MODELING FOR COMPOSITE STRUCTURES BY FINITE ELEMENT METHOD

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Keywords: Finite element modeling, laminated composite structures, laminate with a hole, and layer lumping.

Abstract
In using finite element method to analyze composite structures, special attentions on modeling are required to be considered. This paper investigates the stress effects due to meshing isotropy, using 1D, 2D, or 3D modeling, boundary condition and material property used for lumped layer in modeling for composite structures. Laminates with and without a hole were used to study the effect of the above mentioned issues on the magnitude and location of the peak stress. It is concluded that applying the experience on modeling structures made of isotropic material to the laminated composite structures may lead to a significant error if the structural characteristics of composites is ignored.

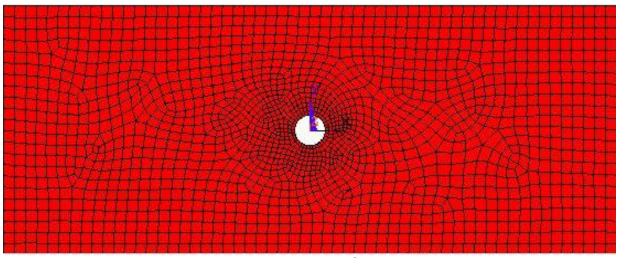
1 Introduction
The finite element method was initially developed for analyzing structures which are made of isotropic materials. Applications of this method to composite structures have been widely adopted. Numerous works in this area have been published in many journals and textbooks [1, 2]. Experiences in using this method to model structures made of isotropic materials were often directly transformed to structures with laminated composites. In so doing, the special characteristics of composite structures such as coupling effects due to ply orientation and staking sequence are often ignored in modelling composite structure. Furthermore, a three-dimensional state of stress is induced at the free-edge of the laminate or the edge of the cut-out even if the laminate is subjected to an in-plane load. Hence, selection of 1D, 2D and 3D model for modelling composite structures depends on the interest of the state of the stresses.
The purpose of this paper is to address the issues that needed special attention when using finite element method in analyzing composite structures. This paper is not intended to compare the types of elements used in modelling composite structures but to emphasize the effect of the stress results due to meshing isotropy, 1D, 2D or 3D modelling, boundary condition and material properties that are commonly used in modelling of composite structures. Laminates with and without a hole were used to study the effect of the above mentioned issues on magnitude and location of the peak stress. It is believed that these issues have not been widely discussed in composite structure modelling.

2. FINITE ELEMENT MESHING

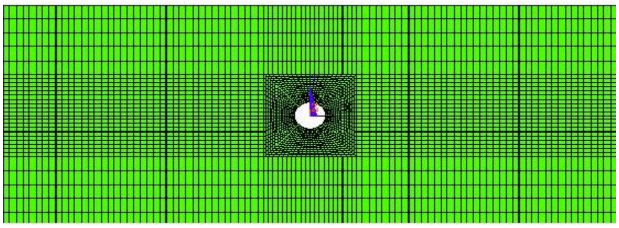
2.1 Geometric Isotropy

The orientation of the mesh can affect the accuracy of finite element solution. In particular, geometric isotropy results when the selected mesh violates the symmetry of the problem resulting in a less accurate solution compared to one aligned with the symmetry of the problem. Triangular mesh is used because of its complete polynomial representation to the corresponding order and its flexibility in representation of geometric complexity. However, it has been known that geometric anisotropy arises with triangular element as it has fewer lines of symmetry when compared to rectangular element [3]. This effect is more pronounced in applications of laminated structures, as demonstrated by Chan and Chen [4].

8-node shell elements were used to study the stress effect in modeling a laminate of $[\pm45/0/90]_s$ subjected to tensile load by using free and mapped meshes, respectively. The free mesh modeling is an automatic mesh generation often used in the modeling isotropic structures and the mapped mesh is a preferred mesh with geometric isotropy with respect to the structural configurations. The normalized layer stresses away from the hole are shown in Table 1. It is indicated that using the mapped mesh in modeling the laminate gives better results when compared against analytical solution.



Free Mesh



Mapped Mesh

Figure1. Free mesh versus Mapped Mesh

	Classical Lamination Theory (Normalized stresses)			Free Mesh (Normalized stresses)			Mapped Mesh (Normalized stresses)		
	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}
45	0.633	0.366	0.417	0.628	0.367	0.414	0.633	0.366	0.417
-45	0.633	0.366	-0.417	0.628	0.367	-0.418	0.633	0.366	-0.417
0	2.559	-0.009	0	2.52	-0.007	-0.0002	2.559	-0.009	0
90	0.174	-0.724	0	0.172	-0.690	-0.0002	0.174	-0.724	0

Table 1. Normalized ply stresses for a 2D plate using free and mapped mesh.

3 1-D, 2-D and 3-D Modeling

3.1 One-Dimensional Modeling

In classical lamination theory, composites are modeled with two dimensional properties. And in one dimension modeling, equivalent properties are used and incorporated conventionally as Equation 1.

$$\overline{E_x} = \frac{1}{a_{11}t};\ \overline{E_y} = \frac{1}{a_{22}t};\ \overline{G_{xy}} = \frac{1}{a_{66}t};\ \overline{\upsilon_{xy}} = -\frac{a_{21}}{a_{11}}; \tag{1}$$

This conventional method, however, violates the zero curvature assumption for equivalent property for unsymmetrical and/or unbalanced laminate. Chen and Chan [4] suggested a modification of the equivalent properties to Equation 2.

$$E_x = \frac{1}{\left(P_{11} - \frac{P_{16}^2}{P_{66}}\right)t}; \quad E_y = \frac{1}{\left(P_{22} - \frac{P_{26}^2}{P_{66}}\right)t}; \quad \upsilon_{xy} = \frac{P_{12} - \frac{P_{16}P_{26}}{P_{66}}}{P_{11} - \frac{P_{16}^2}{P_{66}}}; \tag{2}$$

$$G_{xy} = \frac{1}{\left[P_{66} - \frac{1}{\Delta_1}\left(P_{16}^2 P_{22} - 2P_{12}P_{26}P_{16} + P_{26}^2 P_{11}\right)\right]t}$$

where $[P] = [a] - [b][d]^{-1}[b^T]; \ \Delta_1 = P_{11}P_{22} - P_{12}^2$

The conventional method and the modified method [4] for equivalent properties were computed for symmetric and unsymmetrical layups of $[\pm 45/0/90]_s$ and $[\pm 45/0/90]_{2T}$. It is noted that for a symmetric laminate, Equation 2 gives the same results as Equation 1 does. These results were then input into FEM software to compare the bending rigidity $\overline{D_x}$ for the case of simple supported beam with length 0.0127 m (0.5 in) and a unit load applied at mid length. The bending rigidity is that for general bending, as represented in Equation 3.

$$\upsilon_{max} = \frac{PL^3}{48\overline{D_x}} \tag{3}$$

The material properties for AS4/3501-6 graphite/epoxy are: E_{11} = 150 GPa, E_{22} = E_{33} = 11.0 Pa, $\upsilon_{12} = \upsilon_{13}$ = 0.25, υ_{23} = 0.45, $G_{12} = G_{13}$ = 6.0 GPa and G_{23} = 3.70 GPa. The comparison of equivalent modulus between the conventional method and the modified method [4] is provided in Table 2. There was no deviation between the two methods for the symmetric layup $[\pm 45/0/90]_s$. However, a range of differences from 5% in E_x to 17% in ν_{xy} was observed for the unsymmetrical layup $[\pm 45/0/90]_{2T}$ due to the non-zero curvature assumption.

Equivalent properties		E_x, GPa (Msi)	E_y GPa (Msi)	ν_{xy}	G_{xy} GPa (Msi)
$[\pm 45/0/90]_s$	Conventional Method	58.55 (8.492)	58.55 (8.492)	0.300	22.53 (3.267)
	Modified Method [4]	58.55 (8.492)	58.55 (8.492)	0.300	22.53 (3.267)
$[\pm 45/0/90]_{2T}$	Conventional Method	55.75 (8.086)	51.54 (7.475)	0.351	20.03 (2.905)
	Modified Method [4]	58.55 (8.492)	58.55 (8.492)	0.300	22.53 (3.267)

Table 2. Equivalent properties of symmetric and unsymmetrical laminates obtained by the conventional and the modified methods.

The bending rigidity comparison represented by the difference in displacement in the middle for the conventional and modified method is shown in Table 3. As for the unsymmetrical and balanced layup, the modified method compared better against the closed form solutions (i.e. conventional method: 4%; modified method: 1%).

Displacement in the mid-length, mm (in)		
Symmetric & balanced: $[\pm 45/0/90]_s$	Conventional Method	0.632 (0.0249)
	Modified Method [4]	0.632 (0.0249)
	Closed Form Solution	0.632 (0.0249)
Unsymmetrical & balanced: $[\pm 45/0/90]_{2T}$	Conventional Method	0.663 (0.0261)
	Modified Method [4]	0.632 (0.0249)
	Closed Form Solution	.0638 (0.0251)

Table 3. Displacement obtained by the conventional and modified methods

3.2 2D and 3D Full Model vs. Quarter Model

2D and 3D modeling for laminated composite beams differ in that the former is a lumped property while the latter accounts for interlaminar stresses between layers. 2D modeling was more accurate in predicting stresses in the far field for a plate, whereas 3D modeling was more accurate in predicting the location of maximum stresses near the hole for notched laminated composite plates. The use of quarter-model or full-model in laminated composite beams requires more attention than that for isotropic material systems. The concern lies in the symmetric boundary condition placed on the quarter-model, which affects the orientation of the material axis (a $+\theta^o$ layer will be reflected as a $-\theta^o$ layer) and induced shear deformation and curvature as well. The full-model and quarter-model for both the 2D and 3D analysis were compared for symmetric $[\pm 45/0/90]_S$ to determine the location of the maximum stress for each layer.

For a symmetric $[\pm 45/0/90]_s$, 50.8 mm x 127 mm (2 in x 5 in) plate with a 3.175 mm (0.125 in) radius hole in the middle loaded under tension, the 2D full-model showed the circular hole deforming to an elliptical shape (undeformed: origin of circle (0, 0); deformed: origin of ellipse (0.1248, 0)), whereas the quarter-model showed the circular hole deforming to an elongated quarter circle (same origin before and after deformation; undeformed: y-intercept (0, 0.125); deformed: y-intercept (0, 0.1858)), as shown in Figure 2.

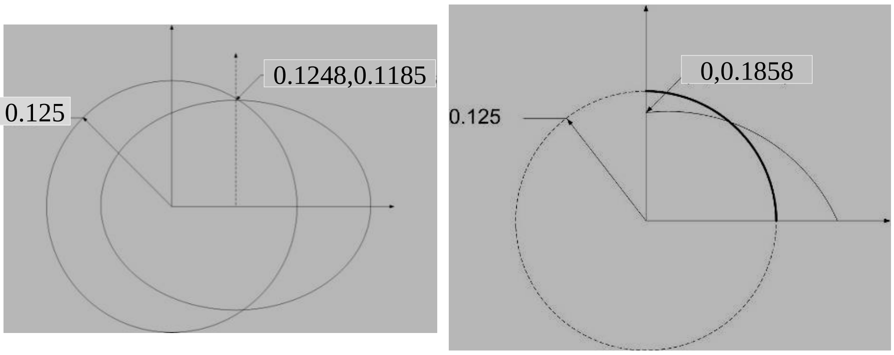


Figure 2. 2D: Deformation of 3.175 mm (0.125 in) radius circular hole in the middle of a plate for a symmetric [±45/0/90]s layup under tension (Left side: full model; Right side: quarter-model).

The maximum stress will occur tangential to the fiber direction, as evident from the 2D and 3D full-models. However, the symmetric boundary conditions in the 2D and 3D quarter-models do not capture the full deformation by enforcing redundant symmetric in the material axis and limiting the shear deformation in the 3D models. As a result of that, there was a shift in location for maximum stress between a full-model and a quarter-model. Table 4 shows the location of maximum stresses for the layers in $[\pm 45/0/90]_s$ comparison between the 2D and 3D full- and quarter- models. Figures 3 and 4 illustrate the stress distribution around the hole for a 2D and 3D full- and quarter-models, respectively.

Layer	**2D: Full-Model** $\theta = \tan^{-1}\frac{y}{x}$	**2D: Quarter-Model** $\theta = \tan^{-1}\frac{y}{x}$	**3D: Full-Model** $\theta = \tan^{-1}\frac{y}{x}$	**3D: Quarter-Model** $\theta = \tan^{-1}\frac{y}{x}$
45	-68.58	90	-53.24	90
-45	68.58	68.50	58.47	68.26
0	90	90	90	90
90	90	90	90	90

Table 4. Location of maximum stress for 2D and 3D symmetric $[\pm 45/0/90]_s$ full- and quarter-models.

For the 2D models, where the shear deformation of each element is negligible, the location of the maximum stress occurs tangential to the fiber direction, as illustrated in the 2D full-model. For the +45^o, the symmetric boundary condition restrained the curvature for the +45^o layer. Instead of having the maximum stress in the direction tangential to the fiber direction, as observed from the full-model, the symmetry constraint modified the actual boundary condition at the true tangential location -68.58^o, and reflected the maximum at the nearest point 90^o with a significantly lower value compared to the full-model. This is illustrated in the quarter-model in Figure 5, where the reflected maximum stress location is away from that observed in the full-model. The symmetric boundary condition affects the other layers (-45^o, 0^o and 90^o) similar but the resultant locations of the maximum stress were not affected. For the 3D full model, where shear deformation of each element is accounted, the location of the maximum stress still occurs tangential to the fiber direction except that the tangential fiber direction for the ±45^o is slightly altered. The 3D quarter-model showed similar results to the 2D quarter-model as a result of the erroneous symmetric in the material axis and limited shear deformation enforced by the symmetric boundary condition.

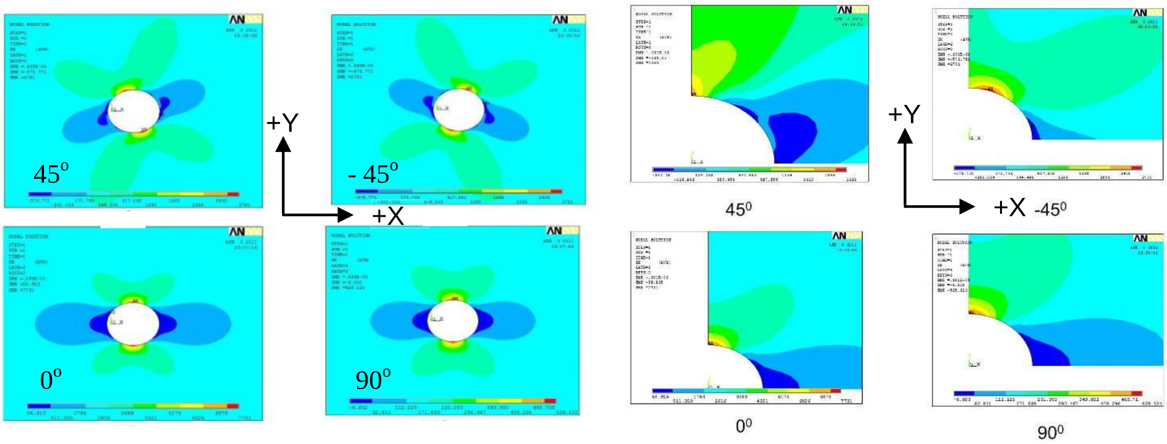


Figure 3. Maximum stress of 2D symmetric $[\pm45/0/90]_s$ full-model (left side) and quarter-model (right side) of plate with a hole.

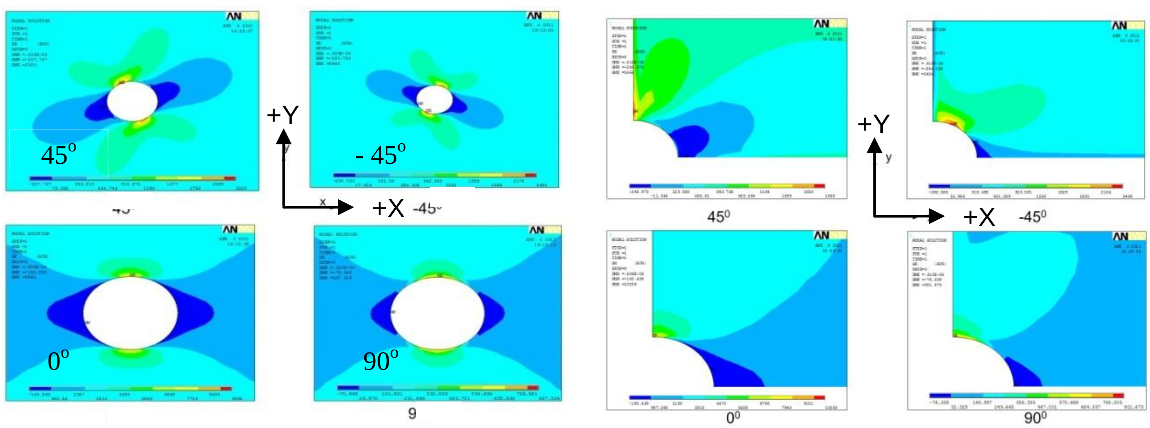


Figure 4. Maximum stress of 3D symmetric $[\pm45/0/90]_s$ full-model (left side) and quarter-model (right side) of plate with a hole.

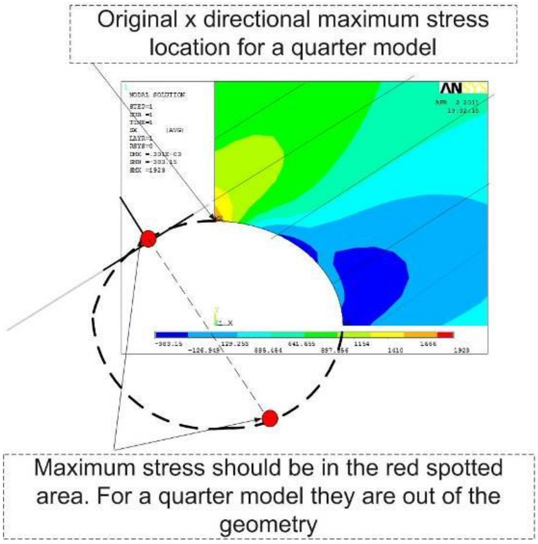


Figure 5. 2D quarter-model in the first quadrant: Differences in the true tangential location and the reflected maximum location for +45°.

The far field stresses, based on comparison with Classical Lamination Theory (CLT), are more accurately predicted with 2D modeling as the plate assumptions in CLT are similar to shell assumption in 2D modeling. Tables 5 and 6 compares the stresses in each layer for the symmetric $[\pm45/0/90]_s$ and unsymmetrical $[\pm45/0/90]_{2T}$, 0.0508 m x 0.127 m (2 in x 5 in) plate between 2D and 3D full- and quarter- models, respectively. In the far field, the symmetric boundary condition imposed by the quarter model did not affect the stresses as significantly as the location of maximum stress discussed earlier. In general, the 2D quarter-model showed negligible differences since the change in stiffness due to the material axis reflection at the symmetric boundary condition is negligible. The 2D models typically exhibit lower stresses compared to the 3D models since the former used a reduced stiffness matrix. And 3D quarter-models had more pronounced edge effects compared to the 3D full-models because of its smaller geometry. These were evident in the stresses for both the symmetric and the unsymmetrical layup. The latter demonstrated a more significant edge effect due to induced extension-bending coupling effect.

	Lamination Theory (Normalized stresses)			**2D: Full-model (Normalized stresses)**			**2D: Quarter-model (Normalized stresses)**		
	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}
45	0.633	0.366	0.417	0.633	0.366	0.417	0.633	0.367	0.417
-45	0.633	0.366	-0.417	0.633	0.366	-0.417	0.633	0.367	-0.417
0	2.559	-0.009	0.000	2.559	-0.009	0.000	2.559	-0.009	0.000
90	0.174	-0.724	0.000	0.174	-0.724	0.000	0.174	-0.724	0.000
90	0.174	-0.724	0.000	0.174	-0.724	0.000	0.174	-0.724	0.000
0	2.559	-0.009	0.000	2.559	-0.009	0.000	2.559	-0.009	0.000
-45	0.633	0.366	-0.417	0.633	0.366	-0.417	0.633	0.367	-0.417
45	0.633	0.366	0.417	0.633	0.366	0.417	0.633	0.367	0.417

	3D: Full-model (Normalized stresses)			**3D: Quarter-model (Normalized stresses)**		
	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}
45	0.634	0.362	0.418	0.625	0.357	0.412
-45	0.634	0.368	-0.418	0.637	0.368	-0.420
0	2.562	-0.011	0.000	2.579	-0.010	0.000
90	0.174	-0.719	0.000	0.176	-0.739	0.000
90	0.174	-0.719	0.001	0.176	-0.735	0.000
0	2.562	-0.011	0.000	2.577	-0.010	0.000
-45	0.634	0.368	-0.418	0.637	0.367	-0.419
45	0.634	0.362	0.418	0.622	0.358	0.406

Table 5. Far field stress comparison for 2D and 3D symmetric $[\pm 45/0/90]_s$ full- and quarter-models.

	Lamination Theory (Normalized stresses)			**2D: Full-model (Normalized stresses)**			**2D: Quarter-model (Normalized stresses)**		
	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}
45	0.478	0.073	0.205	0.478	0.073	0.205	0.472	0.067	0.197
-45	0.642	0.266	-0.396	0.642	0.266	-0.396	0.653	0.276	-0.407
0	2.925	-0.050	-0.005	2.925	-0.050	-0.005	2.931	-0.050	-0.007
90	0.186	-1.130	0.000	0.186	-1.131	0.000	0.186	-1.131	-0.001
45	0.664	0.373	0.442	0.664	0.373	0.442	0.651	0.360	0.427
-45	0.586	0.323	-0.362	0.586	0.323	-0.362	0.603	0.340	-0.381
0	2.445	0.009	0.016	2.445	0.009	0.016	2.449	0.009	0.013
90	0.166	-0.202	0.021	0.166	-0.202	0.021	0.167	-0.202	0.018

	3D: Full-model (Normalized stresses)			**3D: Quarter-model (Normalized stresses)**		
	σ_x	σ_y	τ_{xy}	σ_x	σ_y	τ_{xy}
45	0.481	0.071	0.206	0.520	0.132	0.248
-45	0.637	0.257	-0.389	0.618	0.278	-0.374
0	2.935	-0.500	-0.004	2.810	-0.036	-0.002
90	0.186	-1.145	0.000	0.181	-0.900	-0.001
45	0.660	0.369	0.438	0.698	0.367	0.485
-45	0.587	0.325	-0.364	0.615	0.382	-0.405
0	2.434	0.010	0.015	2.283	0.026	0.011
90	0.165	-0.189	0.020	0.156	-0.258	0.027

Table 6. Far field stress comparison for 2D and 3D unsymmetrical $[\pm 45/0/90]_{2T}$ full- and quarter-models.

4. Conclusions

This paper addresses the issues that need to pay special attention when using the finite element method in analyzing composite structures. Applying experiences on working finite element analysis in isotropic materials to laminated composites should be undertaken with care. The following address specific effects on stress results of finite element analysis on the issues mentioned before:

1. Element Meshing

Using the mapped mesh in modeling the laminate gives better results than by using free meshing when compared against analytical solution.

2. Equivalent properties

Equivalent properties of laminate used in 1D element should take into consideration of contributions due to induced shear deformation and curvature for unsymmetrical and/or unbalanced layups.

3. 2D versus 3D full model

The full 3D model of laminate without a hole gives lower in-plane stress than the 2D does, in particular at the edge of the laminate. A full 3D and full 2D model give a significant peak stress of each ply for laminate with a hole.

4. Full Model versus Quarter Model-Boundary Constraints

Enforcing boundary constraints for symmetry needs to consider both structural configuration and material axis. In 2D model, a full or a quarter model for symmetric and unsymmetrical laminate without a hole gives none or insignificant difference in the ply stress result. However, a quarter 2D model for unsymmetrical laminate with a hole gives significant difference than the results from a full 2D model. For a laminate with a hole, a full and a quarter 3D models give significant different values of the peak stress. Moreover, using a quarter 2D or 3D model fails to predict the right location of the peak stress.

It is concluded that modeling a composite structure requires understanding the structural characteristics of composites.

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