Ply-based Optimization of Laminated Composite Shell Structures under Manufacturing Constraints

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Abstract

This work concerns a new ply-based parameterization for performing simultaneous material selection and topology optimization of fiber reinforced laminated composite structures while ensuring that a series of different manufacturing constraints are fulfilled. The material selection can either be performed on the basis of different materials, and/or consist of discrete selection of the same orthotropic material with different orientations of the fibers. The problem considered is the optimization of a general laminated composite shell structure with respect to maximum stiffness (minimum compliance) with an additional constraint on the maximum allowable amount mass.

1 Introduction

The application of laminated composites in structural design gives the engineers the opportunity to tailor the material properties. Doing so can result in efficient and light weight structures when compared to applying traditional isotropic materials such as steel and aluminum. However, this design freedom also complicates the process from idea to a manufacturable product. When designing structures with complicated geometries and load scenarios as found in boat hauls, aircraft fuselages, and wind turbine blades, the application of efficient finite element analysis has become essential in order to validate the integrity of the structure.

Still, for these large structures where the number of plies may exceed several hundreds at different locations, determining a suitable layup can become a time consuming process. Hence, the application of numerical optimization algorithms in combination with a global finite element model can aid engineers during the design process to determine a suitable layup. Commonly engineers apply a parameterization which coincides with the finite element discretization, or similar grid formulations, where each design region is defined by either a single element or multiple groups of elements, also known as patches. These design regions are assigned a set of design variables e.g. the number of plies, ply-thicknesses, or angular orientation. However, in critical areas such as around ply-drops these global finite element models are not able to capture local states of stress surrounding these areas. In order to capture these local effects full 3D continuum or higher order shell models have to be applied. Nevertheless, the gain in precision is paid with additional time for performing the calculations. As a consequence, engineers tend to apply design guidelines/manufacturing constraints (MC) in combination with traditional shell element models so as to obtain an accurate solution for global quantities of the structure, while avoiding critical local effects through the manufacturing constraints. Implementing manufacturing constraints in an optimization algorithm either as explicit mathematical expressions or implicitly through the choice of parameterization will reduce the amount of time spend on post-processing the optimized design so as to obtain a manufacturable, high performance product.

1.1 Optimum Design under Manufacturing Constraints

Manufacturing constraints have been investigated to some extent, among which are thickness constraints to prescribe a maximum allowable variation in laminate thickness [1], and thickness continuity between layers in adjacent design regions [1-6]. Similarly, fiber continuity between layers in adjacent design regions has also been investigated [3,5,7,8]. Another example is to limit the number of identical contiguous plies so as to avoid matrix cracking [2,9-11]. Many of the referenced authors apply genetic algorithms as either the primary or as a secondary algorithm to determine a suitable layup; however, these algorithms are prone to apply an exhaustive amount of evaluations of the objective function, which typically is dependent upon a solution from a finite element analysis. Hence, these methods are not well suited for application on large industrial products. Other methods and algorithms have also been applied throughout the literature to optimize laminate layups, with and without manufacturing constraints, where the ones with most impact have been summarized in [12,13].

In a recent paper by Sørensen and Lund [14] four manufacturing constraints have been applied for topology and thickness optimization of laminated composites together with the so-called *Discrete Material Optimization* (DMO) method [15]. In their approach they apply generalized SIMP and RAMP interpolation schemes for multi-material interpolation, which rely on a system of sparse linear constraints to ensure that just a single material is chosen among the available candidates [16]. The four presented manufacturing constraints ensure fiber continuity in the individual layers within the design region, limit the maximum rate of ply terminations, limit the number of contiguous plies in each laminate, and prevent intermediate voids from appearing. The presented method is limited to applying a single orthotropic material; hence the material selection problem is instead to determine a suitable orientation of the fibers from a selection of predefined orientations. Also, their presented work is limited to maximum stiffness optimization subjected to a mass constraint.

2 Problem Formulation

The presented work is a generalization of the method developed by Sørensen and Lund [14], which enables the application of different materials e.g. orthotropic and isotropic so as to obtain sandwich like structures. Also, the current implementation enables the application of different objective and constraint functions e.g. maximize buckling load factors subjected to mass and compliance constraints just to name a few options. Furthermore, it permits analysis of general composite structures where the geometry can be subdivided into any number of predefined patches. These patches can either be assigned a unique set of design variables or be setup to share selected variables e.g. all patches can share the material selection variables in selected layers whereas each patch has its own set topology variables distributed to the individual layers.

2.1 Multi-Material Selection with Variable Thickness

The material selection is an integer problem where one single material has to be selected from a finite set, and thus selecting an optimum material is a combinatorial problem. The binary material selection variable can be specified for all layers in all patches as in (1).

$$x_{plc} = \begin{cases} 1 & \text{if material } c \text{ is selected in layer } l \text{ in patch } p \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

However, obtaining a solution using integer optimization methods, even for small problems, becomes computational overwhelming if a solution is to be obtained within a reasonable amount of time. Employing a continuous relaxation of the integer problem, in the form of interpolation functions, makes it possible to solve these problems using standard gradient based optimization algorithms. In this work, such an approach has been selected in the form of the DMO method as described in [16].

The variable thickness design can be obtained by introducing a binary topology variable $\rho_{pl} \in \{0, 1\}$ in all layers for all patches with properties as shown in (2).

$$\rho_{pl} = \begin{cases} 1 & \text{if there is material in layer } l \text{ in patch } p \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Hence, the thickness optimization is achieved by either adding or terminating individual plies. Still, as with the integer material selection problem, this approach also becomes a combinatorial problem. However, by employing a similar relaxation, as with the material selection, and adding a series of different manufacturing constraints, the problem can be successfully solved applying gradient based optimization methods. With these relaxations the effective constitutive matrix for a given layer can be determined as shown in (3) utilizing the generalized RAMP interpolation scheme [16]

$$\begin{aligned} &\mathbf{E}_{el}(\mathbf{x},\rho) = \left(\mathbf{E}_0 + \frac{\rho_{pl}}{1 + r\left(1 - \rho_{pl}\right)} \sum_{c=1}^{n^c} \frac{x_{plc}}{1 + q\left(1 - x_{plc}\right)} \Delta \mathbf{E}_c \right) \\ &\sum_{c=1}^{n^c} x_{plc} = 1 \qquad , \qquad r, q \in \mathbb{R} \mid r, q \geq 0 \end{aligned} \tag{3}$$

where $\Delta\mathbf{E}_c = \mathbf{E}_c - \mathbf{E}_0$ with $\mathbf{E}_0$ assigned the stiffness properties of void such that $\mathbf{E}_c - \mathbf{E}_0 \succ 0$ and $\mathbf{E}_0 \succ 0$. r and q are penalization powers utilized in the RAMP interpolation scheme.

2.2 Manufacturing Constraints

The manufacturing constraints applied in this work are almost identical to those presented in [14], where the changes made are to facilitate the application of patches and additional materials. Nevertheless, they are presented here for reference.

2.2.1 Identical Patch Material: MC1

Manufacturers typically apply fiber mats which have been cut to fit a specific region. Hence, applying a patch size which fits with the manufacturer's processing facilities will reduce the amount of time spent on post-processing the optimized design. This manufacturing constraint is implemented in how the optimization model is setup; hence, no explicit mathematical constraints are required. It is recognized, that applying a fixed patch layout will limit the design space significantly when compared to a setup where all elements are parameterized individually. However, from a practical point of view, patches are more desirable.

2.2.2 Thickness Variation Rate: MC2

To avoid problems with delamination and matrix cracking, the variation in laminate thickness can be constrained so a specified number of plies only can be dropped between adjacent patches. The constraint is setup according to how the patches are located, as shown in Figure 1.

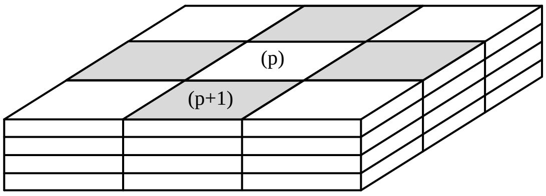

Figure 1: Thickness variation among adjacent patches

Here patch no. (p) has four neighboring patches among which is patch no. (p+1). The thickness variation is defined from a slope perspective, where the allowable number of layers, which can be terminated, is limited by (4) where $S \in \mathbb{N} \mid 0 \leq S \leq n^l - 1$.

$$-S \leq \sum_{l=1}^{n^l} \rho_{pl} - \rho_{(p+1)l} \leq S \quad \forall\ p \tag{4}$$

The constraint is dependent upon the specified patch layout, where each patch can have a unique number of neighbouring patches with the upper limit being that one patch has all other patches as neighbour's i.e. $(n^p - 1)$.

2.2.3 Limit Contiguous Fiber Orientations: MC3

If many contiguous plies with the same fiber orientation are stacked on top of each other, matrix cracking can become an issue. Hence, a constraint limiting the number of equal contiguous layers has been implemented. In the current implementation, the constraint is set to disregard isotropic material candidates. The specified limit of contiguous layers is defined as $CL \in \mathbb{N} | 1 \leq CL \leq (n^l - 1)$. The constraint is formulated as:

$$\sum_{l=z}^{z+CL} x_{plc} \leq CL \qquad \forall (p), | c \not\subseteq isotropic \qquad , \qquad z = 1, 2, \ldots, n^l - CL \tag{5}$$

2.2.4 Preventing Intermediate Void: MC4

From a manufacturing point of view closed intermediate voids inside a layered composite structure are undesirable. In order to avoid this phenomenon the following constraint scheme has been adopted. In [14] the specific constraint is referred to as scheme 4, and it allows for a smooth transition of the topology variables in contiguous layers given a limit threshold denoted by T as shown in (6).

$$\rho_{p(l+1)} \leq \begin{cases} y_1 = a_1 \rho_{pl} & \text{if} \quad \rho_{pl} \leq (1-T) \\ y_2 = a_2 \rho_{pl} + b_2 & \text{else} \end{cases}$$
$$a_1 = \frac{1}{a_2} = \frac{T}{1-T} \qquad , \qquad b_2 = 1 - a_2 \qquad , \qquad 0 \leq T \leq 0.5 \tag{6}$$

This approach results in the layers gradually being "build" from the bottom up. The constraint modifies the bounds of each topology variable and so has to be updated in each design iteration.

3 Numerical Examples

In the following two examples will be presented. Both examples utilize the same loading and boundary conditions which are of a corner hinged plate subjected to a point force in the center of the plate as shown in Figure 2.

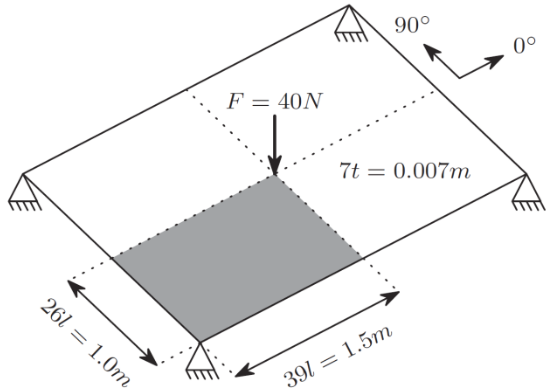

Figure 2: Corner hinged plate with point force

The plate is modeled using symmetry conditions; hence fiber angle continuity across symmetry lines is only satisfied for 0° and 90° orientations. However, the examples still serves as a proof of concept. The plate has seven equally thick layers, which are numbered so the bottom layer is number one; hence the top layer is number seven. The dimensions of the quarter plate are 1m by 1.5m which has been subdivided into 26x39 equally sized, square, nine-node shell elements. All layers in each element have been assigned a topology variable. The material selection is done on a patch level, where each layer spanning all elements is to be assigned a unique material candidate. The objective is to minimize compliance within a maximum allowable mass constraint. The bottom layer is constrained to have full mass. The optimization is thus focused on the selection of the material and its distribution within the bounds of the geometry. The examples have been solved using the SLP algorithm available in SNOPT 7.2, which has been implemented in the in-house finite element program suite named *MUltidisciplinary Synthesis Tool* (MUST) developed by the Department of Mechanical and Manufacturing Engineering at Aalborg University.

3.1 Example 1: Determination of fiber orientations and topology

This example is presented in [14] and is repeated so to compare the obtained results. The problem is to determine the optimum fiber orientations for all seven layers, where each layer is to be assigned a unique orientation throughout the span of the plate. The material is unidirectional GFRP with the following properties: E_1 = 34GPa, E_2 = E_3 = 8.2GPa, ν_{12} = 0.29, G_{12} =G_{13} = 4.5GPa, G_{23} = 4.0GPa, and ϱ = 1910.0kg/m3. The void material is defined as a massless isotropic material with a stiffness of $E=10^{-6}E_1$ and with Poisson's ratio ν = 0.29. The optimization specifications are as follows: The number of candidate orientations is set to four: 45º/-45º/90º/0º. The allowable thickness change is ±1 ply thickness i.e. S=1 in (4). The allowable number of consecutive layers is 2 i.e. CL=2 in (5). Intermediate void is not allowed, where the threshold parameter is set to T=0.1 in (6). Finally the mass constraint is set to half of the plate's full mass determined as when there is material in all layers.

3.2 Example 2: Determination of material, fiber orientation, and topology

This example is an expansion of the example 1, where the difference lies in the material selection now also contains an isotropic foam material. The foams material properties are: E_1=65Mpa, ν_{12} = 0.47, ϱ = 200.0kg/m3. The mass constraint has been modified so the limit is set to a mass equivalent to four GFRP plies and three ply-thicknesses of foam material.

4 Results

In this section the results from examples 1 and 2 are presented. For each example a 3D plot shows the obtained solution. Each plot is accompanied by a table showing the fiber orientations for the orthotropic material in the respective layers together with the final compliance and mass value. For both examples, the solutions have converged to full 0-1 designs for both material selection and topology variables.

4.1 Example 1

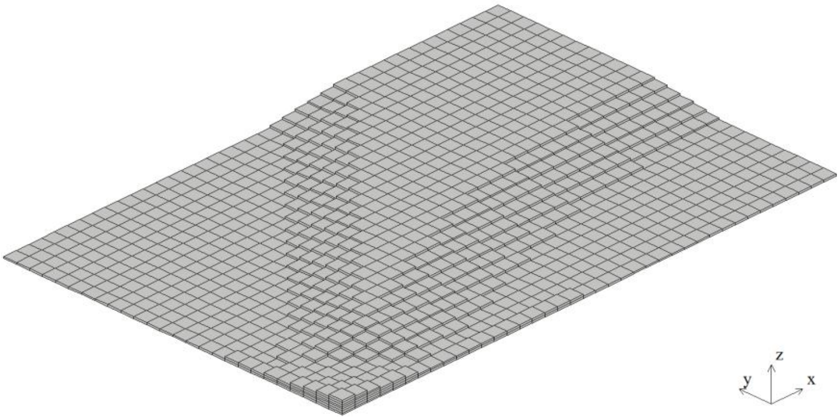

Figure 3 Density Distribution of Corner Hinged Plate Example 1 (Symmetric Modeling)

Layer	1	2	3	4	5	6	7
Orientation	+45	0	+45	0	+45	+45	0
Compliance	0.399354	Mass ≤ Max Mass		10.0275 ≤ 10.0275			

Table 1 Layup, Final Compliance, and Obtained Mass for Example 1

4.2 Example 2

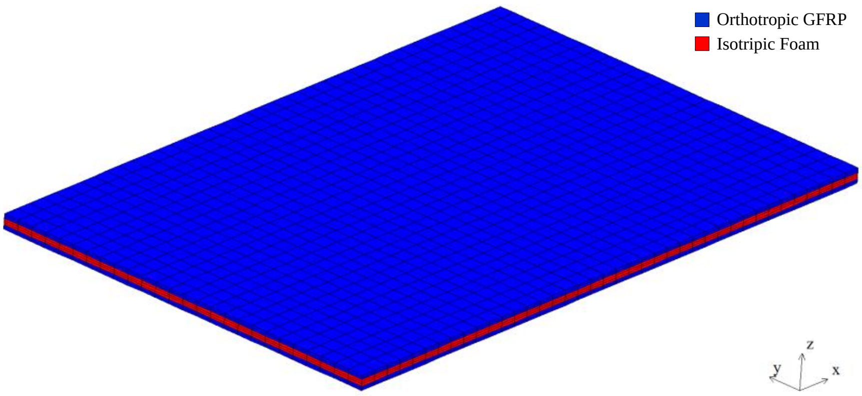

Figure 4 Material Distribution of Corner Hinged Plate Example 2 (Symmetric Modeling)

Layer	1	2	3	4	5	6	7
Orientation	0	0	-	-	-	0	0
Compliance	0.227554	Mass ≤ Max Mass		12.3600 ≤ 12.3600			

Table 2 Layup, Final Compliance, and Obtained Mass for Example 2

5 Discussions

5.1 Optimum results from example 1

The topology obtained in this example is quite similar to the solution shown in [14]. However, the layups are different between the two solutions. In [14] layers 1 and 2 are swapped in accordance with table 1, thus obtaining a symmetric layup. However, the solution is sensitive to the chosen penalization scheme and move-limit strategy. Thus a similar design could possibly be obtained by applying different schemes. Nevertheless, the results show that method is able to capture similar topologies as presented in [14]. For reference, the final compliance value obtained in [14] is 0.390443 for the example.

5.2 Optimum results from example 2

As can be seen from Figure (4) the optimized design represents a sandwich structure, where all layers in all patches have full density. The two top and two bottom layers have been assigned the unidirectional GFRP material orientated at 0 degrees in accordance with Figure (2). The three center layers have all been assigned the foam material; hence a sandwich design has been obtained. This design is in accordance with the specified mass constraint, which allows for such a design. Reducing the allowable amount of mass will thus result in a tapered sandwich design, however, further constraints on the material selection has to be implemented in order to control that the top and bottom layers in each element always represents face sheets made from fiber reinforced material. Otherwise, the foam material has the option for being the top layer, which would make an ill posed sandwich structure. This additional constraint is left for future work.

6 Conclusions

A generalization of the method developed by Sørensen and Lund [14] has been presented. The method is capable of performing simultaneous material selection and topology optimization on general laminated composite structures. For fiber angle and topology optimization, the results obtained are in good coherence with the results presented in [14] giving similar topologies. However, the obtained layups in the presented example differ slightly in comparison. For material, fiber angle, and topology optimization the method is capable of determining a sandwich design which fits with the assigned mass constraint. In the present work, compliance (maximum stiffness) optimization has been shown throughout; however, other criteria can be applied e.g. mass minimization, buckling load factors, or eigenfrequencies. Results obtained applying such objective functions will be outlined in the presentation at ECCM15.

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