COMPARISON OF DIFFERENT TYPE COMPOSITE STRUCTURES SUBJECTED TO BLAST LOAD

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Keywords: composite structures, finite difference method, in-plane, blast load

Abstract
Nonlinear dynamic response of composite rectangular plates impacted by time-dependent external blast pulses is studied by use of numerical techniques. The theory is based on classical lamination theory for laminated and hybrid structures and sandwich plate theory for sandwich structures including the large deformation effects, such as geometric nonlinearities, in-plane stiffness and inertias, and shear deformation. The equations of motion for the plate are derived by the use of the virtual work principle. Approximate solutions are assumed for the space domain and substituted into the equations of motion. Then the Galerkin Method is used to obtain the nonlinear differential equations in the time domain. The finite difference method is applied to solve the system of coupled nonlinear equations. The results of theoretical analyses are summarized.

1 Introduction
With the advent of new composite materials, there is a need to reconsider the structural problems subjected to time-dependent external pulses such as blast load, sonic boom, gust, etc. In many advanced engineering applications, composite structures are widely used in industry because of their high strength/weight ratio, good fatigue life and corrosion resistance. In addition, their behavior under blast load has been of significant concern in engineering.
Several studies related to the effects of air blast loading on the panel structures are reviewed in the literature. Kazancı and Mecitoğlu [1] have studied on the nonlinear damped vibrations of a laminated composite plate subject to blast load. Kazancı and Turkmen [2] have considered in-plane stiffness and inertias in the analytical solution of the laminated composite plates subjected to blast load. Tanrıöver and Şenocak [3] have performed analytical-numerical approach on the large deflection analysis of unsymmetrically laminated composite plates. Librescu et al. [4] have addressed the problem of the dynamic response of sandwich panels exposed to blast loadings. Türkmen and Mecitoğlu [5,6] have performed some experimental, analytical and numerical studies on the nonlinear structural response of laminated composites subjected to blast load. Kazancı [7] has investigated the problem of the dynamic response of sandwich panels exposed to time-dependent external pulses.

Present work includes the analyses of laminated composite and sandwich plates subjected to blast load taking into account the in-plane stiffness and inertia effects. The geometric nonlinearity effects are taken into account with the von Kármán large deflection theory of thin plates. The equations of motion for the plate are derived by the use of the virtual work principle. Approximate displacement functions are assumed for the space domain by considering the nonlinear static deformations obtained using ANSYS software. They are substituted into the equations of motion and then the Galerkin Method is used to obtain the nonlinear differential equations in the time domain. The finite difference method is applied to solve the system of coupled nonlinear equations. The results of approximate numerical analyses are obtained and detailed discussions are made about the displacement-time histories.

2 Equations of Motion

A mathematical model for the laminated composite plate subjected to blast load is presented. The rectangular plate with the length a, the width b, and the thickness h, is depicted in Fig. 1. The Cartesian axes are used in the derivation.
Weierstrass theorem ensures that the displacement functions of a thin plate can be expanded in the series as follows

$$u = u^0 - z\frac{\partial w^0}{\partial x}\ ,\ v = v^0 - z\frac{\partial w^0}{\partial y}\ and\ w = w^0 \tag{1}$$

ignoring the higher order terms. Here *u, v* and *w* are the displacement components in the x, y and z directions, respectively. $(\ )^0$ indicates the displacement components of reference surface. For sandwich plates, separating the lateral displacements into contributions due to bending and shear as $w = w_b + w_s$ mentioned by several authors [9-11]. The details are summarized in Ref. [7].
If the blast source is distant enough from the plate, the blast pressure can be described in terms of the Friedlander exponential decay equation as [8]

$$P(t) = p_m(1 - t/t_p)e^{-\alpha t/t_p} \tag{2}$$

Using the constitutive equations and the strain-displacement relations in the virtual work and applying the variational principles, nonlinear dynamic equations can be obtained in terms of mid-plane displacements for a laminated and/or hybrid composite plate

$$\begin{aligned}
&L_{11}u^0 + L_{12}v^0 + L_{13}w^0 + N_1(w^0) + \bar{m}\ddot{u}^0 - q_x = 0\\
&L_{21}u^0 + L_{22}v^0 + L_{23}w^0 + N_2(w^0) + \bar{m}\ddot{v}^0 - q_y = 0\\
&L_{31}u^0 + L_{32}v^0 + L_{33}w^0 + N_3(u^0, v^0, w^0) + \bar{m}\ddot{w}^0 - q_z = 0
\end{aligned} \tag{3}$$

and for a sandwich plate

$$\begin{aligned}
&L_{11}u^0 + L_{12}v^0 + L_{13}{w_b}^0 + L_{14}{w_s}^0 + N_1(w^0) + \bar{m}\ddot{u}^0 - q_x = 0\\
&L_{21}u^0 + L_{22}v^0 + L_{23}{w_b}^0 + L_{24}{w_s}^0 + N_2(w^0) + \bar{m}\ddot{v}^0 - q_y = 0\\
&L_{31}u^0 + L_{32}v^0 + L_{33}{w_b}^0 + L_{34}{w_s}^0 + N_3(u^0, v^0, w^0) + \bar{m}\ddot{w}^0 - q_z = 0\\
&L_{41}u^0 + L_{42}v^0 + L_{43}{w_b}^0 + L_{44}{w_s}^0 + N_4(u^0, v^0, w^0) + \bar{m}\ddot{w}^0 - q_z = 0
\end{aligned} \tag{4}$$

where L_{ij} and N_i are linear and nonlinear operators, respectively. $\bar{m}$ is the mass of unit area of the mid-plane, q_x, q_y and q_z are the load functions in the axes directions. The explicit expressions of the operators can be found in Refs. [1,7,12].

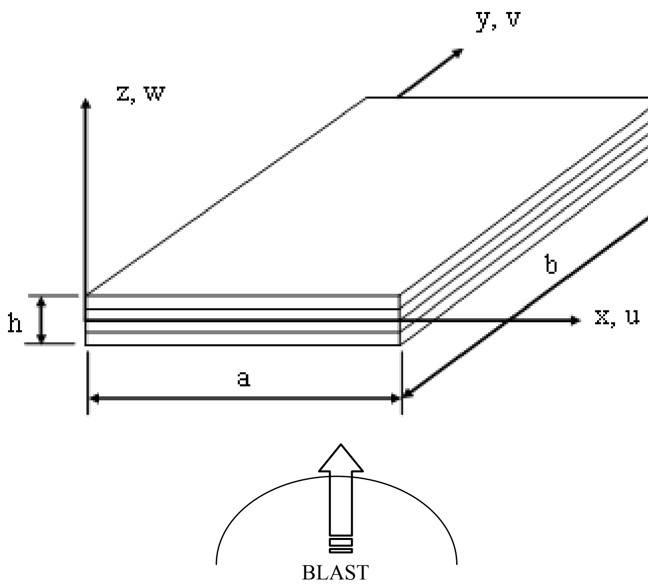

Figure 1a. Laminated composite plate

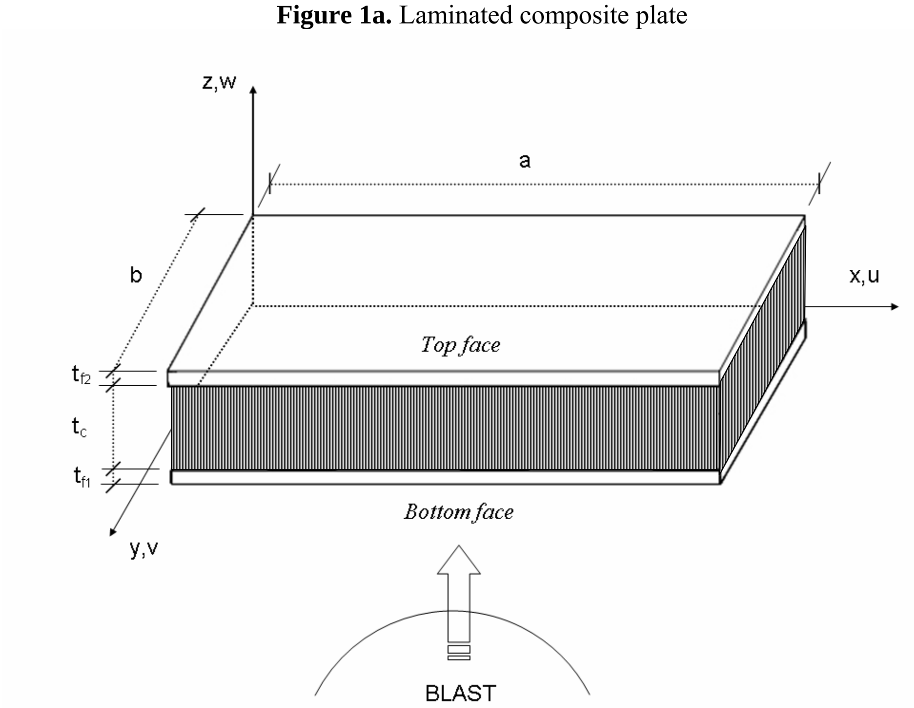

Figure 1b. Laminated sandwich plate

The boundary conditions are in the following forms:
All edges simply supported

$$u^0 = v^0 = w^0 = 0 \text{ at x=0,a and y=0,b}$$

$$M_x = 0 \text{ at x =0,a} \tag{5}$$

$$M_y = 0 \text{ at y =0,b}$$

all edges clamped

$$
\begin{aligned}
&u^0(0,y,t)=u^0(a,y,t)=u^0(x,0,t)=u^0(x,b,t)=0\\
&\frac{\partial u^0}{\partial x}(0,y,t)=\frac{\partial u^0}{\partial x}(a,y,t)=\frac{\partial u^0}{\partial y}(x,0,t)=\frac{\partial u^0}{\partial y}(x,b,t)=0\\
&v^0(0,y,t)=v^0(a,y,t)=v^0(x,0,t)=v^0(x,b,t)=0\\
&\frac{\partial v^0}{\partial x}(0,y,t)=\frac{\partial v^0}{\partial x}(a,y,t)=\frac{\partial v^0}{\partial y}(x,0,t)=\frac{\partial v^0}{\partial y}(x,b,t)=0\\
&w^0(0,y,t)=w^0(a,y,t)=w^0(x,0,t)=w^0(x,b,t)=0\\
&\frac{\partial w^0}{\partial x}(0,y,t)=\frac{\partial w^0}{\partial x}(a,y,t)=\frac{\partial w^0}{\partial y}(x,0,t)=\frac{\partial w^0}{\partial y}(x,b,t)=0
\end{aligned} \tag{6}
$$

and initial conditions are given by

$$
\begin{aligned}
&u^0(x,y,0)=0, \qquad v^0(x,y,0)=0, \qquad w^0(x,y,0)=0\,,\\
&\dot{u}^0(x,y,0)=0, \qquad \dot{v}^0(x,y,0)=0, \qquad \dot{w}^0(x,y,0)=0
\end{aligned} \tag{7}
$$

The approximation functions are selected so as to satisfy the natural boundary conditions.

$$u^0=\sum_{i=1}^{I}\sum_{j=1}^{J}U_{ij}(t)\phi_{ij}\left(x,y\right) \tag{8}$$

$$v^0=\sum_{k=1}^{K}\sum_{l=1}^{L}V_{kl}(t)\psi_{kl}\left(x,y\right) \tag{9}$$

$$w^0=\sum_{m=1}^{M}\sum_{n=1}^{N}W_{mn}(t)\chi_{mn}\left(x,y\right) \tag{10}$$

The simplest multi term approximations even results in the hundreds of integral terms during the application of the Galerkin procedure and therefore they are impractical. Therefore, one term approximation functions for the displacement components are used in this study.
The approximation function should closely resemble the first mode of the plate. It can be determined by considering the results of static large deformation analysis of laminated composite plate under the uniform pressure load by using ANSYS software. The approximation functions are determined by examining the finite element results obtained from the static large deformation results. It can be found from the Refs. [1,6,7].
Accounting only the first terms of displacement functions and applying the Galerkin method to the equations of motion, the time dependent nonlinear differential equations can be obtained for laminated and/or hybrid composite plates

$$a_0\ddot{U}+a_1U+a_2V+a_3W+a_4W^2+a_5=0 \tag{11}$$

$$b_0\ddot{V}+b_1V+b_2U+b_3W+b_4W^2+b_5=0 \tag{12}$$

$$c_0\ddot{W}+c_1W+c_2W^2+c_3W^3+c_4U+c_5V+c_6UW+c_7VW+c_8=0 \tag{13}$$

for sandwich structures

$$a_0\ddot{U} + a_1U + a_2V + a_3W_b + a_4W_s + a_5W_b^2 + a_6W_s^2 + a_7W_bW_s + a_8 = 0 \tag{14}$$

$$b_0\ddot{V} + b_1U + b_2V + b_3W_b + b_4W_s + b_5W_b^2 + b_6W_s^2 + b_7W_bW_s + b_8 = 0 \tag{15}$$

$$\begin{aligned} c_0\ddot{W}_b + c_1U + c_2V + c_3W_b + c_4W_s + c_5W_b^2 + c_6W_s^2 + c_7W_bW_s + c_8UW_b + c_9UW_s + c_{10}VW_b + c_{11}VW_s + c_{12}W_b^3 \\ + c_{13}W_s^3 + c_{14}W_b^2W_s + c_{15}W_bW_s^2 + c_{16} = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} d_0\ddot{W}_s + d_1U + d_2V + d_3W_b + d_4W_s + d_5W_b^2 + d_6W_s^2 + d_7W_bW_s + d_8UW_b + d_9UW_s + d_{10}VW_b + d_{11}VW_s + d_{12}W_b^3 \\ + d_{13}W_s^3 + d_{14}W_b^2W_s + d_{15}W_bW_s^2 + d_{16} = 0 \end{aligned} \tag{17}$$

where the coefficients of the equations are calculated for every different boundary condition. The dot denotes the derivative with respect to time. The initial conditions can be expressed as

$$\begin{aligned} &U(0) = 0\ ,\ \mathrm{V}(0) = 0\ ,\ \mathrm{W}(0) = 0 \\ &\dot{U}(0) = 0\ ,\ \dot{\mathrm{V}}(0) = 0\ ,\ \dot{\mathrm{W}}(0) = 0 \end{aligned} \tag{18}$$

Nonlinear-coupled equations of motion are solved by using finite difference method. Solution method and the coefficients of the equations are given in Refs. [1,7,12].

3 Numerical Results

Analyses are performed for laminated, hybrid and sandwich plate structures subjected to blast load. Material properties and loading conditions are summarized in each section separately.

3.1 Case I: Laminated composite plate (simply-supported)

A seven-layered fiber-glass fabric with (0º/90º) fiber orientation angle for one layer is used in the numerical analyses. Ply material properties used in the analyses are given as E_1 = 24.14 GPa, E_2 = 24.14 GPa, G_{12} = 3.79 GPa, ρ = 1800 kg/m^3, and ν_{12} = 0.11. The dimensions of the plate are a = 0.22 m, b = 0.22 m, and h = 1.96(10^{-3}) m. The analyses are performed for the uniform blast pressure. The maximum blast pressure p_m is taken to be 28.9 kPa for the plate all edges simply supported. The other parameters of the Friendlander's exponential decay function are chosen as α = 0.35 and t_p = 0.0018 s. Time history of dimensionless central deflection is shown in Fig. 2. The displacement-time histories are closely similar and the vibration frequencies are in a good agreement. The vibration amplitudes obtained from the approximate-numerical solutions are higher than those of the ANSYS results.

3.2 Case II: Laminated hybrid composite plate (clamped)

A six-layered hybrid composite laminated plate is chosen. Ply material properties are given in Table 1. Every ply has the same thickness which is 0.4×10^{-3} m. The stacking sequence of lamination is demonstrated in Fig.3. Fiber orientation is taken as [45/-45/45/-45/45/-45]. The dimensions of the plate are a = 0.3 m, b = 0.3 m, and h = 2.4×10^{-3} m. For Friedländer exponential decay equation, maximum blast pressure P_m is taken to be 30 kPa. Parameters are chosen as α = 0.35 and t_p = 0.0018 s. Analysis results are shown in Figure 4. There was a discrepancy after strong blast effect compared to the Finite Element Method (ANSYS) results.

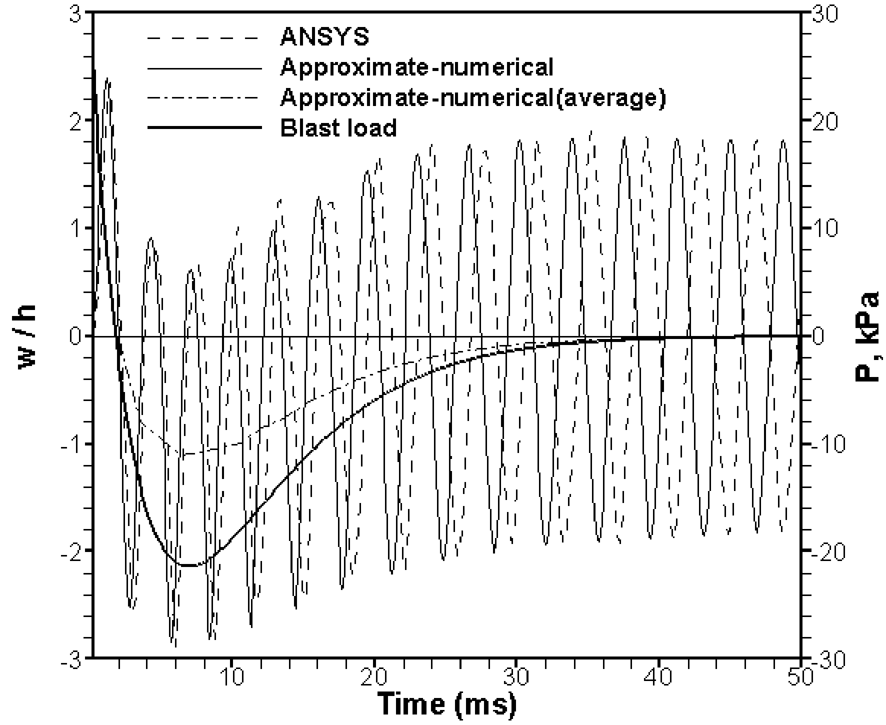

Figure 2. Deflection of laminated plate [13]

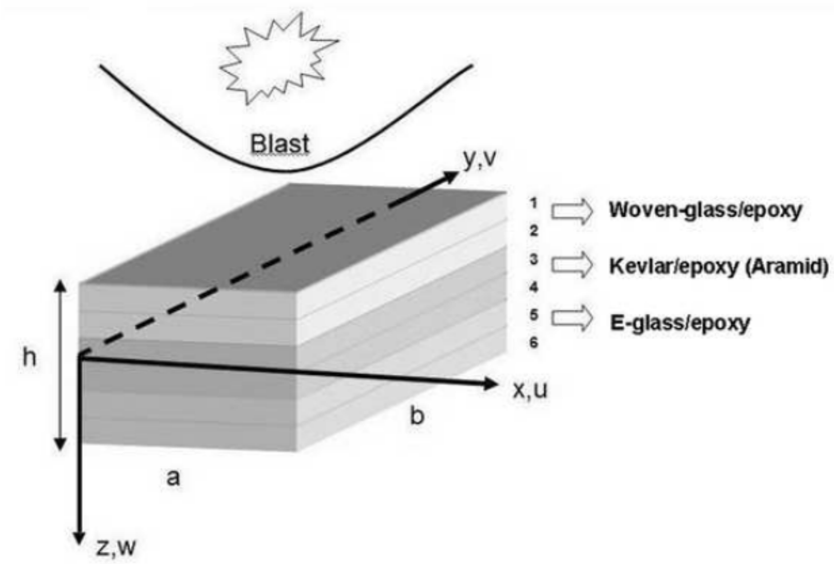

Figure 3. Laminated hybrid composite plate [14]

Ply material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12} (Poisson Ratio)	Density (kg/m^3)
Woven-glass/epoxy	29.7	29.7	5.3	0.17	2200
Kevlar/epoxy(Aramid)	87	5.5	2.2	0.34	1380
E-glass/epoxy	39	8.6	3.8	0.28	2100

Table 1. Material properties for laminated hybrid composite plate

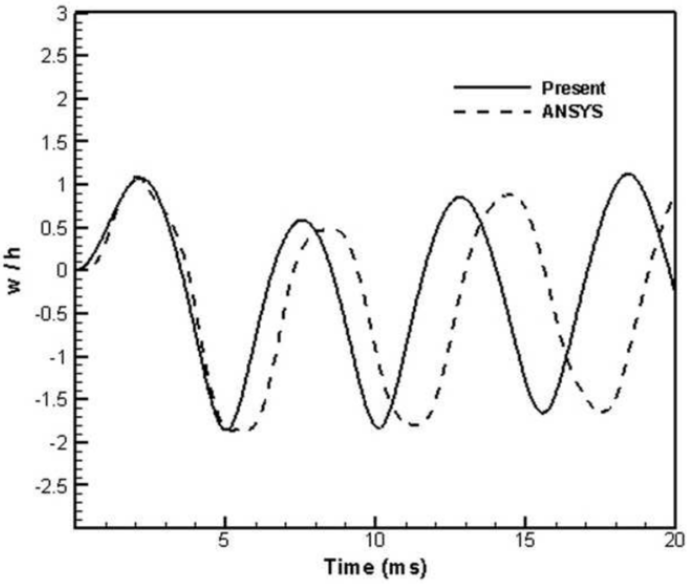

Figure 4. Deflection of laminated hybrid composite plate [14]

3.3 Case III: Composite sandwich plate (simply-supported)

A sandwich plate has a honeycomb core and face sheets of one layer fiber-glass fabric with (0º/90º) fiber orientation angle. Ply material properties used in the analyses are given as E_1 = 10 GPa, E_2 = 10 GPa, G_{12} = 4 GPa, ρ = 1800 kg/m3, and ν_{12} = 0.18. The thickness of the face sheets is 0.23 mm. The material properties of the core material are taken as E_{c1} = 29.6 MPa, E_{c2} = 14.5 MPa, G_{c12} = 14 MPa, ρ_c = 32 kg/m^3 and ν_{c12} = 0.3. The thickness of the core is 5.08 mm. The dimensions of the plate are a = 225 mm, b = 225 mm. The maximum blast pressure is taken to be 28.9 kPa for the plate all edges simply supported. The other parameters of the Friendlander's exponential decay function are chosen as α = 0.35 and t_p = 0.0018 s. Analysis results are shown in Figure 5.

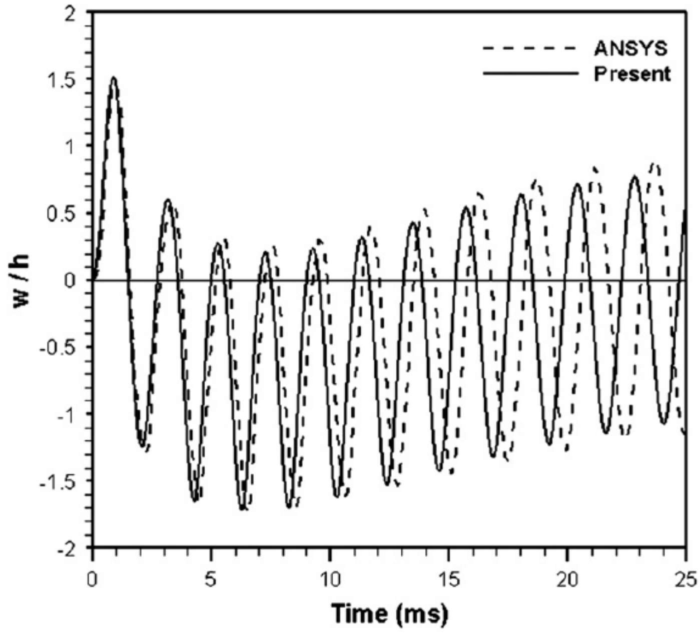

Figure 5. Deflection of sandwich plate [7]

3 Conclusions

Nonlinear dynamic response of laminated, hybrid and sandwich structures subjected to blast load are analyzed and summarized by use of numerical techniques. The geometric nonlinearity effects are taken into account with the von Karman large deflection theory. The finite difference method is applied to solve the system of coupled nonlinear equations. Different boundary conditions are considered. Simply-supported plates show better convergence than clamped ones for the chosen structures. It is shown that different type composite structures can be solved by use of present method. However, there was a discrepancy after the strong blast effect. One term approximation functions are used in the numerical analyses. This may be the reason for the discrepancy.

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