FATIGUE DIAGNOSIS IN COMPOSITES-A ROBUST BAYESIAN APPROACH

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Abstract
A general bayesian framework is proposed for the statistical description of the fatigue damage evolution in composite materials. A parameterized Markov chain model is proposed for the evolution of fatigue damage and a bayesian inverse problem is formulated leading to a rational way to incorporate full information from fatigue data to model parameters for a specific parameterization. This methodology has been validated for damage data from the literature, considering damage as a stiffness reduction over several open hole quasi-isotropic glass-fiber composite coupons under tension-tension fatigue loads.

1 Introduction

Fatigue in composite materials is a complex-multiscale cumulative damage process, starting at the beginning of the lifespan [1]. During 80s and 90s, fatigue damage covered an important area of the composites research topic and nowadays there are a wide spread of fatigue models, all of them valid in its range of application [2]. Recently, probabilistic damage approaches are emerging as a suitable tool for fatigue in composites materials [3]. Among them, Markov chain models have shown ability to account the uncertainty in the fatigue response in composites along complete fatigue process [4, 5]. However, the increasing need to improve the model predictability for fatigue diagnosis in a robust sense, requires updating the initial belief on stochastic models using field data.

In this work, a bayesian framework to infer a non stationary Markov chain model from damage data is presented, conferring the main step to achieve fatigue diagnosis in composites materials. To this end, the likelihood function for a nonstationary Markov Chain is imported from the mathematical literature [6, 7] and adapted for the parameterization originally proposed. This framework is validated against a data set of damage, considered as a progressive stiffness reduction, for several open hole quasi-isotropic glass-fiber composite coupons under tension-tension fatigue loads. As result, the *a posteriori* information about a set of Markov chain parameters can be obtained and further used for the robust reconstruction of the damage growth process.

This approach confers an efficient way to update the initial believe on a particular fatigue model using measured data, and in general, to treat evolutive random processes in composites.

2 Methods

The evolution of fatigue damage as a function of cycles is proposed to be modeled by Markov chains, under the main hypothesis established by the *Markov property*, which states that the *future* of the process only depends on its *present* state, which is independent of the *past*. This phenomenological stochastic approach is based on the theory of Markov Chains [8] and assumes the following underlying assumptions [9, 10]:

1. Damage is a nondecreasing random variable and it passes through an integer and finite number of states, $j = 1, 2, ..., s$, until the "absorbing" state s is reached.
2. The time period N over which damage may accumulate is discretized in integer units of duty cycles (DC), $n = 0, 1, ..., N$.
3. Damage is only considered at the beginning and the end of a DC, without taking into account what is happening within a DC.
4. Damage can only increase within a DC from the state at the start of that DC to the next state.

It follows from the previous remarks that the proposed model is a finite-state (1), discrete-time (2), embedded (3) Markov process in which the damage accumulation mechanism is of the unit-jump type (4). At each integer time n, there is an integer-valued random variable (rv) D_n called the *damage state* at time n and the damage process is family of rv's $\{D_n; n \geqslant 0\}$.

Let then the rv D_n represents the damage state at time or duty cycle n. Thus the probability of D_n to be in state j at time n is denoted by

$$\mathrm{P}\left[D_n = j\right] = p_n(j) \tag{1}$$

The probability mass function of the rv D_n at time n is given by the vector

$$\mathbf{p_n} = \{p_n(1), p_n(2), ..., p_n(s)\} \tag{2}$$

where

$$\sum_{j=1}^{s} p_n(j) = 1 \tag{3}$$

From the theory of stochastic processes, the probability density function (PDF) of damage after a given number of duty cycles N, $\mathbf{p}_N$, is determined by the PDF of the initial damage state, $\mathbf{p}_0$, and the probability transition matrices (PTM), P_n, as

$$\mathbf{p}_N = \mathbf{p}_0 \prod_{n=0}^{N} P_n \tag{4}$$

The PTM summarizes the allowed transitions between damage states. Thus they adopt the form:

$$\mathbf{P}_n = \begin{pmatrix} p_1(n) & q_1(n) & & & \\ & p_2(n) & q_2(n) & & \\ & & \ddots & \ddots & \\ & & & p_{s-1}(n) & q_{s-1}(n) \\ & & & & 1 \end{pmatrix} \quad (5)$$

where the $p_j(n)$ and $q_j(n)$ are conditional probabilities that determine if the current damage state remains or proceeds to the next state at time n, respectively.

2.2 Model Parameterization

For the purpose of inference, the fatigue model described above must be parametrized by setting the transition probability matrix $\mathbf{P}^{\boldsymbol{\theta}}$ dependent on a vector $\boldsymbol{\theta}$ of model parameters. So, let define $\mathbf{P}^{\boldsymbol{\theta}} = p_{ij}^{\boldsymbol{\theta}}(n)(i,j = 1,\ldots,s;\; n = 0,\ldots,N)$ the probability of state j at time n given state i at time $n-1$. A valid parameterization of this stochastic model can be obtain as follows:

$$\mathbf{P}^{\boldsymbol{\theta}} = \mathbf{p}_0 \begin{pmatrix} \theta_5 & 1-\theta_5 & & & \\ & \theta_5 & 1-\theta_5 & & \\ & & \ddots & \ddots & \\ & & & \theta_5 & 1-\theta_5 \\ & & & & 1 \end{pmatrix}^{\alpha} \quad (6)$$

with $\alpha = n \times PMS^1\,(\theta_1, \theta_2, \theta_3, \theta_4),\;\; 0 \leqslant \theta_j \leqslant 1 \quad j = 1\ldots 5.$

In this model, the nonstationarity is accounted by means of an unitary time transformation while the probabilities of transition between states $p_{ij}^{\mathcal{M}}$ remain time-invariant.

2.3 Bayesian Inverse Problem

In Bayesian statistics, two fundamental states of the information are used for the statistical inference: the *a priori* information, that gives the initial relative belief of the model that is obtained independently of the results of measurements, and the *likelihood function* which express the information of idealized relationship between data and model in the sense of how good a model is in explaining the data. In bayesian literature, both together form the basis of a model class $\mathcal{M}$ [11]. It follows that the a priori information, the likelihood function and model parameters can all be described using probability densities. The former is defined by $p(\boldsymbol{\theta}|\mathcal{M})$ whereas the likelihood function is given by $p(\mathcal{D}|\boldsymbol{\theta}, \mathcal{M})$, in both cases provided by the model class $\mathcal{M}$.

The Baye's Theorem over probability densities, combines them to yield the *a posteriori* probability $p(\mathcal{D}|\boldsymbol{\theta}, \mathcal{M})$ of the hypothetical model specified by $\boldsymbol{\theta}$ in the class $\mathcal{M}$,

[1]Monotonic Cubic Spline

$$p(\boldsymbol{\theta}|\mathcal{D},\mathcal{M}) = c^{-1}p(\mathcal{D}|\boldsymbol{\theta},\mathcal{M})p(\boldsymbol{\theta}|\mathcal{M}) \tag{7}$$

where c is a normalizing constant that is needed for $p(\boldsymbol{\theta}|\mathcal{D},\mathcal{M})$ to fulfill the Theorem of Total Probability:

$$\int_{\boldsymbol{\Theta}} p(\boldsymbol{\theta}|\mathcal{D},\mathcal{M})d\boldsymbol{\theta} = c^{-1}\int_{\boldsymbol{\Theta}} p(\mathcal{D}|\boldsymbol{\theta},\mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})d\boldsymbol{\theta} = 1 \tag{8}$$

Notice that Bayes' Theorem takes the initial quantification of the plausibility of each model specified by $\boldsymbol{\theta}$ in the model class $\mathcal{M}$, which is expressed by the prior probability distribution, and updates this plausibility by using the information in the data $\mathcal{D}$ expressed through the likelihood function. For the case of a discrete time Markov chain stochastic model, the likelihood function is based on the Whittle formula [6].

The constant c is a normalizing constant in Bayes' Theorem and so it does not affect the shape of the posterior distribution. By means of this property, stochastic simulation based on Markov Chain Monte Carlo methods can be introduced to obtain the solution of Equation 7.

3 Results

The proposed framework is illustrated in an example considering stochastic damage data from literature [12] for sixteen quasi-isotropic open-hole S2-glass laminates subjected to a constant amplitude tension-tension fatigue loading ($R = 0.1, f = 5Hz, \sigma_{max} = 0.5 \times \sigma_u$). In this experiment, the observed data $d^{(k)} \in \mathcal{O}$ came from measurements of relative stiffness decreases for each k laminate defined as follows:

$$d^{(k)}(n) = \frac{E_0^{(k)} - E^{(k)}(n)}{0.6E_0^{(k)}} \tag{9}$$

E_0 is the initial longitudinal stiffness, $E(n)$ is a stiffness sample measurement in n. For this data, the most suitable value for duty cycle n was considered to be 500 load cycles with a Markov chain assembly of $s = 25$ states. The total number of duty cycles results in $N = {}^{213900}/_{500} = 428$.

For simplification in obtaining the posterior of model parameters, the non-informative distribution $p(\boldsymbol{\theta}|\mathcal{M}) = \mathcal{U}(0,1)$ is assumed which means that, without lack of generality, the parametric inference is contributed solely by the likelihood function.

To simulate the process, the Metropolis-Hastings algorithm has been implemented with $N = 10^4$ trials and proposal variance $\sigma = 0.02$. The algorithm configuration was verified to ensure the chain is ergodic and hence converges to Equation 7 by choosing the first sample distributed according the target PDF. And also by observing that the sample stabilizes (to the expectation to target distribution) after the burn-in period, in this case of 115 samples.

The results for the inference of posterior parameters given data are shown in Figure 1.

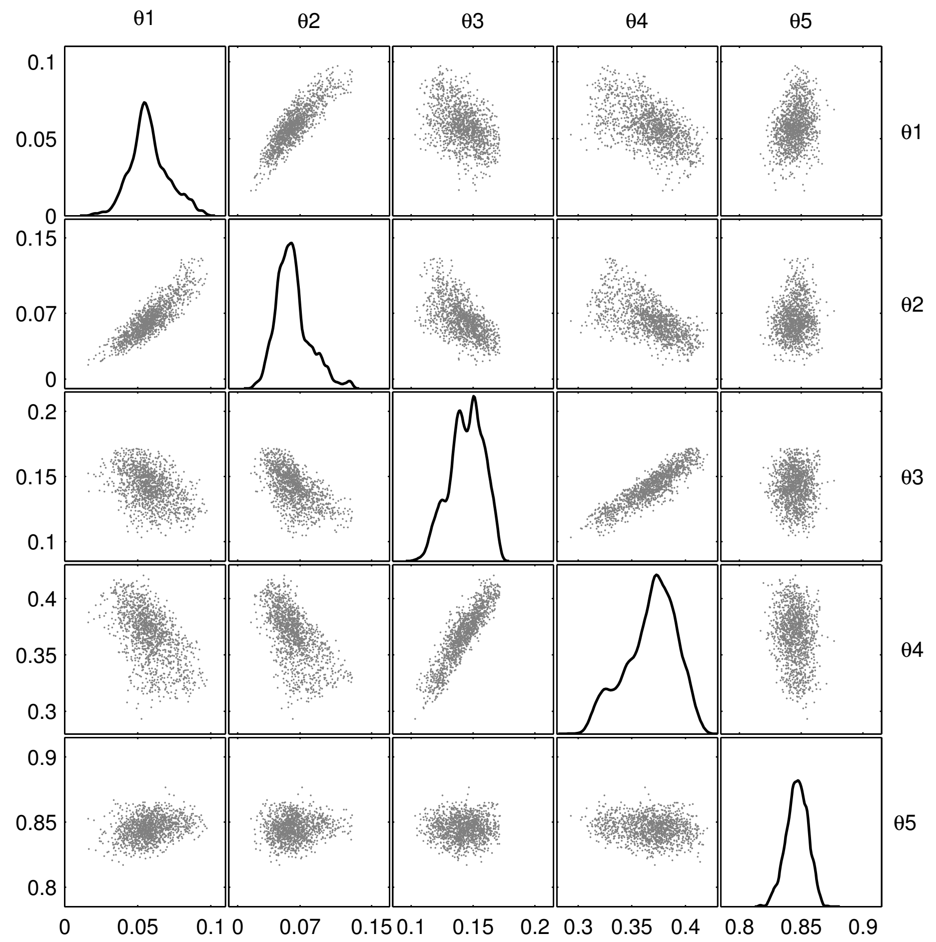

Figure 1: Plots of the samples in the $\boldsymbol{\Theta}$ space when updating model class $\mathcal{M}$ with fatigue data $\mathcal{D}$. In the diagonal, histograms and kernel density estimate construction for parameters.

The damage diagnosis of the fatigue process, as a result of the robust reconstruction of damage over the parameter space $\boldsymbol{\Theta}$, can be obtained as follows,

$$p(X|\mathcal{D},\mathcal{M}) = \int_{\boldsymbol{\Theta}} p(X|\boldsymbol{\theta},\mathcal{D},\mathcal{M})p(\boldsymbol{\theta}|\mathcal{D},\mathcal{M})d\boldsymbol{\theta} \tag{10}$$

where $p(X|\mathcal{D},\mathcal{M})$ is the PDF of damage considering data $\mathcal{D}$ and $p(X|\boldsymbol{\theta},\mathcal{D},\mathcal{M})$ represent the PDF of damage given model parameter $\boldsymbol{\theta}$ and data $\mathcal{D}$.

In Figure 2, the damage reconstruction of the fatigue process based on Equation 10 is represented at several damage steps.

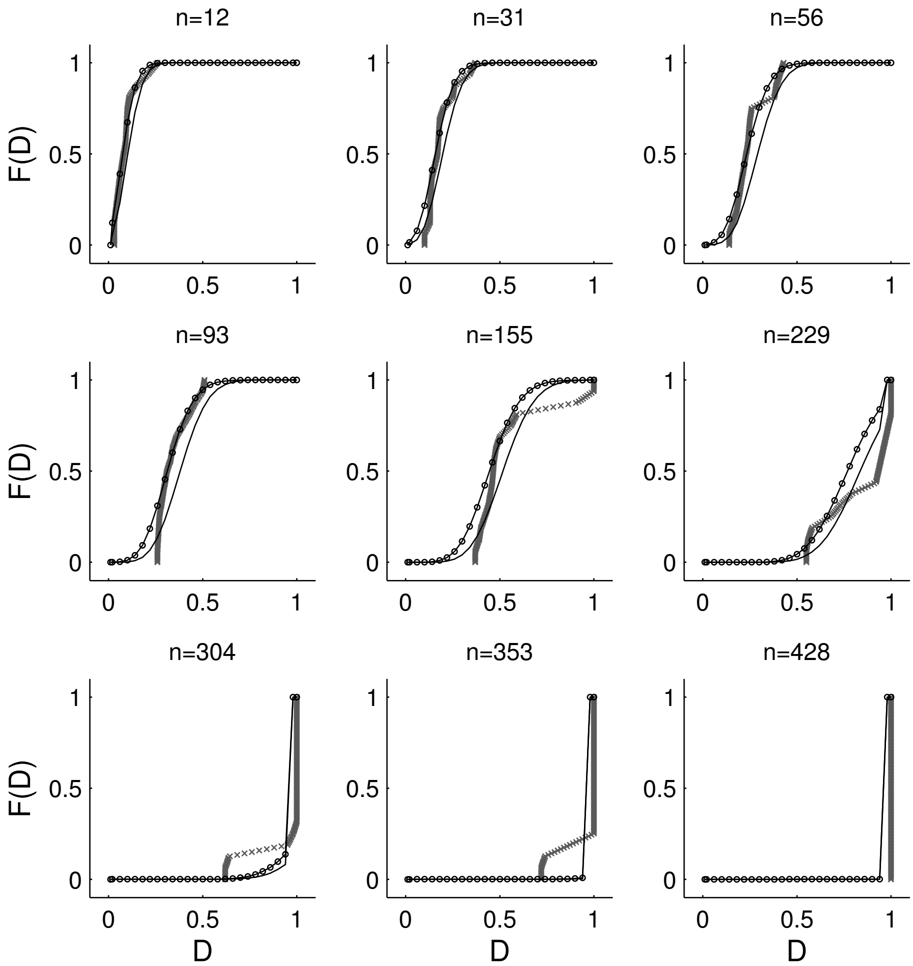

Figure 2: Representation of cumulative distribution functions (CDF) of damage for different values of duty cycles (n), when using the proposed parametrized Markov model. In grey: empirical CDF; dot-line: robust model CDF; solid line: optimum model CDF

4 Conclusions

A new framework is proposed to infer the fatigue-based damage evolution in composites, as solution of a general bayesian inverse problem. This framework has the versatility for accounting for all possible information about data, model and relation between both. Given the required simplifications, this capability can be used for parameter estimation in fatigue testing, for model updating with monitoring damage data, or even for selecting the model classes with best evidence for a specific material data set. The methodology has been validated on a example for obtaining the posterior PDF of model parameters from two nonstationary models, in terms of fitting to stochastic damage data. It has been shown that the posterior PDF and the associated model evidence can be obtained using a Markov Chain Monte Carlo method like Metropolis Hastings algorithm with a moderate computational cost. Other phenomena in composites like porous density, crack growing intensity, etc., that imply cumulative processes can benefit by applying this method by only obtaining a set of data from a state variable observed through time.

Further work is needed to extrapolate this method to Continuos Time Markov Process that would allow to incorporate whatever heterogeneous set of data is available.

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