C_0 FE MODEL FOR THE ANALYSIS OF LAMINATED SOFT CORE SANDWICH PLATES

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Abstract
A new C_0 finite element (FE) model has been proposed in this paper for the static analysis of laminated soft core sandwich plates based on higher order zigzag theory (HOZT). The variation of the in-plane displacements are considered to be cubic for both the face sheets and the core, while the transverse displacement is assumed to vary quadratically within the core and remains constant in the faces beyond the core. It satisfies the conditions of transverse shear stress continuity at the layer interfaces as well as satisfies the zero transverse shear stress condition at the top and bottom of the plate. As very few elements based on this plate theory (HOZT) exist and they possess certain disadvantages, an attempt has been made to develop this new element. The nodal field variables are chosen in an efficient manner to overcome the well-known problem of continuity requirement of the derivatives of transverse displacements in the present refined plate theories. A nine node C_0 quadratic plate finite element is implemented to model the HOZT for the present analysis. Numerical examples covering different features of laminated composite and sandwich plates are presented to illustrate the accuracy of the present model.

1 Introduction
Laminated composites are relatively weak in shear due to their low shear modulus compared to extensional rigidity and this becomes very complex in sandwich structures as the material property variation is very large between the core and face layers. Thus the effect of shear deformation is quite significant and needs special attention. For these kinds of structures, exact three dimensional (3D) solutions are required to predict its response accurately. Such 3D solutions are available only for simple boundary conditions and geometries [1]. 3D finite element (FE) solution is computationally expensive and often intractable. Hence, the development of an appropriate mathematical two dimensional (2D) model to represent the behavior of fiber reinforced composite laminates has drawn a considerable amount of attention. These plate theories (2D) can be broadly divided into two categories based on their assumed displacement fields: 1) Single layer theory and 2) Layer-wise theory.

The structural behavior of sandwich laminate cannot be accurately predicted by first order shear deformation theory (FSDT) as it assumes uniform transverse shear strain over the entire plate thickness requiring a shear correction factor [2]. The further improvement comes in the form of higher order shear deformation theory (HSDT), where the higher order variation of in-plane displacement through the thickness is considered to represent the actual warping of the plate cross-section due to which it (HSDT) becomes free from shear correction factor [3]. It [HSDT] gives a continuous variation of shear strain across the thickness, which gives discontinuity in the shear stress distribution at the layer interfaces due to different values of shear rigidity at the adjacent layers. But the actual phenomenon is just the opposite i.e., the shear strain is discontinuous and the shear stress is continuous at the layer interfaces [4]. In order to consider above aspect, the refined plate theories developed starting with layer-wise plate theories [5]. In layer-wise plate theories the unknown displacement components are taken at all the layer interfaces, which give a zigzag pattern of through thickness variation for the in-plane displacement to represent the desired shear strain discontinuity at the layer interfaces. As the number of unknowns increases directly with the increase in the number of layers due to which these (layer-wise) theories required huge computational effort. The problem of layer wise theories has been overcome by considering the unknowns at all the interfaces are expressed in terms of those at the reference plane. This is achieved by satisfying the condition of shear stress continuity at the layer interfaces.These theories are known as zigzag theories (ZZT) in general. In some improved version of these theories [6-7], the condition of zero transverse shear stresses at the plate/beam top and bottom was also satisfied. The unknown transverse displacement fields across the depth in addition to that in the reference plane are essential for accurate prediction of the variation of transverse deflection.

Based on higher order zigzag theory, Pandit et al. [8] proposed a model for the static and buckling analysis of sandwich plates with soft compressible core. By defining the separate field variable for the derivatives of transverse displacement, the authors [7-8] overcame the problem of C_1 continuity. Due to this some constrains have been imposed which are enforced variationally through penalty approach. However, choosing suitable value for the penalty stiffness multiplier is a well known problem in the finite element method. Recently Chakrabarti et al.[9] has overcome the shortcomings of the FE model presented by the authors [7-8] and proposed a C_0 one dimensional FE model based on higher order zigzag theory for the analysis of soft core sandwich beam structures. The displacement fields are chosen in an effective manner so as no need to impose any penalty stiffness.

Keeping all this in mind, static problems of laminated soft core sandwich plates are solved using a newly developed C_0 FE model based on higher order zigzag theory. The in-plane displacement fields are assumed as a combination of a linear zigzag function with different slopes at each layer and a cubically varying function over the entire thickness. The out of plane displacement is considered to be quadratic within the core and constant in the face sheets. The model satisfies the transverse shear stress continuity conditions at the layer interfaces and the conditions of zero transverse shear stress at the top and bottom of the plate. The isoparametric quadratic plate element has nine nodes with eleven field variables at each node. The displacement fields are chosen in such a manner that there is no need to impose any penalty stiffness in the formulation. The element may also be matched quite conveniently with other C_0 elements.

2. Mathematical formulations

To ensure a piecewise parabolic variation of transverse shear strains across the thickness with discontinuity at the layer interface as expected in a layered plate [1], the in-plane displacement fields [9,14] are chosen as follows:

$$U = u_0 + z\theta_x + \sum_{i=1}^{n_u-1}\left(z - z_i^u\right)H\left(z - z_i^u\right)\alpha_{xu}^i + \sum_{j=1}^{n_l-1}\left(z - z_j^l\right)H\left(-z + z_j^l\right)\alpha_{xl}^j + \beta_x z^2 + \eta_x z^3 \quad (1)$$

$$V = v_0 + z\theta_y + \sum_{i=1}^{n_u-1}\left(z - z_i^u\right)H\left(z - z_i^u\right)\alpha_{yu}^i + \sum_{j=1}^{n_l-1}\left(z - z_j^l\right)H\left(-z + z_j^l\right)\alpha_{yl}^j + \beta_y z^2 + \eta_y z^3 \quad (2)$$

where, u_0 and v_0 denotes the in-plane displacements of any point on the mid surface, θ_x and θ_y are the rotations of the normal to the middle plane about the z-axis, n_u and n_l are number of upper and lower layers respectively, β_x, β_y, η_x and η_y are the higher order unknown, $\alpha_{xu}^i, \alpha_{yu}^i, \alpha_{xl}^j$ and α_{yl}^j are the slopes of i-th/ j-th layer corresponding to upper and lower layers respectively and $H(z - z_i^u)$ and $H(-z + z_j^l)$ are the unit step functions.

The transverse displacement is assumed to vary quadratically through the core thickness and constant over the face sheets [9] and it may be expressed as,

$$\begin{aligned} W &= l_1 w_u + l_2 w_0 + l_3 w_l \quad \text{for core} \\ &= w_u \quad \text{for upper face layers} \\ &= w_l \quad \text{for lower face layers} \end{aligned} \quad (3)$$

The stress –strain relationship of an orthotropic layer/ lamina (say *k*-th layer) having any fiber orientation with respect to structural axes system (*x*-*y*-*z*) may be expressed as

$$\left\{\overline{\sigma}\right\} = \left[\overline{Q_K}\right]\left\{\overline{\varepsilon}\right\} \quad (4)$$

where $\left\{\overline{\sigma}\right\}, \left\{\overline{\varepsilon}\right\}$ and $\left[\overline{Q_K}\right]$ are the stress vector, the strain vector and the transformed rigidity matrix of *k*-th lamina, respectively.

Utilizing the conditions of zero transverse shear stress at the top and bottom surfaces of the plate and imposing the conditions of the transverse shear stress continuity at the interfaces between the layers along with the conditions, $u = u_u$ and $v = v_u$ at the top and $u = u_l$ and $v = v_l$ at the bottom of the plate, β_x, η_x, β_y, η_y, α_{xu}^i, $\alpha_{xl}^i, \alpha_{yu}^i, \alpha_{yl}^i$, $(\partial w_u / \partial x)$, $(\partial w_l / \partial x)$ $(\partial w_u / \partial y)$ and $(\partial w_l / \partial y)$ may be expressed in terms of the displacements $u_0, v_0, \theta_x, \theta_y, u_u, u_l, v_u$ and v_l as

$$\{B\} = [A]\{\alpha\} \quad (5)$$

where,

$$\{B\} = \left\{\beta_x\, \eta_x\, \beta_y\, \eta_y\, \alpha_{xu}^1\, \alpha_{xu}^2 \ldots \alpha_{xu}^{nu-1}\, \alpha_{xl}^1\, \alpha_{xl}^2 \ldots \alpha_{xl}^{nl-1}\, \alpha_{yu}^1\, \alpha_{yu}^2 \ldots \alpha_{yu}^{nu-1}\, \alpha_{yl}^1\, \alpha_{yl}^2 \ldots \alpha_{yl}^{nl-1}\right.$$
$$\left.(\partial w_u/\partial x)(\partial w_u/\partial y)(\partial w_l/\partial x)(\partial w_l/\partial y)\right\}^{\mathrm{T}},\ \{\alpha\} = \left\{u_0\, v_0\, \theta_x\, \theta_y\, u_u\, v_u\, u_l\, v_l\right\}^{\mathrm{T}}$$

and the elements of [*A*] are dependent on material properties. It is to be noted that last four entries of the vector {*B*} helps to define the derivatives of transverse displacement at the top and

bottom faces of the plate in terms of the displacements u_0, v_0, θ_x, θ_y, u_u, v_u, u_l and v_l to overcome the problem of C_1 continuity as mentioned before.
Using the above equations, the in-plane displacement fields as given in equation (1-2) may be expressed as

$$U = b_1u_0 + b_2v_0 + b_3\theta_x + b_4\theta_y + b_5u_u + b_6v_u + b_7u_l + b_8v_l \tag{6}$$

$$V = c_1u_0 + c_2v_0 + c_3\theta_x + c_4\theta_y + c_5u_u + c_6v_u + c_7u_l + c_8v_l \tag{7}$$

where, the coefficients b_i's and c_i's are function of thickness coordinates, unit step functions and material properties.
By imposing four additional conditions obtained by satisfying the in-plane displacements at the top and bottom of the plate, four first order derivatives terms of transverse displacements are replaced in terms of nodal field variables (C_0) in equation (5). Thus the in-plane displacement fields expressed in Eqs. (6) and (7) do not contain any first order derivatives of the transverse displacement and therefore the requirement of C_l continuity of HOZT has been avoided very efficiently without defining new field variables and without using any penalty method.
The generalized displacement vector $\{\delta\}$ for the present plate model can now be written with the help of equations (3), (6) and (7) as

$$\{\delta\} = \{u_0\ v_0\ w_0\ \theta_x\ \theta_y\ u_u\ v_u\ w_u\ u_l\ v_l\ w_l\}^T$$

Using linear strain-displacement relation and equations (1)-(5), the strain field may be expressed in terms of unknowns (for the structural deformation) as

$$\{\bar{\varepsilon}\} = \left[\frac{\partial U}{\partial x}\ \frac{\partial V}{\partial y}\ \frac{\partial W}{\partial z}\ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\ \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\ \frac{\partial V}{\partial z} + \frac{\partial W}{\partial x}\right] \text{ or } \{\bar{\varepsilon}\} = [H]\{\varepsilon\} \tag{8}$$

where,

$$\begin{aligned}\{\varepsilon\} = [&u_0\ v_0\ w_0\ \theta_x\ \theta_y\ u_u\ v_u\ w_u\ u_l\ v_l\ w_l\ (\partial u_0/\partial x)(\partial u_0/\partial y)(\partial v_0/\partial x)(\partial v_0/\partial y)(\partial w_0/\partial x)(\partial w_0/\partial y)(\partial\theta_x/\partial x)\\ &(\partial\theta_x/\partial y)(\partial\theta_y/\partial x)(\partial\theta_y/\partial y)(\partial u_u/\partial x)(\partial u_u/\partial y)(\partial v_u/\partial x)(\partial v_u/\partial y)(\partial w_u/\partial x)(\partial w_u/\partial y)\\ &(\partial u_l/\partial x)(\partial u_l/\partial y)(\partial v_l/\partial x)(\partial v_l/\partial y)(\partial w_l/\partial x)(\partial w_l/\partial y)]\end{aligned}$$

and the elements of $[H]$ are functions of z and unit step functions, as given in Appendix C.
With the quantities found in the above equations, the total potential energy of the system under the action of transverse load may be expressed as

$$\Pi_e = U_s - W_{ext} \tag{9}$$

where U_s is the strain energy and W_{ext} is the energy due to the external transverse static load.
Using equations (3) and (6), the strain energy (U_s) is given by

$$U_s = \frac{1}{2}\sum_{k=1}^{n}\iiint \{\bar{\varepsilon}\}^T[\bar{Q}_k]\{\bar{\varepsilon}\}dxdydz = \frac{1}{2}\iint \{\varepsilon\}^T[D]\{\varepsilon\}dxdy \tag{10}$$

where, $$[D] = \sum_{k=1}^{n}\int [H]^T[\bar{Q}_k][H]dz \tag{11}$$

and the energy due to externally applied distributed transverse static load of intensity $q(x,y)$ can be calculated as

$$W_{ext.} = \iint wqdxdy \tag{12}$$

In the present problem, a nine-node quadratic element with eleven field variables ($u_0, v_0, w_0, \theta_x, \theta_y, u_u, v_u, w_u, u_l$, v_l and w_l) per node is employed. Using finite element method the generalized displacement vector $\{\delta\}$ at any point may be expressed as

$$\{\delta\} = \sum_{i=1}^{n} N_i \{\delta_i\} \tag{13}$$

where, $\{\delta\} = \{u_0\, v_0\, w_0\, \theta_x\, \theta_y\, u_u\, v_u\, w_u\, u_l\, v_l\, w_l\}^T$ as defined earlier, δ_i is the displacement vector corresponding to node i, N_i is the shape function associated with the node i and N is the number of nodes per element, which is nine in the present study.

With the help of equation (13), the strain vector $\{\varepsilon\}$ that appeared in equation (8) may be expressed in terms of unknowns (for the structural deformation) as

$$\{\varepsilon\} = [B]\{\delta\} \tag{14}$$

where $[B]$ is the strain-displacement matrix in the Cartesian coordinate system.

The elemental potential energy as given in equation (9) may be rewritten with the help of equations (10)-(14) as

$$\begin{aligned}\Pi_e &= \frac{1}{2}\iint \{\delta\}^T [B]^T [D][B]\{\delta\}\, dxdy - \frac{1}{2}\iint \{\delta\}^T [B]^T \left[N^w\right]^T q\, dxdy \\ &= \frac{1}{2}\{\delta\}^T [K_e]\{\delta\} - \frac{1}{2}\{\delta\}^T \{P_e\}\end{aligned} \tag{15}$$

where, $[K_e] = \int [B]^T [D][B]\, dx$ (16)

and $\{P_e\} = \iint [N^w]^T q\,\mathrm{dxdy}$ (17)

where, $[N^w]$ is the shape function like matrix with non-zero terms associated only with the corresponding transverse nodal displacements.

The equilibrium equation can be obtained by minimizing Π_e as given in equation (15) with respect to $\{\delta\}$ as, $[K_e]\{\delta\} = \{P_e\}$ (18)

where $[K_e]$ is the element stiffness matrix and $\{P_e\}$ is the nodal load vector.

A numerical code is developed to implement the above mentioned operations involved in the proposed FE model to calculate deflections and stresses in the sandwich plate. The skyline technique has been used to store the global stiffness matrix in a single array and Gaussian decomposition scheme is adopted for the solution.

3. **Numerical results**

To check the accuracy and applicability of the present C_0 plate FE model, various problems of laminated sandwich plates are solved under static loading. The results obtained are compared with publish results and 3D elasticity results [1]. The following non-dimensional quantities are used to show different results in this paper:

Non-dimensional in-plane stresses, Non-dimensional transverse shear stresses, Non-dimensional transverse displacement,

$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_{xy}) = \frac{h^2}{q_0 a^2}(\sigma_x, \sigma_x, \sigma_{xy})$, $(\bar{\tau}_{xz}, \bar{\tau}_{yz}) = \frac{h}{q_0 a}(\tau_{xz}, \tau_{yz})$ and $\bar{w} = \frac{100 E_{Tf} h^3 w}{a^4 q_0}$ respectively, where a, h are the dimension of plate along x-direction and z-direction respectively.

3.1. Four layered laminated composite plate (0/90/90/0) under sinusoidal loading

In this problem, the layers are of equal thickness. The material properties used here and in all subsequent problems are shown in Table 1. The results for the non-dimensional displacements (transverse) and stresses (transverse shear and in-plane normal) are presented in Table 2 to study the rate of convergence and validation of the displacements and stresses by considering different thickness ratio (h/a) ranging from 4 to 100 considering the full plate. It may be observed in Table 2 that the displacements converged at mesh division 8×8. However, more mesh divisions are required for the convergence of the stresses. As such a mesh division of 12×12 is taken for all subsequent analysis to get sufficiently accurate results for displacements and stresses.

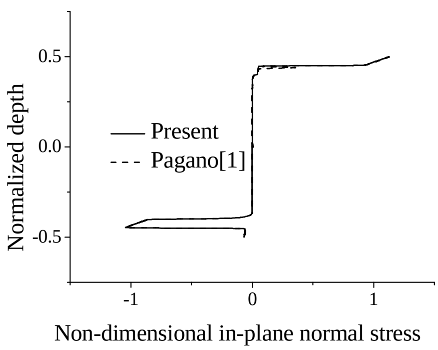
Example	Layer/ sheet	Material properties						
		E_1(psi)	E_2(psi)	E_3(psi)	G_{12}(psi)	G_{13}(psi)	G_{23}(psi)	υ
Composite plates	All layers	25.0E06	1.0E06	1.0E06	0.5E06	0.5E06	0.2E06	0.25
Sandwich plates	Face	25.0E06	1.0E06	1.0E06	0.5E06	0.5E06	0.2E06	0.25
	Core	4.0E06	5.0E04	5.0E04	16.0E04	6.0E04	6.0E04	0.25

Table 1. Material properties for laminated plates

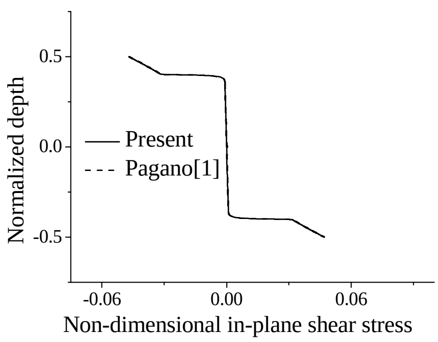
From Table 2, it may be observed that the performance of the present FE model is quite good as compared to other models especially at lower thickness ratio (h/a).

3.2. Un-symmetric laminated sandwich plate with different boundary conditions

An un-symmetrical simply supported laminated sandwich plate (0/90/C/0/90) is considered for the analysis in this example under the sinusoidal loading. The core has a thickness of 0.8h while it is 0.1h each for the two laminated faces, where h (= 1 inch) is the overall thickness of the plate. Material properties are as shown in Table 1. The variations of the in-plane normal stress (at the plate center) and the in-plane shear stress (at the corner) across the depth obtained by the present FE model are shown in Figure 3 and Figure 4 with the results of the 3-D elasticity solution [1]. The variations of the results of these two models are found to match quite well in both the cases.



(a)



(b)

Figure 3. Variation of a) in-plane normal stress and b) in-plane shear stress across the depth of un-symmetric sandwich plate (0/90/C/0/90) (a/h = 10).

a/h	Ref.	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$	$\bar{\tau}_{xy}$
4	Present (4×4)#	2.0015	0.7105	0.6921	0.2216	0.2458	0.0428
	Present (8×8)	1.9297	0.6894	0.6801	0.2198	0.2306	0.0427
	Present (12×12)	1.9296	0.6848	0.6759	0.2160	0.2283	0.0426
	Pagano [1]	1.9367	0.6843	0.6655	0.2193	0.2915	0.0522
	Kapuria and Kulkarni [6]	1.9076	0.7382	0.7023	0.2305*	0.2291*	0.0437
					0.2386**	0.3183**	
	Rodrigues et al. [11]	1.8931	0.6408	0.8506	0.2160	-	0.0436
	Ferreira et al. [12]	1.7095	0.4059	0.5764	0.2825	-	0.0308
	Roque et al. [13]	1.8864	0.6650	0.6295	0.2193	-	0.0343
10	Present	0.7313	0.5595	0.4057	0.3013	0.1904	0.0272
	Pagano [1]	0.7370	0.5591	0.4026	0.3014	0.1960	0.0254
	Kapuria and Kulkarni [6]	0.7366	0.5627	0.4094	0.3148*	0.1460*	0.0274
					0.3049**	0.1980**	
	Rodrigues et al.[11]	0.7227	0.5460	0.4194	0.2978	-	0.0269
	Ferreira et al.[12]	0.6627	0.4989	0.3614	0.3367	-	0.0241
	Roque et al.[13]	0.7136	0.5458	0.3885	0.3186	-	0.0092
100	Present	0.4301	0.5402	0.2697	0.3387	0.1031	0.0212
	Pagano [1]	0.4346	0.5390	0.2681	0.3387	0.1389	0.0213
	Kapuria and Kulkarni[6]	0.4350	0.5407	0.2720	0.3530*	0.1035*	0.0214
					0.3371**	0.1374**	
	Rodrigues et al.[11]	0.4294	0.5364	0.2699	0.3345	-	0.0211
	Ferreira et al.[12]	0.4337	0.5382	0.2705	0.3596	-	0.0213
	Roque et al.[13]	0.4359	0.5403	0.2714	0.3462	-	0.0215

Mesh Size; * Constitutive Relation; ** Post Processed

Table 2. Non-dimensional Deflection and stresses of a simply supported square laminate (0/90/90/0) under sinusoidal load

4. Conclusions

An improved C_0 plate finite element (FE) model has been developed in this paper for the static analysis of laminated sandwich plate with soft core. The continuity requirement of the derivates of transverse displacement is circumvented by effectively choosing the nodal field variables. There is no need to use penalty functions in the formulation as used by many previous researchers. Some numerical examples are solved for different problems of laminated composite and sandwich plates.

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