FLEXURAL PLATES WITH HETEROGENEOUS IN-PLANE STIFFNESS

J. Šliseris[1*], K. Rocēns[1]

[1]*Institute of Building and Reconstruction, Department of Structural Engineering, Riga Technical University, Kaļķu street 1, Riga, Latvia.*
**e-mail: janis.sliseris@gmail.com*

Keywords: plywood plate optimization, discrete stiffness, stress concentration, Genetic Algorithm.

Abstract

A new optimization technique of ribbed plates with discrete varying (heterogeneous) stiffness is proposed. It consists of analysis of optimal continuous varying stiffness, division of plate into discrete domains and sizing optimization of plate's internal structure. The optimal continuous varying stiffness is obtained by minimizing structural compliance and differences of stress fields. The geometry of plate's internal structure is optimized by using trained Feed-Forward Artificial Neural Network and Genetic Algorithm. A method to avoid of stress concentrations in connections of discrete domains is proposed.

1 Introduction

The Plates and shells with variable stiffness are investigated in past two decades. Optimization of fiber orientation angle [1] or thickness optimization [2] problems are usually referred to variable stiffness plates. Optimal properties of plate or shell in cases of complicated objective function or many design variables are obtained by using Genetic Algorithm [3,4] or Ant Colony algorithm [5,6].

A significant material reduction could be reached by using sandwich structures instead of lamina in cases of thick flexural plates [7,8,9]. Sandwich plates could be modeled by using any of plates theory in case if effective elastic properties of core are known. Effective elastic properties of sandwich could be obtained analytically or by using finite element method [10,11].

A numerical simulations and experimental investigation [12] show that properties of core material and skins of flexural sandwich plates should not be uniform. This problem related to multilayer lamina is successfully solved using topology optimization approach [13,14], discrete material optimization method [15], Ant colony algorithm [16] or Genetic algorithm [17, 18].

Sandwich plates with continuously or discrete varying stiffness is not enough investigated yet. Therefore there are proposed a new optimization technique of plates with discrete varying bending and shear stiffness and analyzed stress concentration in connections of discrete domains.

2 Analysis of continuous variable stiffness

Optimal stiffness of orthotropic plate is defined by five independent functions (1). They are used to modify the coupled bending and shear stiffness matrix D^0. The objective function $C(x)$ (2) consists of two parts. The first part is used to minimize the structural compliance function $c(x)$ and second part to minimize stress gradient $\sigma(x)$. The minimization is done using optimal criteria procedure. The second part is necessary to obtain the structure with maximal equally stressed all regions of plate. The minimal stress gradients is achieved by minimizing the absolute value of difference between average stress and stress in each point of plate. Stress gradients of compressions and tension stress components are minimized in the outer planes of plate, but differences of shear stress components are minimized in the middle plane of plate. Each part of objective functions is multiplied by normalized weight parameters h_1, h_2. If there are not special requirements they could be assumed to be equal $h_1 = \frac{1}{2c_{\max}}$, $h_2 = \frac{1}{2\delta\sigma_{kl}^{\max}}$.

$$x = (x^1, x^2, x^3, x^4, x^5) \tag{1}$$

$$\min_x : C(x) = c(x) + \sigma(x) = h_1 U(x)^T \cdot K(x) \cdot U(x) + \dots$$
$$+ h_2 \sum_{i=1}^{Ne} \left(\int_{S^i} \| \sigma_{kl}^i(x) | - | \sigma_{kl}^{av}(x) \| \, dS^i \right) \tag{2}$$

Subjected to:

$$\frac{\int_\Omega x^i d\Omega}{area(\Omega)} = k_i, k_i \in (0,1) \tag{3}$$

$$K(x) \cdot U(x) = F(x) \tag{4}$$

$$0 < x_{i,min} \le x_i \le 1 \tag{5}$$

$$D_{ij} = x_i D_{ij}^0 \tag{6}$$

where D_{ij} - coefficients of material bending stiffness matrix of plate according to first-order shear deformation theory($i = 11,12,21,22,33$ - coefficients of bending stiffness, $i = 44,55$ - coefficients of shear stiffness), Ne - total number of finite elements, $\sigma_{kl}^i(x)$ - function of stress component $kl = 11,22,12,13,23$, $\sigma_{kl}^{av}(x)$ - averaged stress component kl in the structure, $K(x)$ - structures stiffness matrix, $U(x)$ - structures displacement vector, $F(x)$ - structures force vector, $c_{\max}$ - maximal value of compliance field of non-optimized strucutre, $\delta\sigma_{kl}^{\max}$ - maximal difference of stress fields of non-optimized structure.
Optimization problem (1)-(6) is solved by gradient based approach.

3 Discrete domain optimization and sizing optimization

Practically it is difficult to manufacture a constant thickness plate with continuously varying stiffness. Therefore it is proposed to divide the plate into discrete domains with constant stiffness in each domain. A computational method of optimal division of plate into discrete domains will be described in this section.

The method is based on the minimization of function of square of difference between interopolated or approximated stiffness coefficients $X^j(y_1, y_2)$ and averaged discrete stiffness coefficients $-\bar{X}_i^j(\Omega_i)$ in each domain:

$$minF(\Omega_i) = \sum_{j=1}^{5}\sum_{i=1}^{n} w_j \left(X^j(y_1, y_2) - \overline{X}_i^j(\Omega_i) \right)^2 \tag{7}$$

The optimal geometrical properties of plates of internal structure- $p = p(p_1, p_2, ..., p_n)$ are obtained by minimizing the functions $P(p)$

$$minP(p) = k_i \mid MQ^{max}(p)_i - MQ(p)_i \mid, i = 1..5 \tag{8}$$

where $MQ^{max} = (M_x^{max}, M_y^{max}, M_{xy}^{max}, Q_{xz}^{max}, Q_{yz}^{max})$ - maximal bending moments, twisting moment and shear forces that could be carried by current dimensions of cross section, $MQ = (M_x, M_y, M_{xy}, Q_{xz}, Q_{yz})$ - maximal bending moments, twisting moment and shear forces in current discrete domain, $k_i, i = 1..5$ - weight coefficients, calculated according to equation

$$k_i = \begin{cases} \dfrac{MQ(p)_i}{\sum_{j=1}^{5} MQ(p)_j}, \left(MQ^{max}(p)_i - MQ(p)_i\right) > 0 \\ (1+k)\dfrac{MQ(p)_i}{\sum_{j=1}^{5} MQ(p)_j}, \left(MQ^{max}(p)_i - MQ(p)_i\right) < 0, k > 0 \end{cases} \tag{9}$$

where k - coefficient that are used to avoid from cross section with less load bearing capacity than necessary.

The vector MQ^{max} is calculated by using trained feed-forward three layer Artificial Neural Network [19,20]

$$MQ^{max} = f^2\left(W^{2,1} f^1(W^{1,1} p + b^1) + b^2\right) \tag{10}$$

where $p = p(i)$ - vector consisting of geometrical properties of plates internal structure, f^1, f^2 - activation functions, $W^{2,1}, W^{1,1}$ - weight matrix, b^2 - bias vector.

4 Results

Optimization is done for a rectangular plate divided into 16 discrete regions. The internal cell type structure of plate is chosen as shown in (fig. 1, where p_1 - width of cell, p_2 - length of cell, p_3 - thickness of curved rib with constant curvature, p_4 - thickness of straight rib, p_5 - thickness of top skin sheet, p_6 - thickness of bottom skin sheet, p_7 - total thickness of plate. Structural elements of plate are created form plywood with elastic properties- $E_1 = 16400MPa$, $E_2 = 680MPa$, $G_{12} = 890MPa$, $G_{13} = 1540MPa$, $G_{23} = 230MPa$, $\mu_{12} = 0.04$, $\mu_{13} = 0.043$, $\mu_{23} = 0.48$. The plate are simply supported on all and two edges with uniformly distributed load $q = 1KPa$.

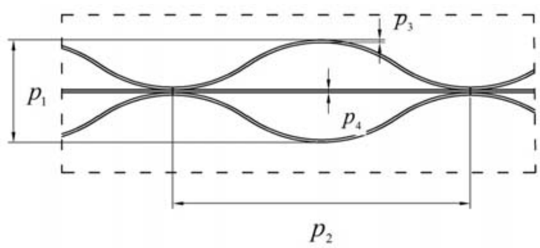

Figure 1. Internal structure of cell type plate [21].

Due to symmetry of structure one quarter- 4 discrete regions are optimized. Discrete domains are defined with two independent parameters $p(x,1), p(y,1)$. Optimal values of these parameters are obtained using Genetic Algorithm. Comparison of maximal deflection of plate with the same material consumption is shown in figure 2. It shows that it is not necessary to divide plate in large number of discrete regions to obtain plate with optimal stiffness.

The dimensions of structural elements are optimized in two steps. In first step is trained ANN but in second step optimal parameters ($p = (p_1, p_2, p_3, p_5)$) are obtained using GA and trained ANN. Optimal number of neurons are 15 and amount of training data should not be less than 300 different cases when input vector consists of 4 parameters and output vector of 5 parameters. Other parameters were assumed to be constant- $p_4 = 0.0065m$, $p_7 = 0.25m$, $p_6 = p_5$. Genetic Algorithm is stochastic method therefore several independent runs were done and most optimal properties were chosen.

The structure with optimized parameters is compared with non-optimized. Case 1-5 indicate plate with suports on all edges and length 6m, width 4.5m, but case 6-9 indicate plate with suports on two edges with span 6m and width 2m. Other geometrical properties of analysed cases are shown in table 1.

Plates are compared by using specific strength criteria of maximal normal stress ($f_{c,0,k}$=27.7 MPa), shear stress ($f_{v,k}$=9.5 MPa) and deflection (maximal deflection- 1/200*span). Results are shown in table 2. and table 3.

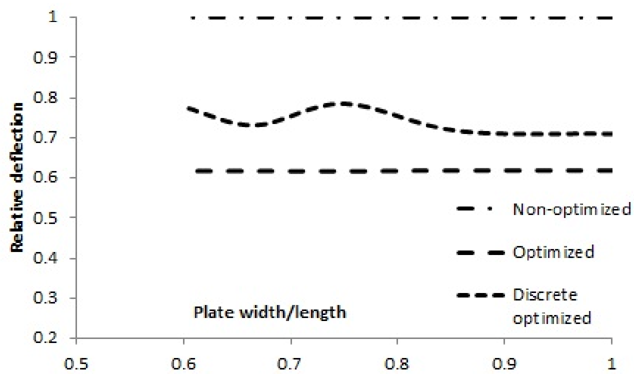

Figure 2. Relative deflection (relative to deflection of non-optimized structure) of optimized, discrete optimized (16 discrete domains) and non-optimized structure depending on ratio of plate width and length.

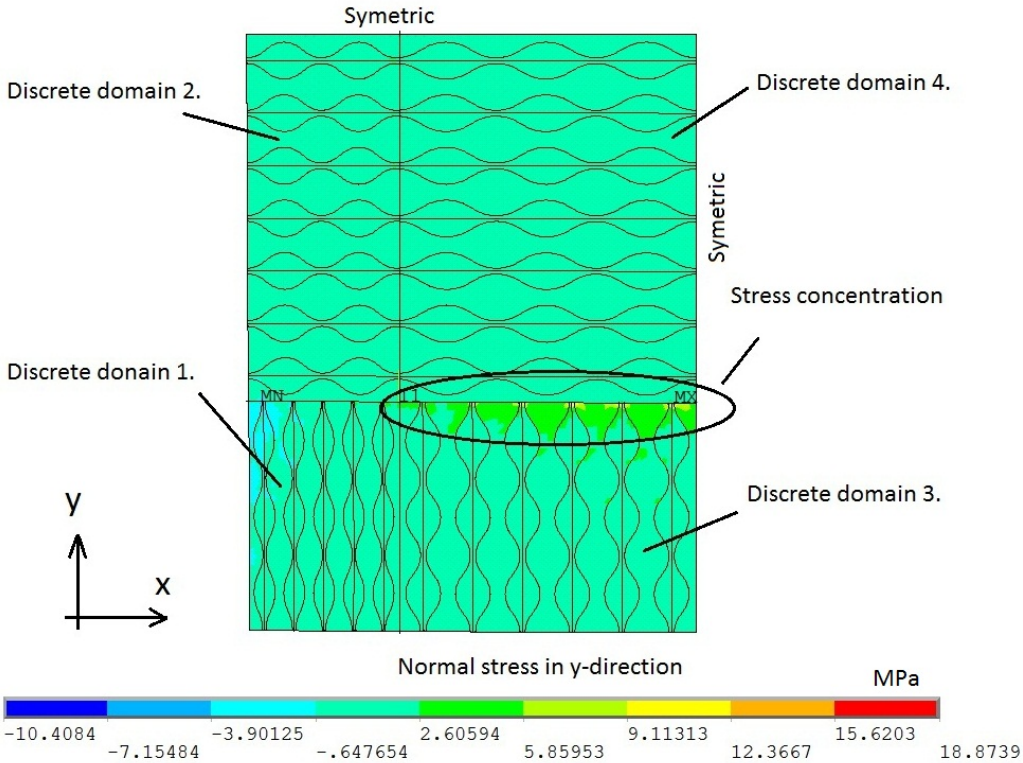

Figure 3. Normal stress field in optimized plate with discrete structure (case 3).

A significant stress concentration appears in connection of domain 3 and 4 as shown in figure 3. It could be explained by the fact that in domain 3 main load carrying element is curved ribs, but in domain 4- top and bottom skins and connection between them are poor. Therefore it is recommended to use thicker skins in the stress concentration zone, or obtain optimal stiffness by changing only ribs and skins keep constant thickness in all plate.

Case no.	Domain 1.					Domain 2.					Domain 3.					Domain 4.				
	p_1, mm	p_2, mm	p_3, mm	p_5, mm	α, [deg]	p_1, mm	p_2, mm	p_3, mm	p_5, mm	α, [deg]	p_1, mm	p_2, mm	p_3, mm	p_5, mm	α, [deg]	p_1, mm	p_2, mm	p_3, mm	p_5, mm	α, [deg]
1	300	560	20.4	3.6	0															
2	300	560	20.4	12	0															
3	249	670.3	31.95	4.473	90	378.9	576.6	29.28	22.46	0	348.7	583	31.95	14.8	90	342.8	616.5	6.518	31.95	0
4	249	670.3	31.95	4.473	90	378.9	576.6	29.28	22.46	0	348.7	583	31.95	7.165	90	342.8	616.5	6.518	31.95	0
5	249	670.3	31.95	22.46	90	378.9	576.6	29.28	22.46	0	348.7	583	31.95	22.46	90	342.8	616.5	6.518	22.46	0
6	300	560	20.4	3.6	0															
7	300	560	20.4	12	0															
8	300	560	12	12	0	300	560	6	12	0										
9	300	560	12	6	0	300	560	6	12	0										

Table 1. Geometrical parameters of plates internal structure, α- orientation angle with x axis (see fig. 3), case 1-5: thickness 0.25m, length (x axis) 4.5m, width (y axis) 6m, plate simply supported on all edges, case 6-9: thickness 0.25m, length (x axis) 6 m, width (y axis) 2m, plate simply supported at x=0 and x=6. Cases 1,2,6,7 are non-optimized, others are optimized.

Case no.	Mass, kg	Specific strength, Kpa/kg		
		Normal stress	**Shear stress**	**Deflection**
1	249.0	0.0079	0.0404	0.0565
2	328.4	0.0224	0.0557	0.0957
3	373.8	0.0030	0.0090	0.0177

4	394.1	0.0051	0.0085	0.0168
5	417.1	0.0470	0.0557	0.0992

Table 2. Specific strength of simply supported on all edges, 5.4x6m, 0.25m thick plate according to normal stress criteria, shear stress criteria and deflection criteria.

Case no.	Mass, kg	Specific strength, Kpa/kg		
		Normal stress	Shear stress	Deflection
6	145.7	0.0434	0.1039	0.1003
7	145.3	0.0294	0.0845	0.0774
8	130.8	0.0473	0.1138	0.1088
9	120.1	0.0253	0.0647	0.0680

Table 3. Specific strength of simply supported on two edges, 2x6m (span 6m), 0.25m thick plate according to normal stress criteria, shear stress criteria and deflection criteria.

According to results shown in table 1 biggest specific strength is achieved by case 5, where the top and bottom skins are constant thickness and only geometry of ribs are varied to obtain necessary stiffness in each domain. The same situation appears when analyze simply supported plate on two edges. Biggest specific strength is achieved by keeping constant thickness skins and modifying only geometry of ribs (case 8) to obtain necessary stiffness in each discrete domain.

5 Conclusions

A new optimization method consisting of analysis of optimal bending and shear stiffness, dividing into optimal discrete domains and optimizations internal structure of plates with discrete varying stiffness is proposed.
Significant normal stress concentration appears into connection between discrete domains of optimized plate. It could be significantly reduced by increasing thickness of top and bottom skins in that zone or by using constant and large enough thickness top and bottom skins in all domains.
The reduction of deflection of rectangular simply supported on all edges plate could be obtained up to 40% in case of optimized internal structure of plate for uniformly distributed load.
In the next work it is necessary to analyze stress concentration zones taking into account material nonlinearity more detail and optimize plate for more complicated load cases.

Acknowledgments

This work has been supported by the European Social Fund within the project "Support for the implementation of doctoral studies at Riga Technical University".

References

[1] Keller D. Optimization of ply angles in laminated composite structures by a hybrid, asynchronous, parallel evolutionary algorithm. *Composite Structures*, **92**, pp. 2781-2790 (2010).
[2] Almeida F.S., Awruch A.M. Design optimization of composite laminated structures using genetic algorithms and finite element analysis. *Composite Structures*, **88**, pp. 443-454 (2009).

[3] Sliseris J., Rocens K. *Rational structure of panel with curved plywood ribs* in "Proceedings of ICBSE 2011 : International Conference on Building Science and Engineering", Venice, Italy, pp. 317-323 (2011).

[4] Sliseris J., Rocens K. Optimization of multispan ribbed plywood plate macro-structure for multiple load cases. *Journal of Civil Engineering and Management* (2012) Not published jet.

[5] Sebaey T.A., Lopes C.S., Blanco N., Costa J. Ant Colony Optimization for dispersed laminated composite panels under biaxial loading. *Composite Structures*, **94**, pp. 31-36 (2011).

[6] Hudson C. W., Carruthers J.J., Robinson A.M. Multiple objective optimisation of composite sandwich structures for rail vehicle floor panels. *Composite Structures*, **92**, pp. 2077-2082 (2010).

[7] Sliseris J., Rocens K. *Non-uniform moisture inuence on multilayer corrugated plywood shell* in "Proceedings of 9th Nordic Symposium on Building Physics- NSB", Tampere, Finland, pp. 723-730 (2011).

[8] Sliseris J., Rocens K. Curvature analysis for composite with orthogonal, asymmetrical multi layer structure. *Journal of Civil Engineering and Management* , **16,** pp. 242-248 (2010).

[9] Sliseris J., Rocens K. *Behaviour of multilayer sheet with technogical imperfection*, in "Proceedings of 1oth International Conference Modern Building Materials, Structures and Techniques", Vilnius, Lithuania, pp. 793-798 (2010).

[10] Chen D.H., Yang L. Analysis of equivalent elastic modulus of asymmetrical honeycomb. *Composite Structures*, **93**, pp. 767-773 (2011).

[11] Manalo A.C., Aravinthan T., Karunasena W. Flexural analysis of discontinuous tile core sandwich structure. *Composite Structures*, **94**, pp. 1524-1532 (2012).

[12] Manalo A.C., Aravinthan T., Karunasena W. Flexural behaviour of glue-laminated fibre composite sandwich beams. *Composite Structures*, **92**, pp. 2703-2711 (2010).

[13] Díaz A., Bendsøe M. Shape optimization of structures for multiple loading conditions using a homogenization method. *Structural and Multidisciplinary Optimization*, **4**, pp. 17-22 (1992).

[14] Bendsøe M. Optimal shape design as a material distribution problem. *Structural and Multidisciplinary Optimization*, **4**, pp. 193-202 (1989).

[15] Stegmann J., Lund E. Discrete material optimization of general composite shell structures. *International Journal for Numerical Methods in Engineering*, **62**, pp. 2009–2027 (2005).

[16] Kaveh A., Hassani B., Shojaee S., Tavakkoli S.M. Structural topology optimization using ant colony methodology. *Engineering Structures*, **30**, pp. 2559-2565 (2008).

[17] Hansel W., Treptow A., Becker W., Freisleben B. A heuristic and a genetic topology optimization algorithm for weight-minimal laminate structures. *Composite Structures*, **58**, pp. 287-294 (2002).

[18] Goldberg D. *Genetic Algorithms in Search, Optimisation and Machine Learning*. Addison-Wesley, USA (1989).

[19] Garson G. Interpreting neural network connection weights. *Articial Intelligence Expert*, **6**, pp. 47-51 (1991).

[20] Man H., Prusty G. Neural network modelling for damage behaviour of composites using full fileld strain measurements. *Composite Structures*, **93**, pp. 383-391 (2011).

[21] Šliseris J., Rocēns K. *Manufactureing technology of cell plastic type ribbed structure based on wood materials and layered composite plate*. Latvian patent No. P-12-52, 2012.