2D-MODELING OF FREE VIBRATION AND IMPACT RESPONSE OF MULTI-LAYER PLATES

A. Lerpinière[1*], V.A. Duong[1], J.F. Caron[1]

[1] *Université Paris-Est, Laboratoire Navier (Ecole des Ponts ParisTech, IFSTTAR, CNRS), 6-8 avenue Blaise Pascal, Cite Descartes, Champs-sur-Marne, 77455 MARNE-LA-VALLEE, France*
**achille.lerpiniere@polytechnique.edu*

Keywords: Composite laminates, Free vibration, Layerwise, Delamination.

Abstract

This paper deals with a finite element approach based on the Multiparticle Model of Mutilayered Materials (M4) designed for delamination study and developed here for the free vibration analysis of multi-layered composite plates and impacted ones. It belongs to the layerwise modeling family. The laminated plate is considered as a superposition of Reissner plate coupled by interfacial stresses. An eight-node per layer element is extended here to the dynamic framework. The results obtained are validated by comparisons with existing analytical and numerical modeling in case of symmetric (0/90/90/0) laminates, antisymmetric $(0/90)_n$ *ones and impacted* $(0_2/45_2/90_2/\text{-}45_2)_S$ *ones. Despite the relatively simple 2D description of the plates, the good results highlight the efficiency of such an approach (even for thick laminates).*

1 Introduction

Aided by the research and demonstration projects funded by industries and governments around the world, laminated composites are finding wider acceptance in industries as aerospace, automobile, civil, marine. These materials have gained the advantage by cost reduction, light weight, desired strength and stiffness parameters achievement. Different studies have been conducted to determine the dynamic properties of composite laminates. Basic theories of plates and shells can be classically classed in five categories: classical thin plate theory (CPT), first-order shear deformation theory (FOST), higher-order shear deformation theory (HOST), layer-wise model and theory of three-dimensional elasticity. The classical lamination theory assumes that straight lines originally normal to the plate median surface are constrained to remain straight and normal during the process of deformation. This Kirchhoff's assumption is equivalent to neglecting transverse shear deformation in the plate. This simply theory can provide reasonably accurate prediction only for relatively thin plate. For thicker plates, Reissner and Mindlin [1] developed the first-order shear deformation theory (FOST) including transverse shear and rotatory inertia effects in the dynamic analysis of plates. Many studies have been carried out using FOST for the free vibration analysis of composite plates. Noor [2] analyzed the free vibration of cross-ply laminated plates. Reddy [3] presented a finite element model based on Yang-Norris-Stavsky theory and validated for the free vibration of antisymmetric, angle-ply laminated plates. Khare and al.[4] presented finite element method and its results for free vibration of thick isotropic plate, cross-ply laminated and antisymmetric angle-ply laminated composite plates, and sandwich plates. Dai, Lim and Chen [5] applied FOST for the shallow conical shell panels. Liew and Lim and al. [6] presented a semi-analytical solution for vibration of discontinuous Mindlin rectangular plates with abrupt thickness variation and studied rectangular plates with central cut-out. Yuan and Dawe [7] developed a B-spline finite strip method (FSM) using FOST for predicting the natural frequencies of rectangular sandwich panels. However, the FOST ignoring the effects of cross-sectional warping leads to an unrealistic variation of the transverse shear stresses through the laminate thickness. So, a family of refined 2D theories has been developed, named higher-order shear deformation theory (HOST). The HOST incorporate higher-order modes of transverse cross-sectional deformation by involving higher-order terms in the Taylor's expansions of the displacements in the thickness coordinate. So, the shear correction coefficients as in the first-order Reissner/Mindlin theory are not required. Kant and al. [8] presented the formulation of a higher-order flexure theory. In [9], they presented an analytical solution based on HOST. Reddy and al. [10] considered angle-ply and cross-ply laminates with two simply supported edges and the other two edges in different combinations of simply supported, free and clamped conditions. To simulate thick laminate construction and to better estimate the 3D stress state in the thickness, the three-dimensional theory could also be used. Among these approaches are included 3D exact analytical solutions and 3D finite element methods. Exact solutions are proposed but for limited cases. Srinivas found an exact solution for vibration of simply-supported homogeneous and laminated thick rectangular plates and of simply supported thick orthotropic rectangular plates and laminate in [11]. Kant [9] proposed analytical solutions based on a higher-order refined theory. Noor [2] used three-dimensional theory of elasticity for obtaining highly accurate predictions of response characteristics of composite plates. Noor and Burton [12] presented solutions for antisymmetric laminated anisotropic plates. Chen [13] developed a semi-analytical method which combines the state space approach with the technique of differential quadrature for free vibration of a cross-ply laminated composite rectangular plate. Otherwise, another model family, called layerwise approaches can also be used to calculate thick laminates. It combines a 2D description of the plate and stress or displacement fields per layer. For the static studies, several references can be found [14], but very few for dynamic development [15]. Initially designed for the study of delamination of composite materials by a relevant estimation of interlaminar stresses, such a model is used in this paper [16]. Inspired from the work of Pagano [17], it was named Multiparticle Models of Multilayered Materials (M4). This model was being already widely validated [18]. A quadrilateral $\mathbf{C}^0$ finite element model is based on this modeling [19]. The present paper deals with developments of this finite element to include the dynamic behaviour of composite and sandwich laminates (free vibration and impact).

2 Theory and formulation of the LS1 (M4-5n)

Construction's details of the Multiparticle Model of Multilayered Materials (M4) were given in [20] and [19]. The LS1 model can be considered as a superposition of Reissner plates coupled by interlaminar shear and normal stresses at the interface and as a Reissner model, 5 generalized displacements describe the kinematic of elementary layer of the n layers composite. This section will describe a global overview of the model in the governing equations.

2.1 Description and notation

The multilayered plate under consideration is then composed of n orthotropic elastic layers perfectly bonded together.

• Each layer i, thickness e_i, is bordered by the bottom surface h_i^- and the top surface h_i^+. The average surface is noted by $\hbar_i$.

• The occupied volume of the plate is $\boldsymbol{\Omega} = \partial\boldsymbol{\Omega} \times [h_1^-, h_n^+]$

• The superscripts i and j, j+1 indicate layer i and the interface between layer j and j+1 (i = 1..n and j = 1..n−1).

• The subscripts o, p, q, r indicate the components in the (x, y, z) space; they are assigned the values 1,2 and 3. The Greek subscripts α, β, γ, δ indicate the components on the (x, y) plane and are assigned the values 1 and 2. The normal direction is noted by indice 3.

• Tensors, matrices and vectors are defined by the bold face characters.

2.2 Governing equations

Details can been found in [20].

The constraint state of each layer i is described by the following generalized efforts: the in-plane stress resultant $N^i_{\alpha\beta}$, the in-plane moment resultants $M^i_{\alpha\beta}$, the out-of-plane shear stress resultant Q^i_α and the interlaminar shear and normal stresses at interface $\tau^{j,j+1}_\alpha$ and $\nu^{j,j+1}$.

$$\begin{aligned}
&N^i_{\alpha\beta}(x,y) = \int_{h_i^-}^{h_i^+} \sigma_{\alpha\beta}(x,y,z)dz \\
&M^i_{\alpha\beta}(x,y) = \int_{h_i^-}^{h_i^+} (z-\hbar_i)\sigma_{\alpha\beta}(x,y,z)dz \\
&Q^i_\alpha(x,y) = \int_{h_i^-}^{h_i^+} \sigma_{\alpha 3}(x,y,z)dz \\
&\tau^{j,j+1}_\alpha(x,y) = \sigma_{\alpha 3}(x,y,h_j^+) \\
&\nu^{j,j+1}(x,y) = \sigma_{33}(x,y,h_j^+)
\end{aligned} \tag{1}$$

In this model, rather than 3D displacements, the 3D stresses are approximated. The approximate membrane stresses $\sigma_{\alpha\beta}$ are chosen linear in the thickness of each layer. Balance equations induce that transverse shear stresses and normal stress are second and third degree polynomial, respectively. The complete expression of the 3D stress field can be found in [30]. Consequently, it has been shown in [30] that the following in-plane deformations $\varepsilon^i_{\alpha\beta}$, the curve $\chi^i_{\alpha\beta}$ and the transverse deformations γ^i_α are the strains associated with the generalized efforts respectively $N^i_{\alpha\beta}$, $M^i_{\alpha\beta}$, Q^i_α. For the same reason, $D^{j,j+1}_\alpha$ and $D^{j,j+1}_3$ are associated with $\tau^{j,j+1}_\alpha$ and $\nu^{j,j+1}$.

$$\begin{aligned}
&\varepsilon^i_{\alpha\beta} = \frac{1}{2}(U^i_{\alpha,\beta} + U^i_{\beta,\alpha}) \\
&\chi^i_{\alpha\beta} = \frac{1}{2}(\phi^i_{\alpha,\beta} + \phi^i_{\beta,\alpha}) \\
&\gamma^i_\alpha = \phi^i_\alpha + U^i_{3,\alpha} \\
&D^{j,j+1}_\alpha = U^{j+1}_\alpha - U^j_\alpha - \frac{e^j}{2}\phi^j_\alpha - \frac{e^{j+1}}{2}\phi^{j+1}_\alpha \\
&D^{j,j+1}_3 = U^{j+1}_3 - U^j_3
\end{aligned} \tag{2}$$

Where the 5n following generalized displacements U^i_α, φ^i_α, U^i_3 are respectively the in-plane displacement, the rotation fields and vertical displacement of layer i.

$$\begin{aligned}
U^i_\alpha(x,y) &= \int_{h_i^-}^{h_i^+} \frac{1}{e^i} U_\alpha(x,y,z)dz \\
\phi^i_\alpha(x,y) &= \int_{h_i^-}^{h_i^+} \frac{12}{e^{i\,2}} U_\alpha(x,y,z)dz \\
U^i_3(x,y) &= \int_{h_i^-}^{h_i^+} \frac{1}{e^i} U_3(x,y,z)dz
\end{aligned} \tag{3}$$

The equilibrium and constitutive equations of model are identified with the help of the Hellinger-Reissner variational approach [20]. The 5n Reissner following equations are firstly obtained:

$$\begin{aligned}
& N^i_{\alpha\beta,\beta} + (\tau_\alpha^{i,i+1} - \tau_\alpha^{i-1,i}) = 0 \\
& Q^i_{\beta,\beta} + (\nu^{i,i+1} - \nu^{i-1,i}) = 0 \\
& M^i_{\alpha\beta,\beta} + \frac{e^i}{2}(\tau_\alpha^{i,i+1} + \tau_\alpha^{i-1,i}) - Q^i_\alpha = 0
\end{aligned} \tag{4}$$

Considering now an orthotropic layer i, the compliance can be classically described by two sub tensors: $S^i_{\alpha\beta\gamma\delta}$ in-plane compliance and $S^i_{\alpha3\beta3}$ shear compliance. It leads to the following constitutive equations:

$$\begin{aligned}
\varepsilon^i_{\alpha\beta} &= \frac{1}{e^i} S^i_{\alpha\beta\gamma\delta} N^i_{\gamma\delta} \\
\chi^i_{\alpha\beta} &= \frac{12}{(e^i)^3} S^i_{\alpha\beta\gamma\delta} M^i_{\gamma\delta} \\
\gamma^i_\alpha &= \frac{6}{5e^i} S^i_{\alpha3\gamma3} Q^i_\beta - \frac{1}{10}(4S^i_{\alpha3\gamma3})(\tau_\beta^{i,i+1} + \tau_\beta^{i-1,i}) \\
D_\alpha^{j,j+1} &= -\frac{1}{10}(4S^j_{\alpha3\beta3})Q^j_\beta - \frac{1}{10}(4S^{j+1}_{\alpha3\beta3})Q^{j+1}_\beta - \frac{e^j}{30}(4S^j_{\alpha3\beta3})\tau_\beta^{j-1,j} \\
&+ \frac{2}{15}(e^j(4S^j_{\alpha3\beta3}) + e^{j+1}(4S^{j+1}_{\alpha3\beta3}))\tau_\beta^{j,j+1} \\
&- \frac{e^{j+1}}{30}(4S^{j+1}_{\alpha3\beta3})\tau_\beta^{j+1,j+2} \\
D_3^{j,j+1} &= \frac{9}{70} e^j S^j_{3333} \nu^{j-1,j} + \frac{13}{35}(e^j S^j_{3333} + e^{j+1} S^{j+1}_{3333})\nu^{j,j+1} \\
&+ \frac{9}{70} e^{j+1} S^{j+1}_{3333} \nu^{j+1,j+2}
\end{aligned} \tag{5}$$

This modeling was firstly designed for the study of the delamination of cross plies and bonding [21]. Several analytical developments were made and criteria of delamination were proposed in [18], [22].

3 Multiparticle finite element model

Based on the previous model, a $\mathbf{C}^0$ finite element model, involving an eight-node isoparametric quadrilateral element with 5n d.o.f at each nodal point and four second-order Gaussian points is formulated. A program called MPFEAP (Multiparticle Finite Element Analysis Program) has been developed for the implementation of the proposed element [19] which permits solution of static laminated plate problem [23]. For instance in [23], the problem of a composite plate with free edges under mechanical an thermal loading is detailed. The development proposed here uses the mixed algorithm - subspace algorithm proposed by Dhatt and Touzot [24] to extend MPFEAP for dynamic problems, in particular to calculate the p first vibration modes and to simulate the impact of an object on a multilayer plate step by step. The geometry is meshed with square master elements defined in the ξ, η space. The element is described by eight nodal points and by the shape function $P_k(\xi,\eta)$. (See [19])

4 Numerical results and discussion

An 8×8 mesh and a 16×16 mesh have been used in the computations for thin and thick simple supported plate respectively for the free-vibration analysis, and a 10×15 mesh has been used for the impacted plate analysis. These choices have been deduced from a convergence study. The details of the convergence study are not presented here. The reduced integration scheme (4 points Gauss) is used.

The following simply supported boundary conditions are applied at each layer i:

$$\begin{aligned} v^i = w^i = \phi_y^i = 0 \rightarrow x = 0, a \\ u^i = w^i = \phi_x^i = 0 \rightarrow y = 0, b \end{aligned} \tag{6}$$

4.1 Cross ply (0/90/90/0) laminated composite plate

The effect of side-to-thickness ratio is considered in the case of cross-ply (0/90/90/0) composites. The orthotropic material properties for the symmetric angle ply laminates considered are $E_1/E_2 = 40$, $G_{12} = 0.6E_2$, $G_{13} = G_{23} = 0.5E_2$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$. The non-dimensional fundamental frequencies presented in Table 1 are compared with those of HOSTs proposed by Kant [8], with the third order shear theory analytical solutions of Reddy [25] and with of the second order shear theory of Whitney-Pagano [26]. Kant developed 2 HOSTs. The models concern with third order shear deformation. When the first supposes a third order displacement in thickness, the second supposes a constant displacement in thickness. The results of M4-5n model to thin and thick plates ($a/h \geq 4$) are in excellent agreement with those of these HOSTs.

a/h	4	10	20	50	100
MPFEAP	9.17966	15.05926	17.63287	18.67047	18.83627
Kant-Model 1 [8]	9.2870	15.1048	17.6470	18.6720	18.8357
Kant-Model 2 [8]	9.2710	15.0949	17.6434	18.6712	18.8355
Reddy [25]	9.3235	15.1075	17.6457	18.6713	18.8357
Whitney-Pagano [26]	9.3949	15.1426	17.66596	18.6742	18.8362

Table 1. Non-dimensionalized natural frequencies $\acute{\omega} = \omega a^2(\rho/(E_2h^2))^{1/2}$ of simply supported cross-ply (0/90/90/0)laminates

4.2 $(0/90)_n$ laminated composite plate

The side-to-thickness of the laminates a/h is equal to 5. The orthotropic material properties in all the laminates considered are E_1/E_2 = open, $E_2 = E_3$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$. The degree of orthotropy of individual layers (E_1/E_2) and the number of layers of antisymmetric cross-ply $(0/90)_n$ are varied (n =1 and 3). The fundamental frequencies obtained by MPFEAP are also validated by comparing with those of 3-D elasticity [2] and finite element results using two higher-order plate formulations given by Khare-Kant [4] and Reddy [10] (see Table 2). The solutions of 3D elastic theory are used as reference.

n	E1/E2	3	10	20	30	40
1	**3-D theory [2]**	0.25031	0.27938	0.30698	0.32705	0.34250
	MPFEAP	(-0.69)	(-0.63)	(-0.61)	(-0.61)	(-0.63)
	Khare – HOST [4]	(-0.65)	(-0.34)	(0.36)	(1.11)	(1.81)
	Reddy [10]	(-0.65)	(0.06)	(1.91)	(4.02)	(6.12)
	CPT	(8.19)	(10.84)	(15.39)	(20.27)	(25.21)
3	**3-D theory [2]**	0.26440	0.33657	0.39359	0.42783	0.45091
	MPFEAP	(-0.99)	(-0.95)	(-0.91)	(-0.89)	(-0.87)
	Khare – HOST [4]	(-0.89)	(-0.59)	(-0.25)	(-0.02)	(0.15)
	Reddy [10]	(-0.82)	(-0.11)	(0.79)	(1.49)	(2.03)
	CPT	(9.55)	(19.48)	(32.71)	(44.83)	(56.04)

Table 2. Non-dimensionalized natural frequencies $\acute{\omega} = \omega(\rho h^2/E_2)^{1/2}$ of simply supported cross-ply square laminates $(0/90)_n$

In the case where n = 1 and for a constant total thickness of the laminate, influence of the degree of orthotropy in the calculations is very critical because layers are thicker than for n > 1, and the cross section is more distorted. Results obtained by MPFEAP where n = 1 show stability of the model LS1. The LS1, as said previously, doesn't try to approximate the displacement fields, but only stresses, which is an easier way to approach the phenomenon. It's surely the reason why results of LS1 are steadier and less influenced by both the number of layers and the degree of orthotropy. MPFEAP provides very good estimations even for high orthotropic material degree E_1/E_2, situation where other approaches are less efficient.

4.3 Impacted plate $(0_2/45_2/90_2/-45_2)_S$

When is considered the study of an impacted plate as in Bouvet and al. [27] with a hertz contact law for the impactor. The plate dimensions are 100×150 mm² and a total thickness of 4.16 mm. The material properties for the impactor are E=207GPa, ν=0.3, ρ=7800kg/m^3 and the experimental indentation coefficient used in the impact law are S_p=0.094, α_p=0.0167.
Calculating the Von-Mises equivalent interface stresses given by:

$$\sigma_{eq}^{j,j+1} = \sqrt{(\nu^{j,j+1})^2 + 3((\tau_x^{j,j+1})^2 + (\tau_y^{j,j+1})^2)} \quad (7)$$

Where $\nu^{j,j+1}$ and $\tau^{j,j+1}_\alpha$ are given in (1).
The results obtained by plotting isovalues of this equivalent stress are very close to the pattern of delamination found in [27] by C-scan and mode I energy release rate calculation as we can see in Figure 1, even though damaging is not taking into account in this study.

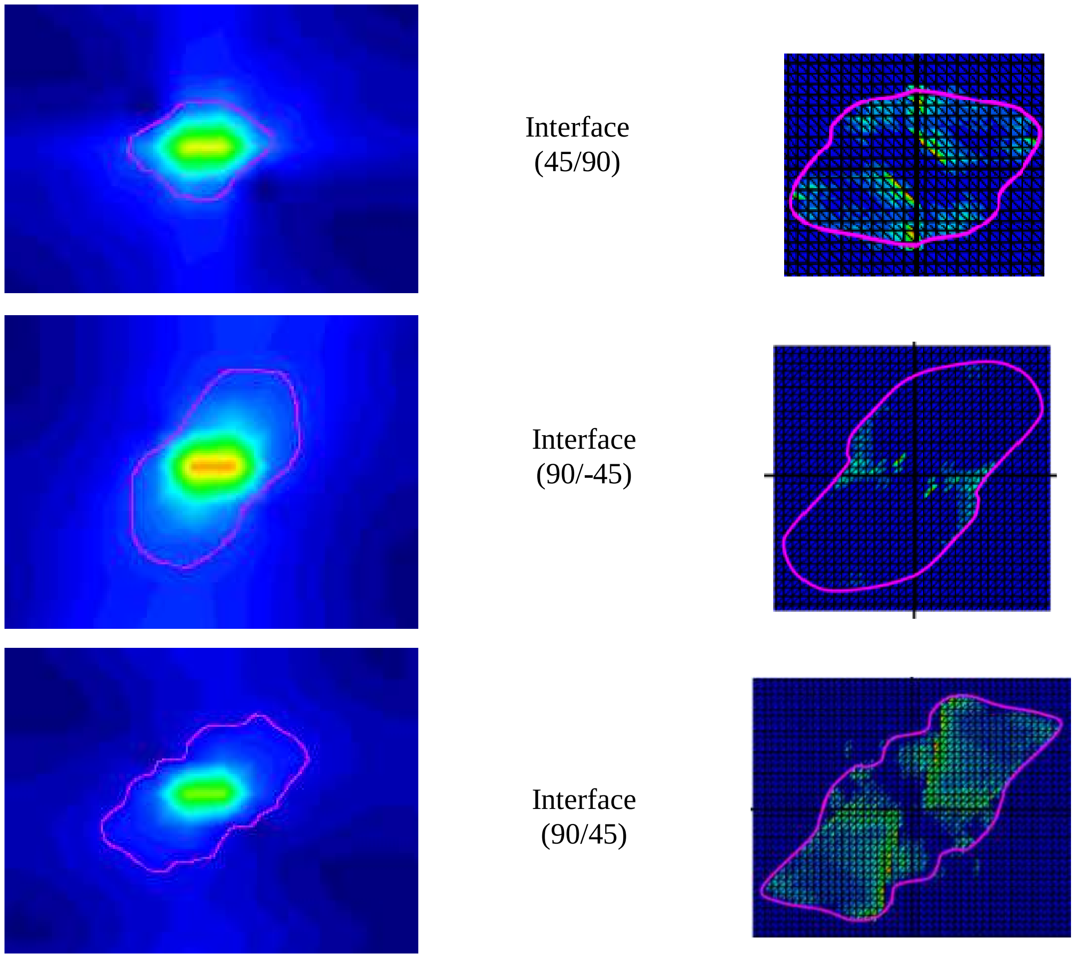

Figure 1. Comparison between equivalent interface stresses given by (7) of MPFEAP (left and purple line corresponding to critical value of σ_{eq}) and delaminated areas found in Bouvet [27] by C-scan (right and purple line) for a $(0_2/45_2/90_2/-45_2)_S$ plate impacted at its center .

4 Conclusion

The approach belongs to the layerwise multilayer plate modeling. It could be described as a superposition of Reissner plate bonding together. Each layer is characterized by the same displacement and constraint fields. The finite element code MPFEAP based on a multiparticle model of multilayered materials is developed here for the dynamic case and tested for free vibration and impacted plates. Solutions of HOSTs and of 3D elasticity models are used as reference in both cases. Considering the influence of orthotropy in $(0/90)_n$ plates, MPFEAP has been more efficient than HOSTs, and not under influence of E_1/E_2. Moreover, it shows some good primary results when studying the impact of a plate to predict delamination, and so delamination criterion and law will be implemented in next MPFEAP version in progress.

References

[1] R.D. Mindlin. Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates. *ASME Journal of Applied Mechanics*, 18:31–38, 1951.

[2] A.K. Noor. Free vibrations of multilayered composite plates. *AIAA Journal*, 11:1038–1039, 1972.

[3] J.N. Reddy. Free vibration of antisymmetric, angly-ply laminated plates including transverse shear deformation by the finite element method. *Journal of Sound and Vibration*, 66(4):565–640, 1979.

[4] R.K. Khare, T. Kant, and A.K. Garg. Free vibration of composite and sandwich laminates with a higher-order facet shell element. *Composite Structures*, 65, 2004.
[5] K.Y. Dai and al. A mesh-free method for static and free vibration analysis of shear deformable laminated composite plates. *Journal of Sound and Vibration*, 269:633–652, 2004.
[6] K.M. Liew, T.Y. Ng, and S. Kitipornchai. A semi-analytical solution for vibration of rectangular plates with abrupt thickness variation. *International Journal of Solids and Structures*, 38:4937–4954, 2001.
[7] W.X. Yuan and D.J. Dawe. Free vibration and stability analysis of stiffened sandwich plates. *Composite Structures*, 63:123–137, 2004.
[8] T. Kant, D.R.J. Owen, and O.C. Zienkiewicz. A refined higher order c^0 plate bending element. *Composite Structures*, 15:177–260, 1982.
[9] T. Kant and K. Swaminathan. Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher-order refined theory. *Composite Structures*, 53:73–85, 2001.
[10] J.N. Reddy and N.D. Phan. Stability and vibration of isotropic, orthotropic and laminated plates according to a higher-order shear deformation theory. *Journal of Sound and Vibration*, 98(2):157–226, 1985.
[11] S. Srinivas and A.K. Rao. Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates. *International Journal of Solids and Structures*, 6:1463–1543, 1970.
[12] A.K. Noor and W.S. Burton. Three-dimensional solutions for antisymmetric laminated anisotropic plates. *ASME Journal of Applied Mechanics*, 57:182–189, 1990.
[13] W.Q. Chen and C.F. Lue. 3d free vibration analysis of cross-ply laminated plates with one pair of opposite edges simply supported. *Composite Structures*, 69: 77–87, 2005.
[14] E. Carrera and S. Brischetto. Analysis of thickness locking in classical, refined and mixed multilayered plate theories. *Computers Structures*, 82:549–562, 2008.
[15] A.J.M Ferreira, C.M.C Roque, and R.M.N Jorge. Free vibration analysis of symmetric laminated composite plates by fsdt and radial basis functions. *Computer Methods in Applied Mechanics and Engineering*, 194:4265–4278, 2005.
[16] J.F. Caron and A. Ehrlacher. Modeling the kinetics of transverse cracking in composite laminates. *Composites Science and Technology*, 57:1261–1330, 1997.
[17] N.J. Pagano. Stresses fields in composite laminates. *International Journal of Solids and Structures*, 14:385–400, 1978.
[18] J.F. Caron, A. Diaz Diaz, A. Chabot, and A. Ehrlacher. Multi-particle modeling for the prediction of delamination in multi-layered materials. *Composites Science and Technology*, 66:755–765, 2006.
[19] V.T Nguyen and J.F Caron. A new finite element for free edge effect analysis in laminated composites. *Computers Structures*, 84:1538–1546, 2006.
[20] R.P. Carreira, J.F. Caron, and A. Diaz Diaz. Model of multilayered materials for interface stresses estimation and validation by finite element calculations. *Mechanics of Materials*, 34:217–230, 2002.
[21] R. Hadj-Ahmed, G. Foret, and A. Ehrlacher. Stress analysis in adhesive joints with a multiparticle model of multilayered materials (M4). *International Journal of Adhesion and Adhesives*, 21:297–307, 2001.
[22] A. Diaz Diaz and J.F. Caron. Interface plasticity and delamination onset prediction. *Mechanics of Materials*, 38:648–663, 2006.
[23] V.T. Nguyen and J.F. Caron. Finite element analysis of free-edge stresses in composite laminates under mechanical and thermal loading. *Composites Science and Technology*, 2007.
[24] D. Gouri and G. Touzot. Une Présentation de la Méthode des Eléments Finis. *Maloine S. A.*, 1984.
[25] J.N. Reddy. A simple higher order theory for laminated composite plates. *Journal of Applied Mechanics*, 51:745–796, 1984.
[26] J.M. Whitney and N.J. Pagano. Shear deformation in heterogeneous anisotropic plates. *Journal of Applied Mechanics*, pages 1031–1036, 1970.
[27] C. Bouvet. Impact damages on composite laminate. *JNC 17*, 2011.