INVESTIGATION OF CONCEPTS DESCRIBING THE INFLUENCE OF STRESS CONCENTRATION ON THE FATIGUE BEHAVIOUR OF SHORT GLASS FIBRE REINFORCED POLYAMIDE

A. Moesenbacher¹*, Ch. Guster¹, G. Pinter², W. Eichlseder¹

¹Chair of Mechanical Engineering, Montanuniversität Leoben, Franz-Josef-Straße 18, 8700 Leoben, Austria
²Institute of Materials Science and Testing of Polymers, Montanuniversität Leoben, Franz-Josef-Straße 18, 8700 Leoben, Austria
*andreas.moesenbacher@unileoben.ac.at

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Abstract
In this paper fatigue behaviour of a short glass fibre reinforced polyamide 66 (PA66-GF30) was investigated. Focus of fatigue tests was studying the influence of stress concentrations whereupon unnotched and different notched specimens manufactured by injection moulding were used. Fatigue tests were conducted on a servo hydraulic test bench and on a rotating bending machine. Results in form of S/N-curves show a distinct support effect of the tested material. For characterization of local fatigue behaviour the concept of highly loaded volume, concept of local stress gradient and a volume corrected combination of both is used. A comparison based on standard deviation of different concepts shows the fitness of each concept. By using parameter optimisation a volume exponent describing statistical size effects could be proposed.

1 Introduction
Today due to demands of customers and engineers the relevance of lightweight constructions increases, as a reason the use of composite materials rises. Especially by bulk production of monotonically or cyclically loaded components short glass fibre reinforced (sgfr) polyamides have a wide application area. These materials have a high specific strength and are processable very cost efficient by injection moulding. Therefore especially in automotive industry components made of metal are more and more substituted by components made of such a material.

But to benefit from sgfr polyamides in lightweight construction and for dimensioning of dynamically loaded components, useful design methods and the knowledge of cyclic material behaviour and its affecting parameters are necessary.

To calculate fatigue lifetime of cyclically loaded parts the concept of local S/N-curves which was developed for metals is widely used and also applicable for parts made of sgfr plastics [1, 2, 3]. In this concept, local S/N-curves are calculated by using parameters like loading type, mean stress, notch sensitivity effects (stress concentrations), size, technological influences, loading order, plastic deformation, multiaxial loading and many more [1, 4, 5]. Regarding components made of sgfr plastics a lot of additional influencing parameters such as
fibre orientation and concentration [6, 7, 8], type of matrix, temperature [9, 10], moisture [10, 10], weld lines [12], and others have to be considered [13]. In this paper fatigue behaviour of a sgfr polyamide regarding influence of stress concentrations which result from geometrical notches was investigated. To characterize the effect of notches with the local S/N-curve concept, a material model has to be deduced from results of fatigue tests. Some reference values have to be determined by using the finite element method. In literature many different empirical hypothesis such as the concept of local stress gradient [1], concept of highly loaded volume [13], theory of critical distances [14] and resultant combinations [15] are present. But it is not clear which hypothesis characterize the influence of stress concentrations on fatigue behaviour of sgfr polyamide the best. Therefore in this work concept of highly loaded volume [13], concept of local stress gradient [1] and a combination of both is compared. Due to anisotropic material behaviour of injection moulded components, local stresses have to be calculated in consideration of anisotropic material properties. So also reference values which are used in material models have to be calculated in an anisotropic way as it is done in the present paper. Anisotropic material properties are calculated by a moulding simulation. An agreement with the associated companies allows no publishing of detailed data.

2 Theoretical background

Stress concentrations have a wide influence on fatigue behaviour of materials. In system of nominal stresses fatigue strength drops as a reason of notches, which is described by notch sensitivity factor $K_f$. In Equ. 1 $K_f$ is defined by the ratio of nominal fatigue strength of unnotched specimen $\sigma_{a,n,\text{unnotched}}$ to nominal fatigue strength of notched specimen $\sigma_{a,n,\text{notched}}$. But in most cases this drop is less than the value of the stress concentration factor $K_t$ (cf. Equ. 2) which is defined by maximum notch stress $\sigma_{\text{max}}$ divided by nominal stress $\sigma_n$. That indicates that the material has a distinct support effect, where local fatigue strength in the notch root (related to maximum stress of stress concentration) is higher than fatigue strength of the unnotched material. This support effect is defined by the supporting factor $n$ (cf. Equ. 3). By using a material model for characterization of supporting factor $n$, endurable local fatigue strength $\sigma_a$ can be calculated by Equ. 4 where $\sigma_{\text{ref}}$ is a characteristic value of fatigue strength which is in most cases fatigue strength of the plain material.

\[
K_f = \frac{\sigma_{a,n,\text{unnotched}}}{\sigma_{a,n,\text{notched}}} \quad (1)
\]
\[
K_t = \frac{\sigma_{\text{max}}}{\sigma_n} \quad (2)
\]
\[
n = \frac{K_t}{K_f} = \frac{\sigma_{a,n,\text{notched}}}{\sigma_{a,n,\text{unnotched}}} = \frac{\sigma_a}{\sigma_{a,n,\text{unnotched}}} \quad (3)
\]
\[
\sigma_a = n \cdot \sigma_{\text{ref}} \quad (4)
\]

Figure 1 shows a schematic description of different empirical hypothesis to characterize the support effect of materials. In concept of highly loaded volume as shown in Figure 1 (1) a linear correlation between endurable local fatigue strength and highly loaded volume $V_{90\%}$ is assumed in double logarithmic scale. Local fatigue strength rises by decreasing volume. $V_{90\%}$ is the volume surrounding the stress concentration which is loaded with a minimum value of 90 % of maximum notch stress. In this concept the influences of stress concentration and statistical size effect are combined. Therefore it is only applicable if both influencing effects are nearly similar [16]. One of the biggest advantages of this concept is that only a minimum
of two $S/N$-curves is necessary to assess a material model able to characterize stress mechanical influence and statistical size effect.

Another method which characterizes only stress mechanical influence is the concept of local stress gradient $\chi'$ as shown in Figure 1 (2) [1]. In this concept support effect of material is characterized by an exponential function whose origin is fatigue strength of plain material $\sigma_{a,tc}$. The trend of exponential function is fixed by fatigue strength obtained from bending tests $\sigma_{a,b}$ and fatigue strength obtained from fatigue tests on notched specimens which is required to find the exponent of stress mechanical influence $K_D$. Local fatigue strength rises by increasing relative stress gradient of the stress concentration. To assess a material model to characterize stress mechanical influence a minimum of three $S/N$-curves is necessary. Statistical size effect is not included in this concept and has to be considered separately.

$$\sigma_a = \sigma_{ref} \left( \frac{V_{90%}}{V_{90% ref}} \right)^{\chi'}$$

$$\sigma_{a,b} = \sigma_{a,tc} \left( \frac{\sigma_{a,b}}{\sigma_{a,tc}} \right)^{-1} \left( \frac{\chi'}{2/b} \right)^{K_D} \left( \frac{V_{90% ref}}{V_{90%}} \right)^{a}$$

*Figure 1.* Schematic description of different hypothesis to characterize the support effect, concept of highly loaded volume (1), concept of local stress gradient (2), combination of both concepts (3)

In [15] a new combination of concepts described above is presented (cf. Figure 1 (3)). The difference to a common combination is that fatigue strengths, for characterization of stress mechanical influence by using concept of local stress gradient, are based on equal volumes. Fatigue strengths at effective highly loaded volumes of different stress concentrations are corrected to equal values of volume by using a power law according to concept of highly loaded volume. Therefore required influence of pure statistical size effect, which is described by volume exponent $a$, is determined by fatigue strengths obtained from axial bending and rotating bending fatigue tests. On the one hand the main advantage of this form of combination is that a separate estimation of stress mechanical influence and statistical size effect is possible. On the other hand there is a minimum of four $S/N$-curves necessary to determine required parameters.

$$\sigma_a (V, \chi') = \sigma_{a,tc} \left( 1 + \left( \frac{\sigma_{a,b}}{\sigma_{a,tc}} \left( \frac{V_{90%}}{V_{90% ref}} \right)^{a} - 1 \right) \left( \frac{\chi'}{2/b} \right)^{K_D} \left( \frac{V_{90% ref}}{V_{90%}} \right)^{a} \right)$$

(5)
3 Experimental procedure

3.1 Material and specimen geometries
The investigated material is a short glass fibre reinforced (sgfr) polyamide 66 with a fibre content of 30% (PA66-GF30). For fatigue tests, different notched and unnotched rotating bending (RB) specimens as shown in Figure 2 are used. This special type of specimen was developed to realize fatigue tests on sgfr injection moulded plastics very cost efficient by using a rotating bending machine. The cross section in the middle of the different types of specimens is equivalent. Because this types of specimens are only for internal use, more information of geometrical details cannot be give.

![Figure 2. Used RB-specimen (l) and their schematic manufacturing process by injection moulding (r)](image)

Parts made of sgfr plastics show a high anisotropic material behaviour, which results from very complex flow conditions during processing and characteristic layer structure depending from flow conditions and wall thickness [6]. Because of anisotropic modulus of elasticity and complex layer structure stress concentrations and reference values used in material models (e.g. \( \chi' \), \( V_{90\%} \)) have to be calculated in consideration of anisotropic material properties. In this paper local stress concentrations and reference values are determined by using anisotropic finite element calculation where information about main fibre orientation and degree of orientation of each finite element follows from a moulding simulation.

3.2 Constant amplitude fatigue tests (S/N-curves)
Fatigue tests were conducted under constant sinusoidal loading with a testing frequency from 1 to 10 Hz. To avoid increasing temperatures due to hysteretic heating temperature was measured continuously and also a fan was placed on the test bench for cooling the specimen. The tests are performed on a rotating bending machine as shown in Figure 3 (l) and a servo hydraulic test bench as drafted in Figure 3 (r). The rotating bending machine was designed at the chair of mechanical engineering and operates according to the four point bending principle so that there is a constant bending moment in the whole test range. So specimens were tested under bending load and also under tension/compression load by using a servo hydraulic test bench. The applied stress ratio \( R = -1 \) was kept constant in both test configurations.

![Figure 3. Functional principle of rotating bending machine (l) and servo hydraulic test bench (r)](image)
Fatigue tests were performed at different stress levels to obtain cycles to failure between $N = 10^4$ and $N = 10^7$. The failure criterion for fatigue tests was fatigue rupture of the specimen. S/N-curves were generated by plotting the amplitude of maximum notch stress $\sigma_a$ over the cycles in double logarithmic scale. Interpretations of slope $k$ and fatigue strength were carried out using log-normal distribution according to [17] with a probability of survival of 50%. Based on these slopes $k$ a model function using relative stress gradient $\chi'$ to characterize slope $k$ of S/N-curves was generated, which is not presented in this paper. Finally test data was analysed a second time by using slope $k$ of the model function.

4 Results and Discussions
In Figure 4 results of fatigue tests are shown. As a reason of avoiding hysteretic heating only low testing frequencies at tension/compression tests were performed, therefore only few points were measured. The unnotched specimens tested under tension/compression load exceeding $10^5$ cycles to failure were ruptured in the clamping area. Therefore test points are considered only at less than $10^5$ cycles to failure. The S/N-curve of this test condition is estimated by using valid test points and slope $k$ calculated by generated model function.

![Figure 4. Results of fatigue tests by tension/compression load (l) and rotating bending load (r)](image)

The trends of fatigue behaviour of both testing methodologies are very similar. It is shown that the investigated material has a very small statistical scatter and has also no fatigue limit in the test range. The slopes $k$ of S/N-curves depend on sharpness of notches whereupon S/N-curve of notched specimen is steeper than the unnotched one. It is also shown that endurable fatigue strength related to maximum notch stress rises with sharpness of notches. Stress concentration is enforced by bending load so local S/N-curves determined by rotating bending tests show higher values than from tension/compression tests.

In Figure 5 local fatigue strength in the notch root is plotted against different reference values of notches to characterize the influence of stress concentrations. In the left image endurable fatigue strengths at $10^6$ cycles to failure are fitted by using the concept of highly loaded volume [13]. In the right image concept of local stress gradient is used. To compare the quality of different concepts the standard deviation $SD$ of fatigue strengths is used. It is shown that the concept of local stress gradient which has a standard deviation $SD$ of 3.2 MPa fits test data better than concept of highly loaded volume whose standard deviation is 4.5 MPa. But it has to be taken into account that the concept of local stress gradient considers only the support effect of material induced by stress concentrations. A possible fault as a reason of the statistical size effect is not considered in this concept.
To uncouple influences of highly loaded volume and support effect which are based on completely different mechanisms, a volume corrected combination of both concepts is shown in Figure 6 (r) as presented in [15]. Because of missing fatigue tests under axial bending load statistical size effect of material described by the volume exponent $a$ cannot be determined by testing. Therefore the volume exponent $a$ is estimated by parameter optimisation as shown in Fig 6 (l) whereupon command variable is standard deviation of Equ. 5.

![Figure 5](image1.png)  ![Figure 6](image2.png)

**Figure 5.** Concept of highly loaded volume (l) and concept of local stress gradient (r) to characterize influence of stress concentration

**Figure 6.** Influence of volume exponent $a$ on quality of volume corrected stress gradient concept (l) and area of endurable fatigue strength by using volume exponent $a$ of 0.05 (r)

The area of endurable fatigue strength of volume corrected stress gradient concept is shown in Figure 6 (r). As shown in Figure 6 (l) best fit of test data is done by using volume exponent $a = 0.05$. But consideration of statistical size effect brings no significant improvement of standard deviation. So difference to concept of local stress gradient is only 0.02 MPa. Anyway the volumetric add-on is a very good possibility to consider statistical size effect. An adverse implication is that evaluation of statistical size effect needs more experimental tests and for characterization two different reference values ($V_{90\%}$ and $\chi'$) are necessary.

## 5 Conclusion

Fatigue behaviour of a short glass fibre reinforced polyamide 66 (PA66-GF30) was investigated relating to influence of stress concentrations and different test methodologies. For the characterization of the influence of stress concentration factor injection moulded
specimens with different geometrical notches were designed and tested on a servo hydraulic test bench as well as on a rotating bending machine. Results in form of $S/N$-curves show that the investigated material hasn’t a distinct fatigue limit or changing of slope in test range. But slopes and local endurable fatigue strength in notch root is significantly influenced by stress concentration whereupon $S/N$-curves of notched specimens are steeper and higher than from unnotched. To consider influence of stress concentration in concept of local $S/N$-curves a mathematical formulation of material behaviour (support effect), based on a reference value describing sharpness of notches, is required. In literature many different formulations are available but in computer aided fatigue life calculation concepts reference values such as highly loaded volume and local stress gradient are widely used. Therefore concept of highly loaded volume [13], concept of local stress gradient [1] and a combination of both [15] are compared in accuracy and complexity within this paper.

On the one hand the concept of highly loaded volume shows the highest standard deviation but on the other hand the realization is easy because a minimum of only two $S/N$-curves are necessary. In the present case better result is shown by concept of local stress gradient. In this concept only stress mechanical influence is considered and a minimum of three $S/N$-curves are necessary for meaningful characterization. By using volume corrected combination of both concepts no distinctive improvement of standard deviation is shown. But using this combined concept it is possible to consider the statistical size effect as well as the gradient effect. To characterize statistical size effect normally an additional $S/N$-curve is necessary. Alternatively in combined concept the influence of this statistical size effect can be estimated by parameter optimization as shown in this paper.

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