FATIGUE OF LAMINATED COMPOSITE STRUCTURES

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Abstract

A model defined on the ply scale to describe the rupture in the fibre direction for statics and fatigue loadings was proposed. This model is based on a reduction of strength in the fibre direction for high levels of transverse damage. This phenomenon of reduction of strength in the fibre direction for high levels of transverse damage can also be observed on 0° tubular specimen solicited in fatigue in torsion up to a level of high damage followed by a tensile test in the fibre direction. In addition, an original approach based on a Fracture Characteristic Volume has been developed to predict the fibre failure of laminated structures with stress concentrations for static loadings. The FCV is a cylinder defined at the ply scale on which an average stress is calculated. The parameters of the FCV are identified starting from a homogeneous test and a test which presents a stress concentration.

1 Introduction

The rupture of laminated composite structures is due to many mechanisms acting on various scales. The models based on damage mechanics can describe the progressive diffuse damages induced by small cracks parallel to the fibres direction [1]. Even if the sizes of these cracks correspond to the thickness of the ply, they do not lead in general to the rupture of the laminate. On the other hand, the rupture of one ply in the fibre direction is in general catastrophic for the laminate and the structure.

A model defined on the ply scale to describe the rupture in the fibre direction for statics and fatigue loadings was proposed [2-3]. This model is based on a reduction of strength in the fibre direction for high levels of transverse damage. For unidirectional carbon fibre plies, loaded in the fibres direction only, strength is not appreciably modified in fatigue [2]. For balanced woven plies with glass or carbon fibres, strength in the fibres direction decreases in fatigue. Indeed, the woven plies can be regarded as a [0°, 90°] laminate with "virtual" unidirectional plies which undergo a transverse positive traction stress in the 0° "virtual" unidirectional ply for a tensile test on the woven ply [3]. This phenomenon of reduction of strength in the fibre direction for high levels of transverse damage can also be observed on 0° tubular specimen solicited in fatigue in torsion up to a level of high damage followed by a tensile test in the fibre direction.

In addition, an original approach based on a *Fracture Characteristic Volume* (FCV) has been developed to predict the fibre failure of laminated structures with stress concentrations for static loadings [4]. The FCV is a cylinder defined at the ply scale on which an average stress is calculated. The parameters of the FCV are identified starting from a homogeneous test and a test which presents a stress concentration. The fatigue model and the nonlocal criterion of rupture were introduced in Abaqus. In the case of fatigue, the strongly damaged material corresponds to another material (compared to healthy material) and can require a new identification of FCV. This talk presents some comparisons between tests on structures in fatigue (perforated plates in traction) and the model.

2 Modeling of the damageable behavior at the level of the ply

2.1 Assumptions

A model based on the Continuum Damage Mechanics (CDM) has been developed to describe the damageable behaviour of composite material at the ply scale [1,2,3]. The damage is assumed to be uniform in the thickness of the ply. The modeling of both static and fatigue loadings with the same model is allowed by the use of a non-linear cumulative law which describes the damage evolution according to the maximal load and the amplitude of the cyclic loading.

In the fibre direction, the UD ply shows linear elastic behaviour in response to tension loading until the final brittle failure. In the transverse and shear directions, the behaviour is non linear due to the damages which lead to a decrease in stiffness. The damage kinematics was described by three internal damage variables:

- d_1, whose evolution represents the linear elastic behaviour and the brittle failure of the fibres observed in response to tension loading applied in the longitudinal direction
- d_2, which models the lost of transverse stiffness resulting from transverse and shear loadings
- d_{12}, which describes the decrease in shear stiffness due to transverse and shear loadings.

The gradual development of both damage d_2 and d_{12} depends on the tension load as well as on the shear load, which generates the matrix cracks. Assuming the existence of plane stresses and small perturbations, the strain energy in the ply can be written as follows [1]:

$$E_D^{ps} = \frac{1}{2}\left[\frac{\langle\sigma_1\rangle_+^2}{E_1^0(1-d_1)} + \frac{\langle\sigma_1\rangle_-^2}{E_1^0} + \frac{\langle\sigma_2\rangle_+^2}{E_2^0\left(1-d_2\right)} + \frac{\langle\sigma_2\rangle_-^2}{E_2^0} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2 + \frac{\sigma_{12}^2}{G_{12}^0\left(1-d_{12}\right)}\right] \tag{1}$$

where $< . >_+$ is the positive part and $< . >_-$ is the negative part. The tension energy and the compression energy are split in order to describe the unilateral nature of the damage process due to the opening and closing of the cracks. The thermodynamic forces associated with the internal tension and shear variables d_1 , d_2 and d_{12} are defined as follows:

$$\begin{cases} Y_{d_i} = \dfrac{\partial E_D^{ps}}{\partial d_i} = \dfrac{\langle\sigma_i\rangle_+^2}{2E_i^0(1-d_i)^2} \qquad with\ i = 1, 2 \\ \\ Y_{d_{12}} = \dfrac{\partial E_D^{ps}}{\partial d_{12}} = \dfrac{(\sigma_{12})^2}{2G_{12}^0(1-d_{12})^2} \end{cases} \tag{2}$$

The development of internal variables depends on these thermodynamic forces. Under tension loading conditions, d_1 has to develop suddenly to model the brittle behaviour in the fibre direction. So, d_1 is defined as:

$$\begin{cases} d_1 = 0 \quad if\ Y_{d_1} < Y_1^{\max} \\ d_1 = 1 \quad if\ Y_{d_1} \geq Y_1^{\max} \end{cases} \tag{3}$$

where $Y_1^{\max}$ is the parameter defining the ultimate force in the fibre direction

2.1 Damage evolution laws

The cumulative damage evolution law is defined as follow where the damage variables in the transverse and the shear directions, denoted respectively d_2 and d_{12}, are obtained by adding the terms due to static and fatigue loadings.

$$d_2 = d_2^s + d_2^f \tag{4}$$

$$d_{12} = c\, d_2 \tag{5}$$

In the case of static loading, the tension/shear coupling during the development of d_2^s and d_{12}^s is accounted for by the following equivalent thermodynamic force:

$$Y_{eq} = a\left(Y_{d_2^s}\right)^n + b\left(Y_{d_{12}^s}\right)^m \tag{6}$$

where a, b, m and n are material parameters specifying the tension/shear coupling. The evolution law for the damage is written as:

$$d_2^s = \left\langle 1 - e^{-\left(Y_{eq} - Y_0^s\right)}\right\rangle_+ \tag{7}$$

where the constant parameter Y_0^s corresponds to the threshold value of the development of d_2^s (which ranges from 0 to 1) and d_{12}^s is proportional to d_2^s.

In the case of fatigue loading, damage evolution depends on the maximal load $Y_{d_i^f}$ and the amplitude of the loading $\Delta Y_{d_i^f}$ during a cycle. The damage evolution law is then written as:

$$\frac{\partial d_2^f}{\partial N} = (1 - d_2)^\gamma \left\langle a_f\left(Y_{d_2^f}\right)^{\beta_1}\left(\Delta Y_{d_2^f}\right)^{\beta_2} + b_f\left(Y_{d_{12}^f}\right)^{\beta_3}\left(\Delta Y_{d_{12}^f}\right)^{\beta_4} - Y_0^f\right\rangle_+ \tag{8}$$

Where Y_0^f is the threshold value of the development of d_2^f

$$\Delta Y_{d_2^f} = \frac{\left(\left\langle\sigma_2^{\max}\right\rangle_+ - \left\langle\sigma_2^{\min}\right\rangle_+\right)^2}{2\, E_2^0\left(1 - d_2^f\right)^2} \quad and \quad \Delta Y_{d_{12}^f} = \frac{\left(\sigma_{12}^{\max} - \sigma_{12}^{\min}\right)^2}{2\, G_{12}^0\left(1 - d_{12}^f\right)^2} \tag{9}$$

3 Fiber tensile failure criterion

3.1 Damage influence on the fiber failure

During the loading, crack density increases in the transverse direction and this damage leads to a decrease of the stiffness. In the longitudinal direction, the damage disrupts the load transfer between fibers. Fiber failure can then occur even if the maximal stress usually measured in the case of homogeneous tension test is not reached. This phenomenon was not observed in the case of static loading due to the weak crack density compared to the case of fatigue loading where the damage can reach a very high level.

Experimental tests were performed to study this phenomenon which can lead to premature failure of laminate. Specifics tubes were manufactured with Glass/Epoxy unbalanced woven ply. The tube shape was studied so that the strains field was homogeneous in the central area (Figure 1). The lay-up in the central area was $(0)_3$. Torsion cyclic loading was applied to the tubes to generate matrix damage. The rotation was limited to avoid applying load on fibers. Stress/Strain curves were plotted to evaluate the decrease of stiffness which characterize the evolution of the damage as we can see on Figure 2. Various levels of damage were obtained according to the number of cycles applied. Then, the tubes were loaded in static tension until the final failure to estimate the residual strength of the fibers. Figure 3 shows the influence of the damage on the strength at failure in the fiber direction. Because of specimen complexity and test duration, the results are limited but it appears clearly that a high level of damage leads to a decrease of the strength in the fiber direction.

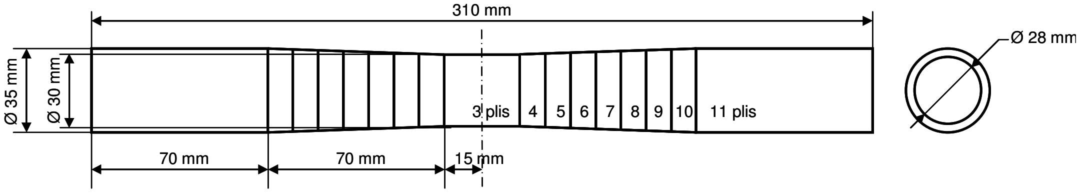
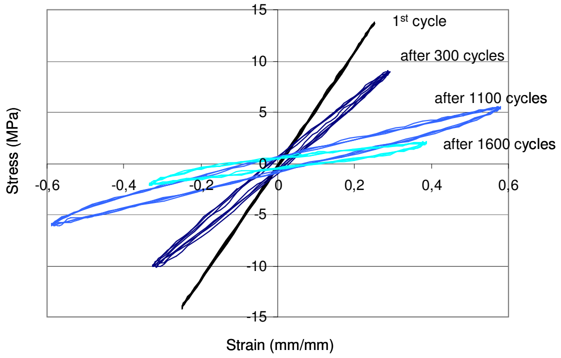


Figure 1. Geometry of the tube



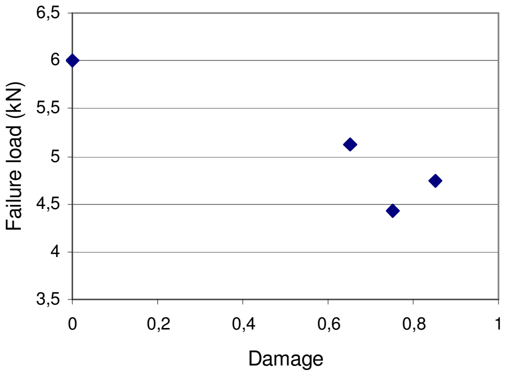


Figure 2. Stiffness decrease during the cycles **Figure 3**. Strength at failure according to the damage

The Figure 4 confirms that the decrease of strength is mainly due to matrix damage. The Load/Displacement curves were plotted for safe and damaged tubes. Fiber damage would affect the stiffness of the specimen.

Tests showed the strong influence of the matrix damage on the fiber failure. Because of matrix cracking and matrix/fibre debonding, the load transfer between fibers could not be fully. Failure of the ply occurred although the stress in the fiber direction was lower than the tensile failure strength.

For a fiber failure model, the matrix damage is evaluated and its influence on the fibers strength can be taken into account with the following simple criterion:

$$Y_{d_1} \le Y_{d_1}^{\max}(d_2) \tag{10}$$

where the thermodynamic force Y_{d_1} is proportional to the longitudinal stress (Refer to eq.(3))

and d_2 is the transverse damage in the UD ply. $Y_{d_1}^{\max}$ evolves sharply between two values according to a threshold value of d_2 as it was shown in Figure 5. In the case of static loading, the level of damage does not usually reach the threshold and the criterion (10) is equivalent to a maximal stress criterion. But in the case of fatigue loading with a high number of cycles, the level of the damage cannot be neglected.

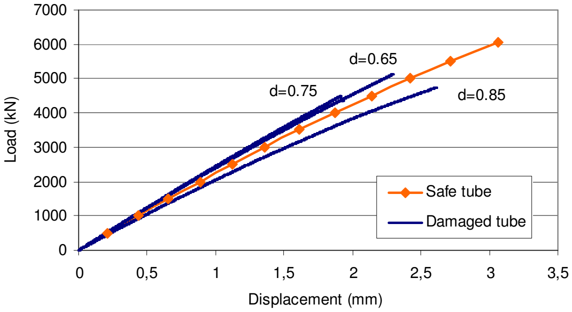


Figure 4. Tension tests on safe and damaged tubes

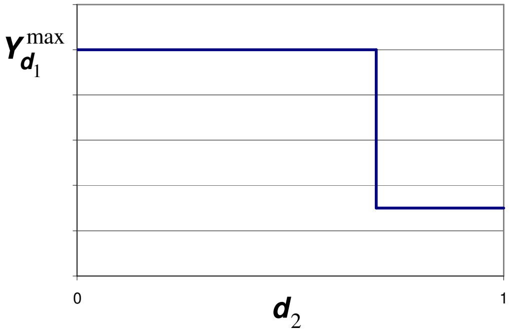


Figure 5. $Y_{d_1}^{\max}$ evolution law according to the damage d_2

3.2 Non local failure criterion

Previous studies led to observe a strong underestimation of the failure strength when a local criterion was used to predict the failure of laminated structure [5] with stress concentration. An original approach based on a *Fracture Characteristic Volume* (FCV) has been developed to predict the failure of laminated structures with stress concentrations 4]. The FCV was a cylinder defined at the ply scale as the volume $V = hS$, where h corresponds to the thickness of the ply and S is the in-plane area (see Figure 6).

Non local fracture criterion was defined in the case of static loading as:

$$\overline{Y_{d_1}} = \frac{\left(\frac{1}{V}\int_V \langle\sigma_1\rangle_+ \, dV\right)^2}{2\,E_1^0} \qquad and \qquad \overline{Y_{d_1}} < Y_{d_1}^{\max} \tag{11}$$

where $\overline{Y_{d1}}$ is the average thermodynamic force associated to the damage variable in the longitudinal direction d_1 and $Y_{d_1}^{\max}$ is a material property which needs to be identified.

The extension to fatigue loading led to modify the criterion and take into account the influence of matrix damage on the tensile fibre failure. The criteria (10) and (11) have become:

$$\begin{cases} \overline{Y_{d_1}} = \dfrac{\left(\dfrac{1}{V_f}\int_{V_f} \langle\sigma_1\rangle_+ \, dV\right)^2}{2\,E_1^0} & and \qquad \overline{Y_{d_1}} < Y_{d_1}^{\max}(\overline{d_2}) \\ \overline{d_2} = \dfrac{1}{V_f}\int_V d_2 \, dV & \end{cases} \tag{12}$$

In the case of fatigue, the strongly damaged material corresponds to another material (compared to healthy material) and can require a new identification of FCV.

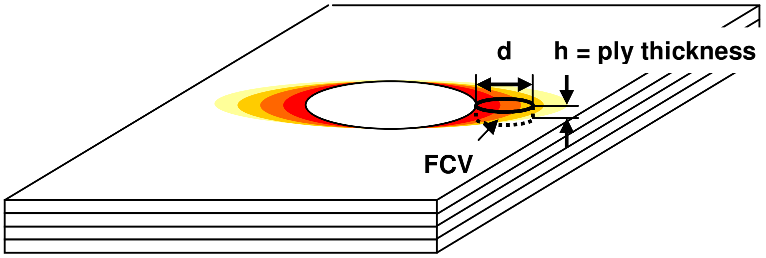


Figure 6. Non local criterion based on *Fracture Characteristic Volume* (*FCV*)

4 Open hole plate

Open hole plate were manufactured with Glass/Epoxy unbalanced woven ply. The geometry of the plate is presented in Figure 7. Two different laminates were studied $(0,-45,+45,90)_S$, noted as (QI), and $(+18,-18)_{2S}$. Cyclic loading was applied on the plates. The ratio between the minimal stress and the maximal stress was equal to 0.1. Different values of the maximal stress were studied.

The model was applied to the open hole plates. Failure of the laminates structures was defined by the criterion (12). A first ply failure strategy was applied. Experimental data and numerical simulations are compared in Figures 8.

The model matched quite well the experimental tests, which means that both damage and stress concentration have to be taken into account to predict the failure of laminated structures in the case of tensile fatigue loading.

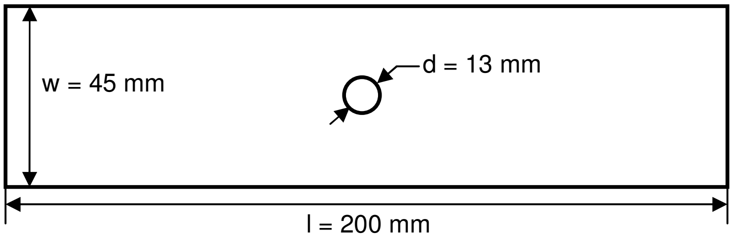


Figure 7. Geometry of the open hole plate

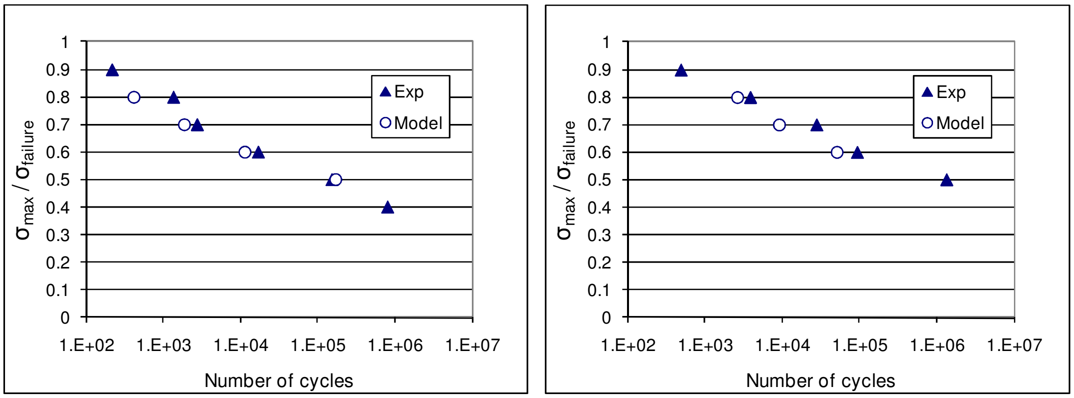


Figure 8. Comparison of experimental and numerical S-N curves for QI (left) and $[+18,-18]_{2S}$ open hole plates

5 Conclusion

A CDM model which describes the behavior of laminates under static and fatigue loadings was developed. The simple criterion based on a Fracture Characteristic Volume previously developed to predict the failure of laminated structure under static loading was extended to the case of fatigue loading. The effects of the matrix damage and the stress concentration were investigated and taken into account in a new criterion. Experimental tests on open hole plate were performed. The results obtained with the model matched the experimental data fairly satisfactorily.

The relation between the damage, the stress concentration and the tensile fiber failure needs to be studied with more accuracy. It seems that the size of the FCV depends on the level of transverse damage.

References

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