EFFECT OF THE BOUNDARY CONDITIONS ON THE HYDROELASTIC IMPACTS OF COMPOSITE PLATES

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Abstract. *Hydroelasticity is a phenomenon occurring during the interaction between water and a deformable structure. Structural deformations can modify the fluid motion, introducing difficulties in the calculation of the impact-induced stresses. Predicting the structural deformations and stresses during the water entry of flexible structures is a major challenge and suitable computer-aided design tools are currently being developed and validated. This work investigates the water impact of deformable wedges. Experimental and numerical results on the parameters affecting hydroelasticity are evaluated for deformable plates with different boundary conditions. The occurrence of hydroelastic effects is shown to depend on the ratio between the natural frequency of the structure and the characteristic wetting time. The conditions for which fluid-structure interactions are significant and must be accounted for in the analysis are identified, while outside this range the structure can be modeled as a rigid body.*

1. Introduction

Hull slamming is a phenomenon occurring when the forefoot of a ship rises above the water surface and then submerges again with high vertical velocity. Impulsive pressure loads act on the hull, introducing dynamic excitation to the structure due to the large force applied for a short period: the slamming event is in the order of milliseconds. This work focuses on hull slamming; however, there are three more phenomena that are defined as slamming in marine applications: (i) the impact of the bow on water induced by the ship motions in waves, (ii) the horizontal impact of steep waves or breaking waves on the ship hull and (iii) the water impact induced by water run-up and green water on the deck.

In order to fully describe impact forces and resulting structural response, various phenomena (trapped air, hydroelastic interaction, compressibility effects, and non-linear free surface mechanics) must be correctly modeled [1]. This work concentrates on the investigation of hydroelastic interaction.

Structural analysis of vessels has relied for years on a static or quasi-static approach in which the actual dynamic load is replaced by an equivalent static pressure uniformly distributed over the panels. When the duration of the pressure pulse is considerably longer than the natural period of the panels, this pressure can simply be taken as the spatial average of the real hydrodynamic load. Alternatively, if the panel is expected to show a non negligible dynamic response, the

equivalent pressure would be defined as that pressure which, if applied to the panel, will result in the same deformation and same maximum stress produced by the actual loading.
Several scientists ([2–7]) investigated the water impact of elastic structures, showing that the hydroelastic effects are governed by the deadrise angle, panel's thickness and impact velocity. Hydroelasticity is a challenging problem to be solved analytically due to the difficulties in coupling the fluid motion and the elastic deformation and a reliable numerical solution is particularly needed due to its flexibility to deal with complex shapes and coupling between deforming bodies and fluid motion. Panciroli at al. [8] recently studied by experiments and numerical simulations the hydroelastic effects of symmetric wedges composed by two cantilever plates connected at the keel. They showed that the water entry of deformable bodies introduces hydroelastic effects in terms of oscillations of the impact load due to the interaction between elastic deformations of the structure and the fluid flow. The occurrence of hydroelastic effects was drawn for this particular geometry and the capability of the SPH numerical method of treating hydroelastic impacts was validated. The ratio between the wetting time and the first natural period of the structure was found to be the governing parameter deciding weather or not hydroelastic effects should be taken into account for the evaluation of the structural deformation.
In this work we concentrate on the evaluation of the initiation of the hydroelastic effects varying various impact parameters like: boundary condition, deadrise angle, impact velocity and structural stiffness. The objective is to develop a reliable formula capable of estimating the maximum stress reached during the water entry of deformable bodies. As the hydroelastic effect increases, the maximum impact-induced stress lowers (up to one order of magnitude) if compared to the value calculated by a simplified quasi-static approach.

2. Von Karman's approach

The first analytical solution to solve the impact dynamics of rigid bodies entering the water was presented by Von Karman [9], who developed a formula capable to predict the maximum force acting on a rigid body entering the water. As an example, to study the water entry of a rigid wedge, Von Karman considered a wedge of unit thickness, mass M, and deadrise angle β entering the water with initial velocity V_0. Von Karman's work is based on some simplification, i.e.: (i) the flow is inviscid and irrotational, (ii) surface tension, gravity and structural elasticity effects are neglected, (iii) no air is entrapped between the structure and the fluid. In this method, as the body hits the water it is assumed that the mass of a half disk of water of radius r is moving with the wedge , resulting in an added mass $m = \frac{\pi}{2}\rho r^2 = \frac{\pi}{2}\rho\frac{\xi^2}{tan^2(\beta)}\gamma^2$, where γ is a coefficient accounting for the water pile up at the intersection with the free surface that varies with the deadrise angle. The value of γ can be evaluated as suggested in [10], for example. In this model, velocity and acceleration of the body are given by:

$$\dot{\xi} = \frac{V_0}{1 + \frac{\pi}{2}\rho\frac{\gamma^2\xi^2}{Mtan^2(\beta)}} \quad ; \qquad \ddot{\xi} = \frac{\pi\rho\gamma^2}{MV_0tan^2(\beta)}\xi\dot{\xi}^3 \tag{1}$$

The impact force reaches its maximum value $F^* = \left(\frac{5}{6}\right)^3 \frac{V_0^2}{tan(\beta)}\sqrt{\frac{2\pi}{5}\rho M\gamma^2}$ when the velocity is $\dot{\xi}^* = \frac{5}{6}V_0$, the penetration depth is $\xi^* = \sqrt{\frac{2M}{5\pi\rho\gamma^2}}tan(\beta)$ and the time is $t^* = \frac{16}{15}\frac{\xi^*}{V_0}$ Thus, the maximum force increases with the square of the velocity and the square root of the mass of the wedge. F^* is inversely proportional to $tan(\beta)$ so that it decreases as β increases and it becomes infinite as the deadrise angle tends to zero. When β becomes small, r becomes very large, the added mass becomes infinite and the wedge stops instantly. The velocity is 5/6 times the initial velocity when the force reaches its maximum and the penetration depth at that particular instant is proportional to the square root of the mass and to $tan(\beta)$ ($tan(\beta) = 0$ implies no penetration). Combining the last two equations we obtain:

$$t^* = \frac{16}{15}\frac{\tan(\beta)}{V_0}\sqrt{\frac{2M}{5\pi\rho\gamma^2}} \tag{2}$$

This shows that the force reaches its maximum at a time that is inversely proportional to the initial velocity and increases with $tan(\beta)$.
The next section presents a simplified analytical formulation based on a single degree of freedom (SDOF) system to evaluate the effect of the impulsive nature of the impact on the maximum structural deformation.

3. SDOF system to evaluate the dynamic structural deformation

Approximating the impact load as a sine pulse of duration $t_d = 2t^*$, this can be applied to a SDOF system with mass m and stiffness k, where k represents the static rigidity of the panel composing the wedge and m is its equivalent mass. If no damping is considered, the motion is governed by:

$$m\ddot{x} + kx = F \sin \Omega t \tag{3}$$

assuming $\Omega = \frac{2\pi}{t_d}$ and $\omega = \sqrt{k/m}$, the solution is

$$x = \frac{x_s}{1 - \frac{\Omega^2}{\omega^2}} \left(\sin \Omega t - \frac{\Omega}{\omega} \sin \omega t \right) \quad \texttt{or} \tag{4}$$

$$x = \frac{x_s}{1 - (T - t_d)^2} \left(\sin \frac{2\pi t}{t_d} - \frac{T}{t_d} \sin \frac{2\pi t}{T} \right) \tag{5}$$

where $x_s = F/k$ is the elastic deflection. Introducing the adimensional time $\tau = \frac{2\pi t}{t_d}$ we obtain

$$\bar{x} = \frac{x}{x_s} = \frac{1}{1 - \left(\frac{T}{t_d}\right)^2} \left(\sin \tau - \frac{T}{t_d} \sin \frac{t_d}{T}\tau \right) \tag{6}$$

Plotting this function for several values of the ratio $\frac{t_d}{T}$ we find that the deflection lowers as $\frac{t_d}{T}$ decreases. Figure 1 shows the deflections for various ratios of $\frac{t_d}{T}$.
Figure 2 (solid line) shows the maximum response of the SDOF system varying the ratio $\frac{t_d}{T}$. When the maximum response of the system equals one means that the solution behaves quasi-statically and the problem can be solved by a static solution (the deflection is evaluated by $x_s = F/k$). The results show that the response assumes values close to the unity for ratios of $\frac{t_d}{T}$ higher than 5. Note that in the region $1 \leq \frac{t_d}{T} \leq 5$ the quasi static solution over-estimates the dynamic deflection, while in the region $\frac{t_d}{T} \leq 1$ it is under-estimated. In real slamming events the structural natural frequencies are lowered during the impact due to the presence of fluid, which can be considered as a non-structural added mass to the system, as usually done in the literature. This non-structural added mass has the effect of lowering the oscillations. Assuming that the effect of the added mass can be reproduced by the introduction of a damping component, equation 3 becomes:

$$m\ddot{x} + c\dot{x} + kx = F \sin \Omega t \tag{7}$$

or

$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = f_0 \sin \Omega t \tag{8}$$

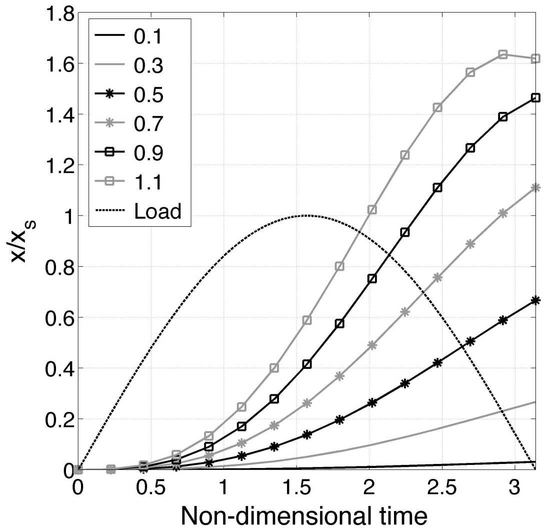

Figure 1: Applied sinusoidal pulse (blue line) and deflection for variable values of $\frac{t_d}{T}$.

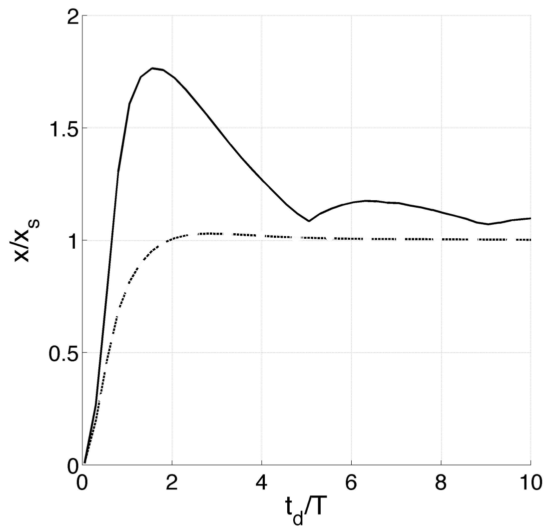

Figure 2: Maximum $\frac{x}{x_s}$ for variable ratios $\frac{t_d}{T}$. Solid line: undamped solution; for $\frac{t_d}{T} \geq 5$ the deflection is close to the unity and can be considered as a quasi-static solution. Dashed line: highly damped solution; in this case hydroelastic effects appears when $\frac{t_d}{T} \leq 2.1$.

In the case of an highly damped system (e.g. $\xi = 0.6$), which is clearly an overestimation of a realistic damping factor, the maximum values of $\frac{x}{x_s}$ for variable $\frac{t_d}{T}$ assumes the trend shown by the dashed line in figure 2. In this case the hydroelastic effects appear for values of $\frac{t_d}{T} \leq 2.1$ (above this value the error of $\frac{x}{x_s}$ is lower than 5%). It is found that in case of highly damped system the deformation is never expected to exceed the quasi-static solution, showing that neglecting the effect of the damping in the SDOF system can be a strong approximation. Assuming that real slamming impacts should behave in between these two cases (undamped and highly damped), hydroelastic effects should be accounted when the ratio between the wetting time and the first natural period is:

$$\frac{t_d}{T} = \frac{2t^*}{T} = \frac{32}{15}\frac{\tan\beta}{V_0}\sqrt{\frac{2M}{5\pi\rho\gamma^2}}\frac{1}{T} \leq 2.1 \sim 5 \tag{9}$$

or:

$$R = \frac{\tan\beta}{V_0\,\gamma}\frac{\sqrt{M}}{T} \leq 88 \sim 210 \tag{10}$$

This equation is similar to the one proposed by Faltinsen [11] but the effect of the total mass M and the first natural period T is shown. The range $88 \sim 210$ indicates the two extreme values of undamped and highly damped system. The actual value is supposed to be within this range. Above the defined ratio the slamming-induced stresses can be evaluated by a quasi static solution, while below this ratio more sophisticated tools accounting for duration and pressure distribution over time should be used to correctly evaluate the maximum stresses.

The next sections present experiments and numerical simulations of the water entry of wedges made by panels with different boundary conditions. The results will be compared with the analytical formulation presented herein.

4. Experimental setup

Experiments were conducted on a drop-weight machine where a sledge capable of holding wedges with different boundary conditions falls with free-fall motion over a water tank. Wedges are made by panels of different materials (aluminum and glass/epoxy composite) hinged at the sledge with two different boundary conditions:

- (C) - Cantilever: the panels are rigidly clamped to the sledge on one single side.
- (SS) - Simply supported: The panels are connected together and to the sledge by pins.

The sketches of the wedges with different boundary conditions are shown in figure 3. The right sketch shows the fully clamped boundary condition. This particular boundary condition has not been tested experimentally yet, but it has been solved numerically and the numerical results are presented in the next section. For given material and panels thickness, the impact variables are: deadrise angle β (range $15°$ to $35°$) and falling height (range 0.25 to 2.5 m). During experiments, structural deformations are recorded by strain gauges located at various positions, while an accelerometer and a laser position sensor record the impact dynamics. Further details about the experimental set-up are better described in [8, 12]. The next section presents the experimental results.

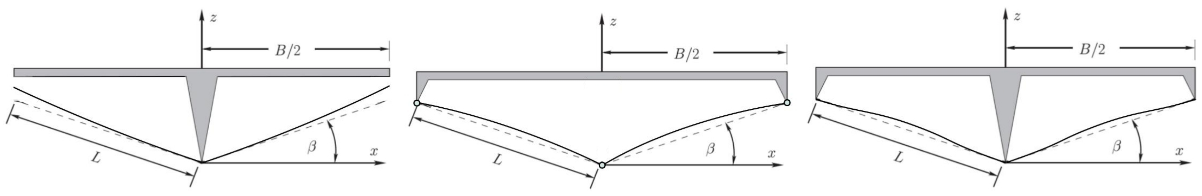

Figure 3: Sketch of wedges made by plates with different boundary conditions. Dashed line: initial panel geometry. Solid line: expected deformation during the water entry. Left: (C) cantilever plates. Center: (SS) simply supported plates. Right: (CC) plates clamped on both sides

5. Experimental Results

By the strain gauges it was possible to evaluate the maximum stress occurring during the water entry. Adimensionalizing this value by the theoretical value that is expected if the maximum impact load is applied quasi statically (i.e $\sigma\frac{\tan(\beta)}{V_0^2\sqrt{M}}S$, where S is the section modulus) and plotting it as a function of the parameter shown in equation 10, we obtain a graph indicating the relative influence of hydroelasticity varying the impact parameter R.

As indicated by equation 10, the higher is the first natural period (T), the higher are the hydroelastic effects (lower values of R). For this reason the hydroelastic effects are higher for the wedges made by cantilever plates than for the simply supported ones. The hydroelastic effects are negligible for values of $R > 100$. For lower values the impulsive nature of the load is highly affecting the overall deformation, leading to a maximum deformation that can be one order of magnitude lower than expected if the load is applied quasi-statically. Although it seems reasonable that hydroelastic effects are negligible for values of R greater than 100, the graphs of figure 4 and 5 should be expanded to account for values of R greater than 200, to have stronger confirmation of the proposed results.

The next section presents the results of a campaign of numerical simulations on the water entry problem of wedges. The boundary condition of the panels is fully clamped on both sides to allow higher values of R.

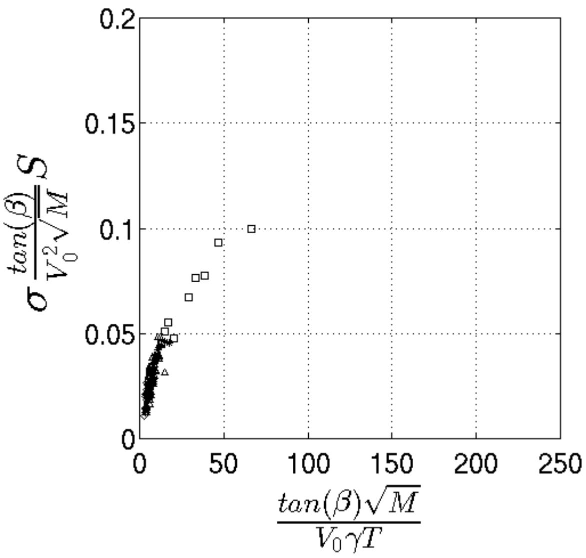

Figure 4: Non-dimensional stress as a function of a parameter that is proportional to the ratio between the impulse time of a rigid wedge and the first structural natural period. Experimental results of cantilever panels.

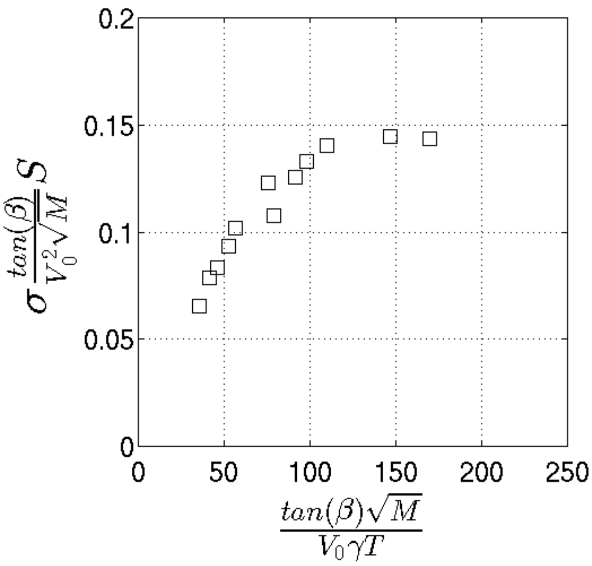

Figure 5: Non-dimensional stress as a function of a parameter that is proportional to the ratio between the impulse time of a rigid wedge and the first structural natural period. Experimental results of SS panels.

6. Wedges with clamped panels - numerical results

To evaluate the structural deformations occurring during slamming of clamped wedges, coupled FEM/SPH 2D numerical simulations were performed. Due to symmetry (pure vertical entry velocity) only one half of the model is considered. The fluid is modeled as a region $0.2\ m$ depth and $0.8\ m$ large filled with particles of 0.25 mm of radius equally spaced. For a detailed explanation of the numerical model, its optimization and validation please refer to the book chapter by Panciroli [12]. The wedge is modeled as a shell $0.3\ m$ long with unitary width divided in 100 elements. The first and the last elements of the wedge are set to move together downward and the rotations of the boundary nodes are constrained to simulate clamped boundary conditions. Concentrated masses are added to the wedge tip to simulate the impact of wedges with the same total mass as during experiments. The variables of the numerical simulations campaign are:

- Elastic modulus (20 GPa, 70 GPa, 120 GPa)
- Shell thickness (2 mm, 4 mm and 6 mm)
- Impact velocity (2 m/s, 3 m/s, 4 m/s and 6 m/s)
- Deadrise angle (20° and 30°)

Figure 6 and 7 show the results of wedges with deadrise angle of 20° and 30° respectively. Results are adimensionalized as shown in the previous section. In these cases the panels are stiffer than during experiments and the parameter R can reach higher values due to the lower natural period. The trend of the numerical results is very similar to the one obtained for the others boundary conditions. The observations done for the experimental results are confirmed: hydroelastic effects appears for values of $R < 200$ (as predicted by the analytical formulation without damping) but can be considered negligible until $R \approx 100$, when hydroelastc effects become considerably large. These values agree extremely well with the ones predicted by eq. 10, considering the simplification adopted in the analytical formulation.

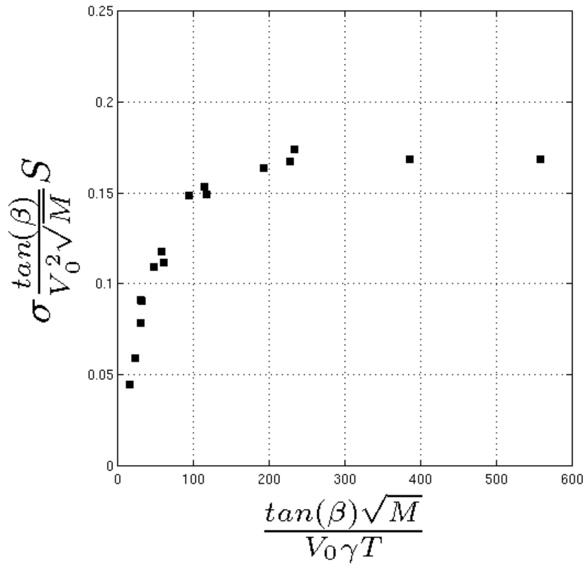

Figure 6: Non-dimensional results of clamped wedges with $\beta = 20°$.

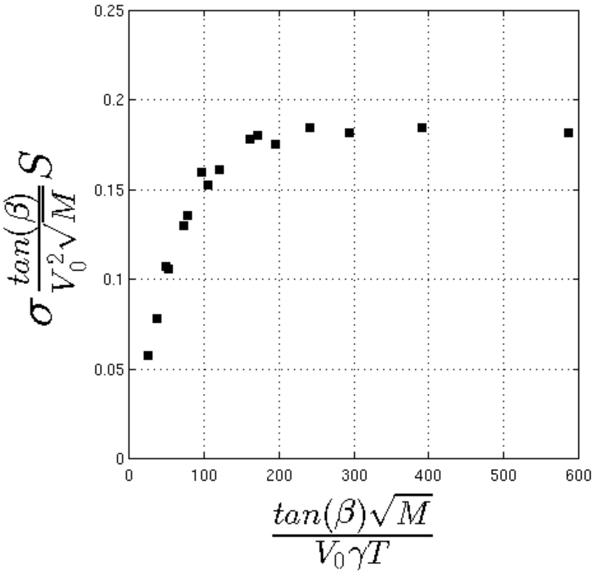

Figure 7: Non-dimensional results of clamped wedges with $\beta = 30°$.

7. Conclusions

We have studied the water entry of deformable wedges made by panels presenting various boundary conditions by using experiments and coupled FEM/SPH numerical simulations. The water entry of deformable structures has been previously treated in the literature, however, on the contrary with their studies, this work investigates extremely flexible structures, introducing high hydroelastic effects. Furthermore, in opposition with what is available in the literature, experiments do not focus on the evaluation of the pressure at the fluid/structure interface but on the overall structural deformations and stresses. A simple analytical method to evaluate the onset of the hydroelastic effects is proposed and compared with the experimental and numerical results. Hydroelastic effects are studied as function of different parameters like: deadrise angle, impact velocity and stiffness to area mass ratio. In particular, it is found that hydroelastic effects lowers increasing deadrise angle and plate stiffness, while increase with the impact velocity. It is found that the ratio (R) between the impact characteristic time and the structural natural period is the key parameter for determining whether or not the hydroelastic effects should be taken into account. Hydroelastic effects are found to appear for values of $R < 200$, which fits extremely well with the analytical prediction, and are important for values of R lower than 100.

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