3$] quadrilateral elements in a 96×48×2.208 [$mm^3$] composite plate would be optimum. Another analysis was conducted for optimal positioning of the loads so as to ensure that static failure modes dominate delamination failure mode. For this purpose, the results of a delamination criterion [9] are compared with the results of Tsai-Wu and maximum stress failure criteria for different load positions. Simulations are conducted using the optimal loading conditions in which the most likely failure mode is static failure, not delamination. Because brick elements can account for out-of-plane Poisson's effect and out-of-plane stresses can be calculated, a stress state different from the analytical solution is obtained. Accordingly, the failure criteria yield different predictions.

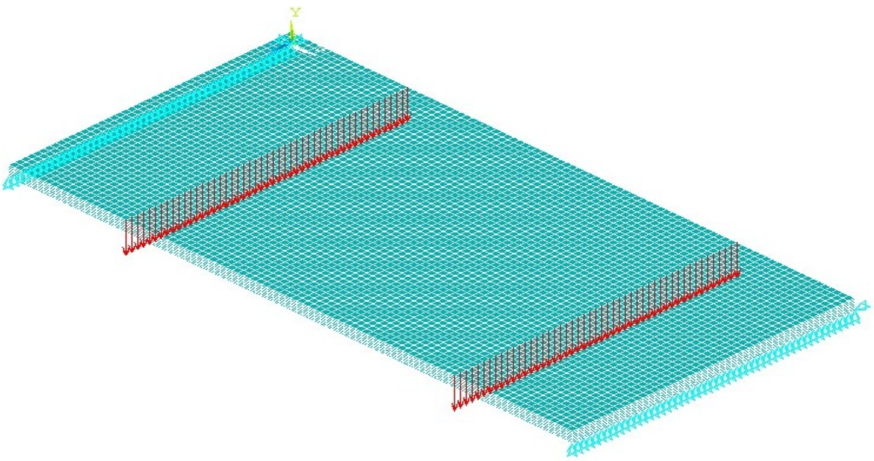


**Figure 1:** Finite element model of the problem showing force and displacement boundary conditions

## 4 Failure Criteria

*4.1 Tsai-Wu Failure Criterion*

According to this criterion [1], the onset of failure under plane stress is estimated by the following equation:

$$\left(\frac{1}{X_t}+\frac{1}{X_c}\right)\sigma_{11}+\left(\frac{1}{Y_t}+\frac{1}{Y_c}\right)\sigma_{22}-\frac{\sigma_{11}^2}{X_tX_c}-\frac{\sigma_{22}^2}{Y_tY_c}+\frac{\sigma_{12}^2}{S_{12}^2}-\frac{\sigma_{11}\sigma_{22}}{\sqrt{X_tY_tX_cY_c}}=1 \tag{2}$$

One can obtain $M_{max}$ by substituting the stresses in native coordinates into Equation (2).

*4.2 Maximum Stress Failure Criterion*

According to this criterion [13], safety of a composite plate under plane stress is ensured if the following conditions are satisfied:

$$X_c \le \sigma_{11} \le X_t \tag{3}$$
$$Y_c \le \sigma_{22} \le Y_t \tag{4}$$
$$|\sigma_{12}| \le S_{12} \tag{5}$$

Corresponding to the limiting values of $\sigma_{ij}$, the maximum allowable moments are evaluated. Their minimum is taken as $M_{max}$.

*4.3 Hashin Failure Criterion*

Hashin [2] proposed a physically based failure criterion that determined the failure modes of fiber – reinforced laminates. According to the criterion, failure occurs under plane stress condition, if one of the following conditions occurs:

*Tensile Fiber Mode ($\sigma_{11}$>0):*

$$\left[\frac{\sigma_{11}}{X_t}\right]^2+\left[\frac{\sigma_{12}}{S_{12}}\right]^2=1 \tag{6}$$

*Compressive Fiber Mode ($\sigma_{11}$<0):*

$$\sigma_{11}=-X_c \tag{7}$$

*Tensile Matrix Mode ($\sigma_{22}$ >0):*

$$\left[\frac{\sigma_{22}}{Y_t}\right]^2+\left[\frac{\sigma_{12}}{S_{12}}\right]^2=1 \tag{8}$$

*Compressive Matrix Mode ($\sigma_{22}$ <0):*

$$\left[\frac{\sigma_{22}}{2S_{23}}\right]^2+\left[\left(\frac{X_c}{2S_{23}}\right)^2-1\right]\frac{\sigma_{22}}{X_c}+\left[\frac{\sigma_{12}}{S_{12}}\right]^2=1 \tag{9}$$

*4.4 Quadric Surfaces Failure Criterion*

The onset of failure of symmetric laminates under plane stress condition is predicted according to this criterion [3], if one of the following conditions is met:

$$\frac{a}{X^2}\sigma_{11}^2+\frac{a}{Y^2}\sigma_{22}^2+\frac{a}{S^2}\sigma_{12}^2+\frac{b}{XY}\sigma_{11}\sigma_{22}+\frac{b}{XS}\sigma_{11}\sigma_{12}+\frac{b}{YS}\sigma_{22}\sigma_{12}+\frac{c}{X}\sigma_{11}+\frac{c}{Y}\sigma_{22}+\frac{c}{S}\sigma_{12}=1 \quad (10)$$

$$\frac{a}{Y^2}\sigma_{22}^2+\frac{c}{Y}\sigma_{22}=1 \quad (11)$$

$$\frac{a}{X^2}\sigma_{11}^2+\frac{c}{X}\sigma_{11}=1 \quad (12)$$

where a=0.98, b=0.49, c=0.02, X, Y, and S are strengths which depend on the sign of respective stresses.

These equations are directly used to calculate $M_{max}$ of the plates analyzed by CLT. On the other hand, 3D versions of these failure criteria are used for laminates analyzed by FEM, because 3D stress state is obtained with the brick elements.

## 5 Results and Discussion

The material chosen for the simulations is AS4/8552. There are catalogue data published by the manufacturer, Hexcel®, and reports [10-11] on the mechanical properties of the material. However, the values for some of the elastic properties are not consistent. In this study, the values provided by the reports are used only when the catalogue values are missing (Table 1).

| Property | Value | Property | Value |
|---|---|---|---|
| $E_1$ | 131.69 GPa | $X_t$ | 2137 MPa |
| $E_2$=$E_3$ | 9.72 GPa | $Y_t$=$Z_t$ | 81 MPa |
| $G_{12}$=$G_{13}$ | 4.826 GPa | $X_c$ | 1531 MPa |
| $G_{23}$ | 3.352 GPa | $Y_c$=$Z_c$ | 268 MPa |
| $\nu_{12}$=$\nu_{13}$ | 0.319 | $S_{xy}$=$Sx_z$ | 91.6 MPa |
| $\nu_{23}$ | 0.45 | $S_{yz}$ | 102.7 MPa |
| $\alpha_x$ | 0.1265 $10^{-6}$ | Cure Temperature | 180 °C |
| $\alpha_y$=$\alpha_z$ | 37.12 $10^{-6}$ | Room Temperature | 20 °C |

**Table 1.** Properties of AS4/8552 [10-11].

Figures 2-10 present the results. In Figure 2, predictions of the failure criteria for UD laminates, $[\theta_6]_s$, based on the analytical model are compared. The figure shows that maximum stress and Hashin criteria predict slight increase in strength as the fiber angle is varied from 0 to 3 - 4 degrees. Aside from the range of angles between 0° and 7°, there are relatively large discrepancies in the predictions of the criteria in the range of 15° to 55°. Tsai-Wu and quadric surfaces criteria predictions for UD laminates are quite consistent; they change smoothly as the fiber angle changes from 0 to 90 degrees.

Because of the discrepancy between thermal expansion coefficients along the fiber direction and transverse to it significant residual stresses develop during manufacturing of multidirectional fiber - reinforced laminates, which may have great effect on their failure behavior. Figures 3 and 4 present $M_{max}$ predictions obtained based on the analytical model for multidirectional laminates, $[\theta_3/-\theta_3]_s$, including and excluding residual stresses, respectively. Failure behavior is not affected by the presence of residual stresses for small and large fiber angles because of their relatively small magnitudes. However, within the range of 10 to 70 degrees, exclusion of residual stresses might decrease the accuracy of predictions. A slight increase in the predictions of Hashin and maximum stresses criteria for small fiber angles θ is also observed in Figures 3 and 4. Figure 5 shows the predictions of $M_{max}$ for UD laminates obtained using the finite element results. The small increase in the strength predictions of Hashin and maximum stress criteria obtained using the analytical stress results for small fiber angles (Figure 2) is not observed for the FEM model. Figure 6 shows the FEM results for multidirectional laminates, $[\theta_3/-\theta_3]_s$, including residual stresses. These results are quite different from the analytical results presented in Figure 3. Figures 7 - 10 compare analytical and FEM model results. In all figures, FEM model $M_{max}$ predictions of all the criteria for $[0_6]_s$ laminates are less than the analytical model predictions. This difference arises from the fact that CLT neglects the out-of-plane Poisson effect. If the Poisson's ratios are set to zero during calculations, FEM and analytical model predictions coincide for $[0_6]_s$ laminate.

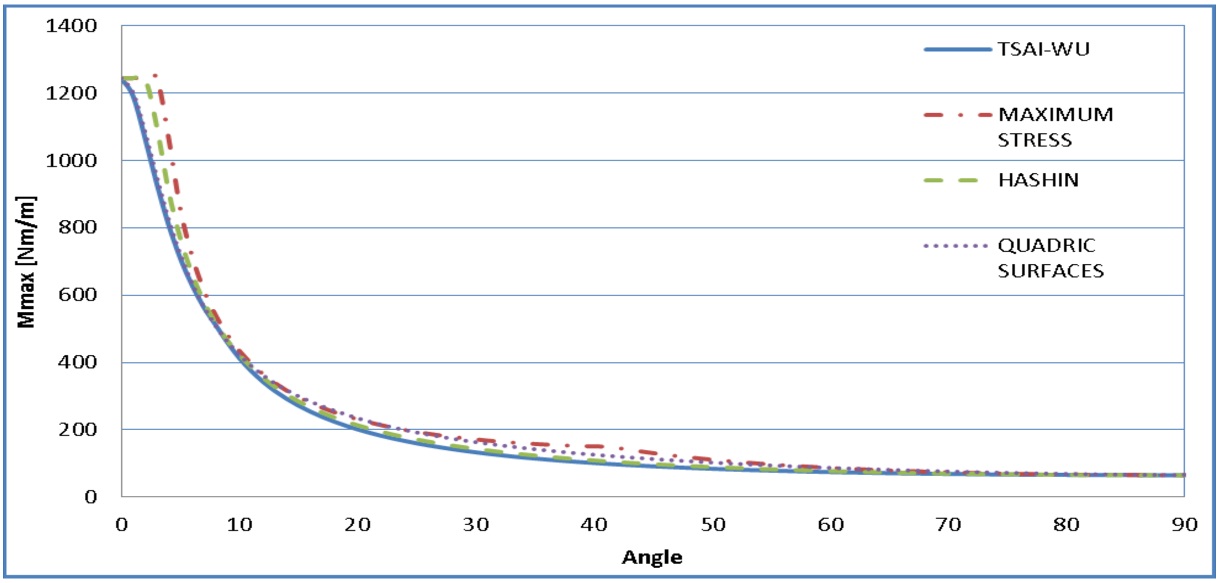



**Figure 2.** $M_{max}$ predictions of the failure criteria for unidirectional $[\theta_6]_s$ laminates based on the analytical model.

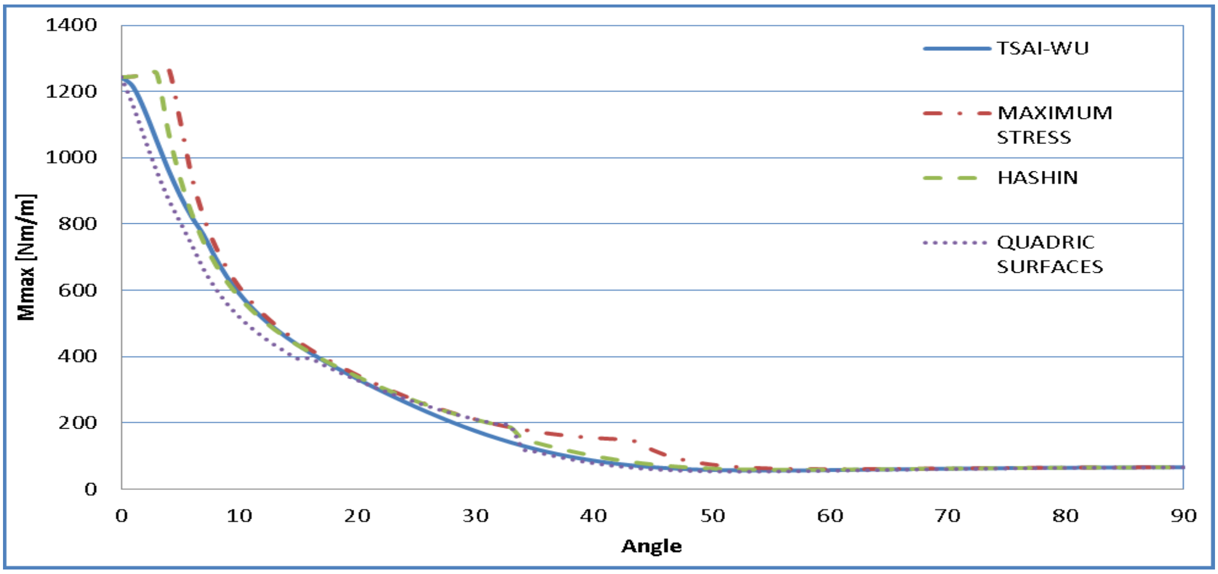



**Figure 3.** $M_{max}$ predictions for $[\theta_3/-\theta_3]$ configuration based on the analytical model including residual stresses

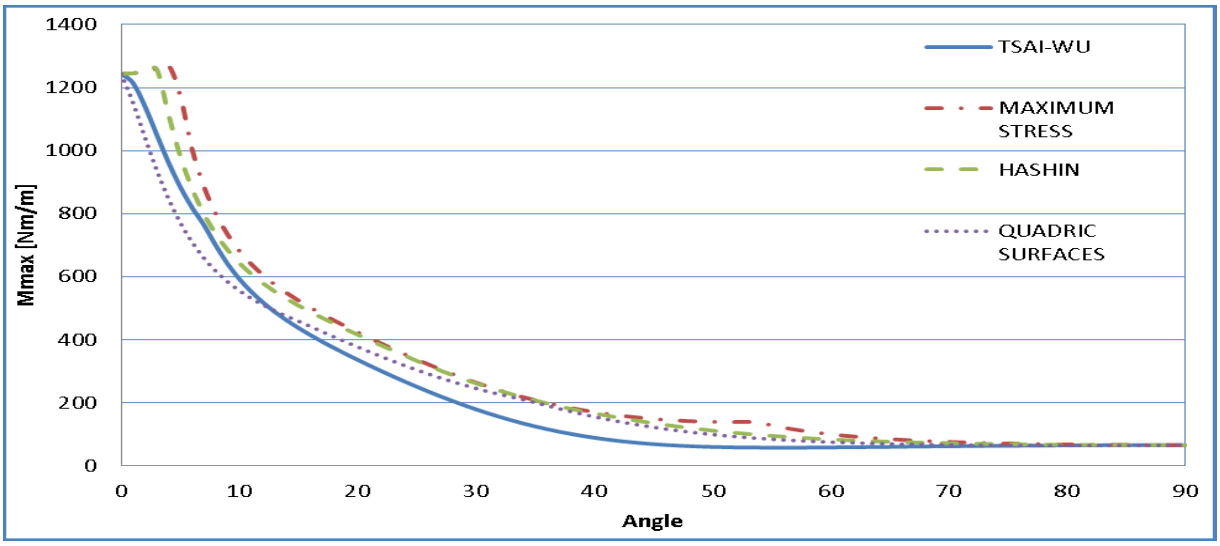



**Figure 4.** $M_{max}$ predictions for $[\theta_3/-\theta_3]_s$ configuration based on the analytical model excluding residual stresses

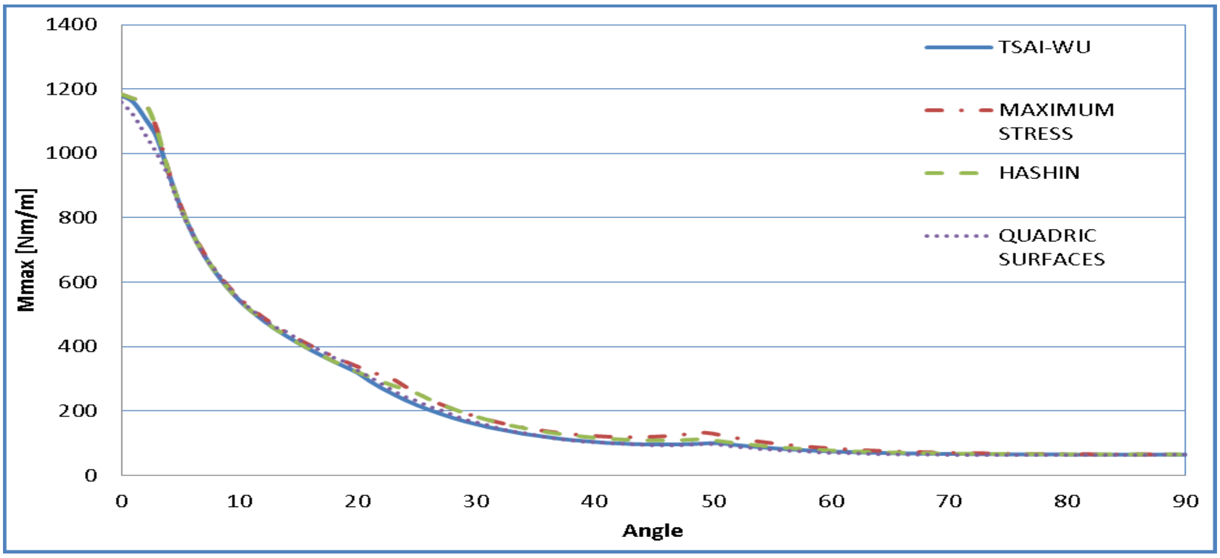



**Figure 5:** $M_{max}$ predictions based on the FEM model for unidirectional $[\theta_6]_s$ laminates.

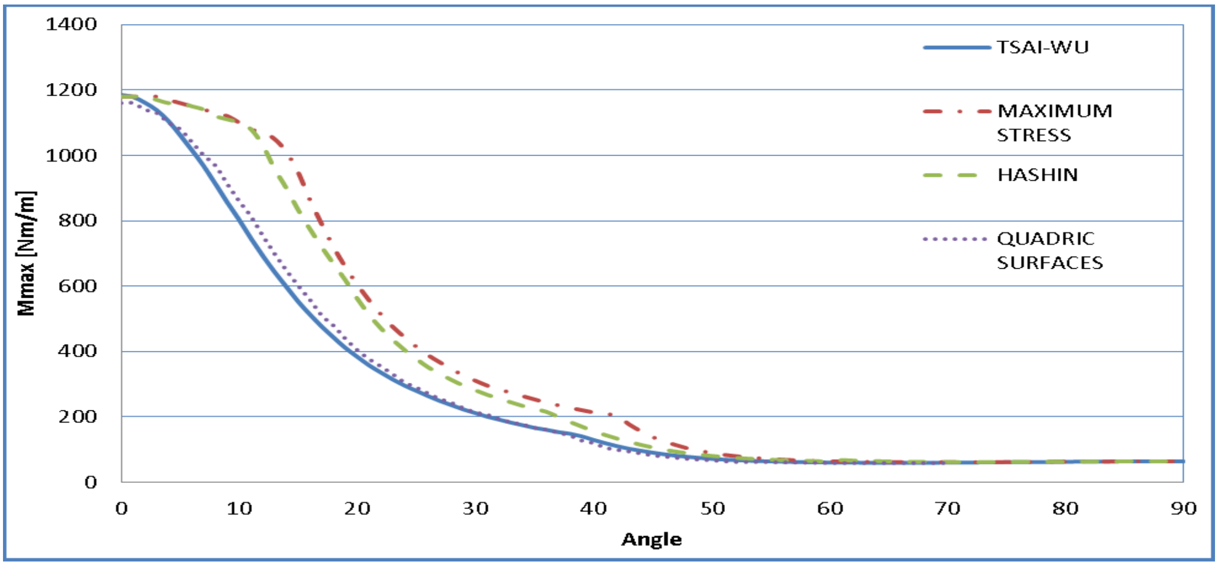
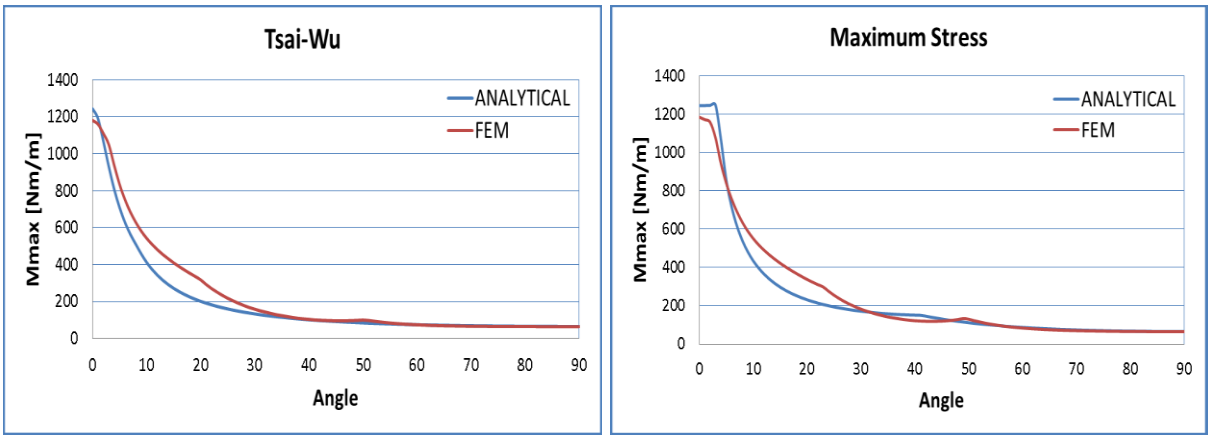
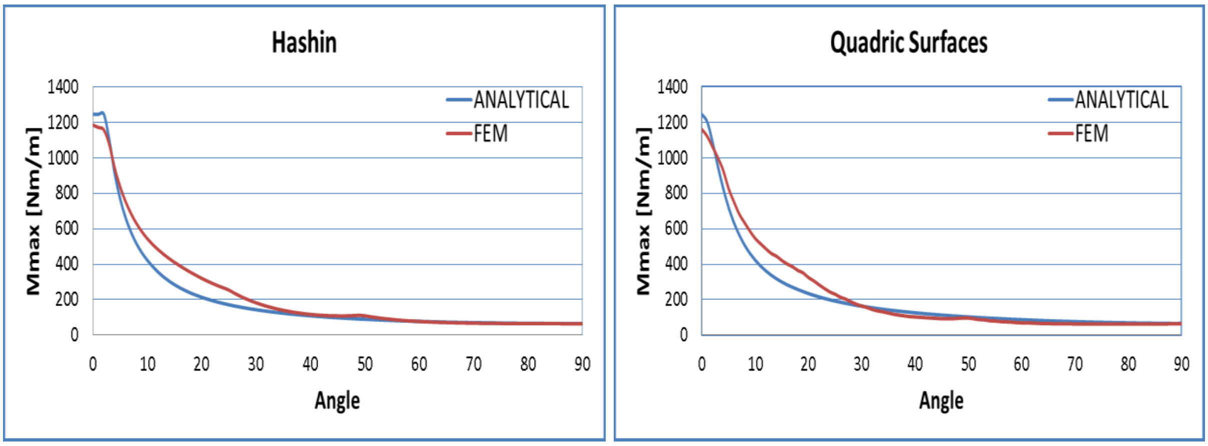



**Figure 6.** $M_{max}$ predictions based on the FEM model for $[\theta_3/-\theta_3]_s$ configuration including residual stresses



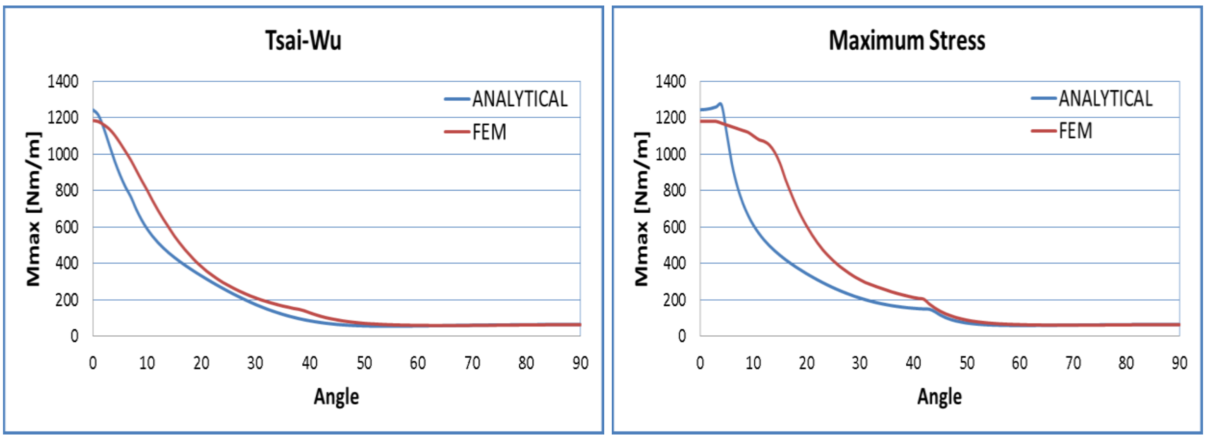

(a) (b)

**Figure 7.** Comparison of $M_{max}$ predictions obtained with analytical and finite element results using (a)Tsai-Wu (b) maximum stress criteria for unidirectional off-axis $[\theta_6]_s$ specimens.



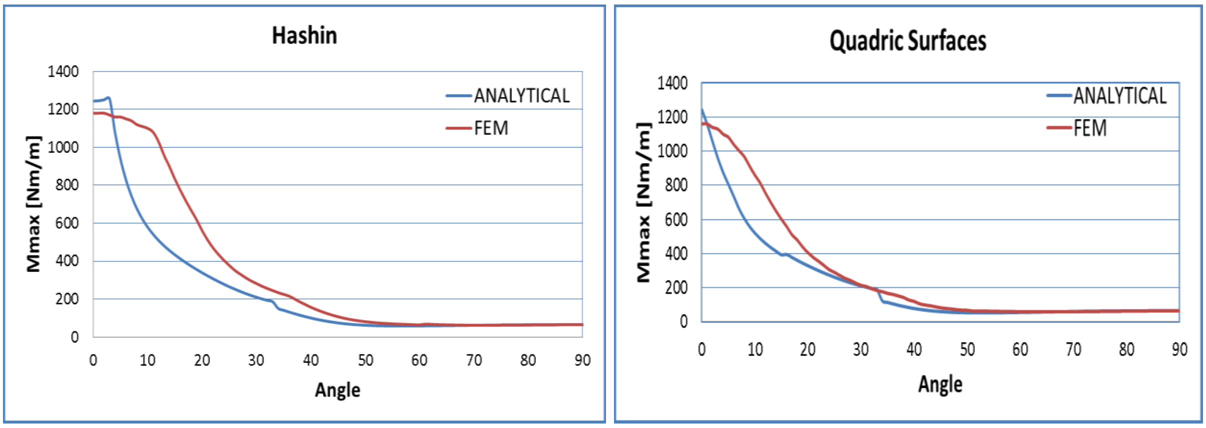

(a) (b)

**Figure 8.** Comparison of $M_{max}$ predictions of analytical and finite element methods with respect to (a)Hashin (b)Quadric Surfaces criteria for unidirectional off-axis $[\theta_6]_s$ specimens




(a) (b)

**Figure 9.** Comparison of $M_{max}$ predictions of analytical and finite element methods with respect to (a)Tsai-Wu (b)Maximum Stress criteria for multidirectional $[+\theta_3/-\theta_3]_s$ specimens




(a) (b)

**Figure 10.** Comparison of maximum $M_{max}$ predictions of analytical and finite element methods with respect to (a)Hashin (b)Quadric Surfaces criteria for multidirectional $[\theta_3/-\theta_3]_s$ specimens

## 6 Conclusions

The predictions for the maximum allowable bending moment obtained using different failure criteria are not consistent for many laminate configurations. Only after comparison with experimental results, one may conclude on the relative merits of the failure criteria. Residual stresses in multidirectional laminates affect the results remarkably, thus, should not be neglected. Predictions are also dependent on the out-of plane Poisson effect, which is neglected in CLT. The future work will involve conducting four - point bending experiments to determine how well each criterion predicts failure.

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