EFFECTIVE STIFFNESS APPROACH OF FRP REINFORCED CONCRETE BEAMS

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Abstract

Current provisions in ACI Committee for calculating flexural structures deflections reinforced with fiber reinforced polymer (FRP) bars or reinforcements require the use of an effective moment of inertia. Many equations have already been developed and they have shown significant difference between them. In this study, CEB-FIP and Benmokrane formula have been compared and the variation between them is principally due to bond character and tension stiffening effects.

For this object, an empirical expression is proposed quantifying the effective moment of inertia for flexural members. Obtained results show a favorite accuracy using the presented effective moment of inertia model in-service loading conditions.

1 Introduction

The use of FRP bars to reinforce concrete structures has received a great deal of attention in recent years. The effectiveness of FRP composites for strengthening beams is attributed for various advantages, such as: high efficiency, high resistance to corrosion attacks under long-term service loading, high tensile capacity, low weight, no conductivity of electricity and ease of application and transport.

FRP reinforcements have been developed to replace steel bars in particular applications mainly in corrosion-prone RC structures. Among rheological disadvantages, compared to steel bars, FRP reinforcements possess a low modulus of elasticity that leads to higher reinforcement strains, wider cracks and larger deflections. Therefore, the serviceability of limit state may often used in the design of FRP reinforcement and steel bars as amount of structure reinforcements. Furthermore, bond and tension stiffening characteristics of FRP re-bars are not well quantified according to experimental data.

The design of FRP- bars reinforced beams requires an important attention in serviceability conditions due to the cracking phenomenon and low stiffness of FRP-bars. For these reasons, this category of structures design is often based on the evolution of stresses of the FRP-reinforcing bars. In the cracking range of the behavior, the low stiffness of FRP-bars, in fact, involves an intensive initiation and propagation of cracks therefore larger widths and greater deflections can be obtained compared to steel reinforced concrete beams.

In this subject, several methods are available in literature for computing deflections in FRP reinforced flexural structures taking into account the nonlinear effects, such as: the cracking of concrete, the tension stiffening propriety and the bond character. Due to the complexities of

computations to be performed, these procedures can't be adopted by engineers. Many works have published methods that more accurately estimate responses of structures and can be easily executed by engineers in this field. Many engineers inspire beams capacities from the fundamental structural analysis and conservative material assumptions. In general, these procedures underestimate the really capacities of beams.
The aim of this contribution presents an empirically expression based on the Branson's equation. Obtained results were carried out to show the behaviour of concrete beams reinforced by FRP-bars. Furthermore, predicted service load-deflections using selected models are compared with maximum deflections at mid-span for four different beams and their corresponding predicted values using CEB-FIP model [1] and Benmokrane model [2] largely used in the literature.

2. Comparing effective moment of inertia relationshipS

The expression of the effective moment of inertia (I_{eff}) is the commonly approach used to compute RC members deflections. Among, two relationships largely used in literature are selected for comparison in this paper: the CEB-FIP approach and the Benmokrane's method. Firstly, the computed deflection according the CEB-FIP model uses the bilinear equation expressed by:

$$\delta_c = (1-\zeta_b)\delta_1 + \zeta_b \delta_2 \tag{1}$$

where $\zeta_b = 1 - \beta_1 \beta_2 \frac{M_{cr}}{M_a}$, which β_1, β_2 are coefficients that depict the bond nature of bars and the time of loading, eventually 0.8 and 0.5 for short and long term loading; respectively. M_a and M_{cr} are the maximum applied moment and the cracking moment, respectively. $\delta_1 = k_{s1}\delta_c$ and $\delta_2 = k_{s2}\delta_c$ are coefficients of the bilinear law and k_{s1} and k_{s2} depend on parameters : $\eta = \frac{E_{FRP}}{E_c}$, $\rho = \frac{A_s}{bd}$ and $\rho' = \frac{A_s'}{bd}$. It should be noted that tension-stiffening contribution in equation (1) is taken into account with an explicitly manner. The cracking moment is computed when the maximum tensile stress of concrete reaches its strength value in tension.

$$M_{cr} = \frac{f_t I_g}{y_t} \tag{2}$$

Where I_g is the moment of inertia of the gross section, y_t is the distance from the neutral axis to the extreme tension fiber and f_t is the tensile strength of concrete.
Computed deflections are primordially based on previsions intended for steel reinforced concrete elements. Thus, ACI 440 code [3] recommends the use of Branson's equation to be consistent with the existing ACI Building Code Requirements [4] for Structural Concrete. The expression used in the cracking range of the behavior is computed by:

$$\delta_{max} = \frac{Pa}{24 E_c I_{eff}}(3L^2 - 4a^2) \tag{3}$$

with P is the applied load, a is the slenderness of the point load, L is the span of the beam, E_c is the elastic modulus of concrete and I_{eff} is the effective moment of inertia.
However, the primary interest is given to the rational evaluation of the effective moment of inertia, I_{eff}. The deflection relationship was empirically derived to represent a gradual digression from the gross un-cracked moment of inertia I_g to the cracked moment of inertia I_{cr}. The following approach was originally proposed by Branson [4], as:

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \le I_g \tag{4}$$

with M_a is the applied service load moment.
Some investigators have already observed that the previous equation was not generally in an agreement with experimental data of FRP reinforced concrete structures. For this object, Benmokrane et al. [2] have introduced two empirical factors in the equation (4) to correlate results with experimental data of reinforced concrete beams reinforced with GFRP bars.

$$I_{eff} = \alpha\, I_{cr} + \left(\frac{I_g}{\beta} - \alpha\, I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g \tag{5}$$

where α and β were determined to be 0.84 and 7, respectively.
Later, Gao [6] further modified the equation (5) to agree obtained deflections with experiment results. This modification feels the object of ACI guides [4] and [7], both codes subsequently recommended modifications to be reconciled with test data.
The new expression of the effective moment of inertia is then given by:

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 \beta_r\, I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \leq I_g \tag{6}$$

with $\beta_r = 0.5\left(\frac{E_{frp}}{E_s} + 1\right)$, E_{frp} and E_s are elastic modulus values of the FRP and steel bars. The coefficient β_r was initially set equal to 0.6 for glass FRP bars [8] and soon after it has been expressed by $\beta_r = \alpha_b\left(\frac{E_{fpr}}{E_s} + 1\right)$ while α_b is the bond coefficient assumed equal to 0.5. However, Yost et al. [9] have neglected the dependence on bond character and set it equal to $\alpha_b = 0.064\frac{\rho}{\rho_b} + 0.13$ to account the reinforcing bars ratio ρ and the balanced reinforcing ratio ρ_b in consideration. Recent changes by ACI 440 [3] recommend using the reduction factor with $\beta_r = 0.2\frac{\rho}{\rho_b}$ based on a comprehensive statistical fit of existing beam deflection data.
An alternative approach improving computed deflections of FRP beams has been proposed to stiff the member by increasing the cubic power in Branson's original equation (Equ.4) to higher value [10]. However, the prediction of results for lower load levels is not further considered [11].
The challenge consists of developing an expression to be satisfactorily applied to different types of FRP beams reinforced with various reinforcement ratios. To improve analytical results, it must be reducing the tension stiffening contribution in Branson's model. This procedure takes into account some variables incorporated in the effective moment of inertia formula. However, different models for the effective moment of inertia, including the effective contribution of cracked concrete and the participation of the tension stiffening option, have been proposed to define flexural behavior of RC beams. The object can be reached by multiplying the gross moment of inertia by the reduced factor β_r (equ.6) that effectively reduces the $\frac{I_g}{I_{cr}}$ ratio to $\beta_r\,\frac{I_g}{I_{cr}}$ [12-13]. The literature has already presented many attempts to modify the Branson's expression basing on the consistence of the tension stiffening and bond effects.

This study develops an empirical expression inspired from the Branson's approach based on the affectation to an appropriate value of the $\frac{I_g}{I_{cr}}$ ratio, which can control the tension stiffening in the Branson model. In addition, the paper identifies the limitations of both the existing and past methods ACI 440 [3,4,7]. Results are also compared to a more general unified approach that can be equally applied for both steel and FRP reinforced concrete [11].

3. Analytical approach

For the moment value less than the cracking one, the beam is assumed un-cracked and behaves homogeneously and elastically. In this range of the behavior, the slope of the applied moment-mid span deflection curve is proportional to the moment of inertia of the un-cracked section. The member firstly cracks when the applied moment reaches the cracking moment value, M_{cr}, then the tensile stress in the extreme fiber reaches the flexural tensile strength of the concrete.

After the formation of the first crack, concrete between adjacent cracks can still resist to tensile forces. This phenomenon known as the tension stiffening effect is primordially characterized by an important change in the curvature of the beam. Really, it's reasonable that there was a balance between two members of the relationship giving the effective moment of inertia.

From the typically response curve of applied moment-mid span deflection, it's seen that is possible to introduce parameters quantifying effective moment of inertia to improve the predicted response of flexural beams reinforced with FRP bars. In the same manner, it's also possible to bring nearer predicted response curves to the experimental ones, which can be obtained considering an increase of the intact section and simultaneously a decrease of the cracking section. In this sense, the Benmokrane's expression has empirically modified introducing two following coefficients of correction that are: $\psi_1 = (1 + \frac{E_c}{E_{FRP}})$ and $\psi_2 = (1 - \frac{E_c}{E_{FRP}})$, respectively.

In this study, an analytical relation of the effective moment of inertia is established addicting these terms of correction. The approach most commonly used to reduce tension stiffening to realistic levels where the gross moment of inertia I_g is multiplied by a correction factor that effectively reduced (M_{cr}/M_a) ratio. Since tension stiffening is directly related to applied moment (M_{cr}/M_a) and the reducing of this ratio will also decreases the tension stiffening effect in the cracking stage. Most attempts to improve the effective moment of inertia formula have been done but not to be consistent with the principal mechanisms underlying the tension stiffening effect. The aim of this paper is to develop an expression for reducing factor (M_{cr}/M_a) based on the quantification of the (M_{cr}/M_a) ratio and controlling the tension stiffening effect contribution. The corresponding equation can be written as,

$$I_{eff} = \psi_1 \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \psi_2 \, (1 - \left(\frac{M_{cr}}{M_a} \right)^3) \, I_{cr} \leq I_g \tag{7}$$

The use of this expression, it's appeared that the balance between components, I_g and I_{cr} of the effective moment of inertia relation is more quantified. The contribution of the cracking behavior is herein very considered and the contribution cracked concrete is strictly evaluated in the cracking range. Eventually, this process leads to the minimization of the tension stiffening contribution.

4. Results of equations

This work is intended to develop an analytical solution for calculating the load-deflection response of FRP reinforced concrete beams using the above expression. Comparisons are made between analytical formula and experimental results of beams subjected to four-point bending.
To show the accuracy of the three analytical approaches, it's necessary to compare their results with experimental data. Geometrical and mechanical properties are summarized in Tables 1 and 2, respectively (Fig. 1). Beams are tested as: Beam 1 [14], Beam 2 [15], Beam 3 [16] and Beam 4 [17].
The Predicted load versus mid-span deflection using bilinear (CEB-FIP) model, Benmokrane's approach, this study and experimental data for beams are shown in Figs. 2-5. The interest of this comparison is to confirm the accuracy of the approach presented and its performance based on the evaluation of ultimate loads is regrouped in the table (3).

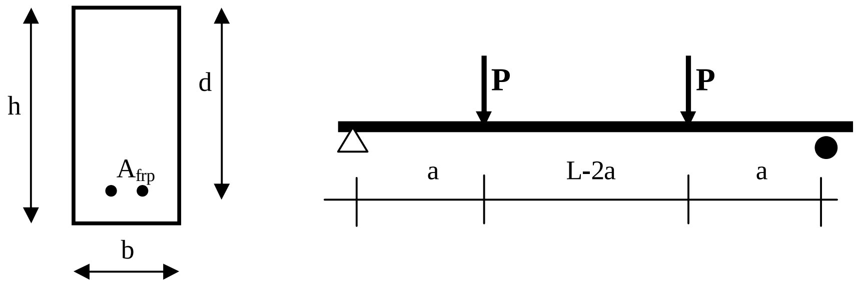


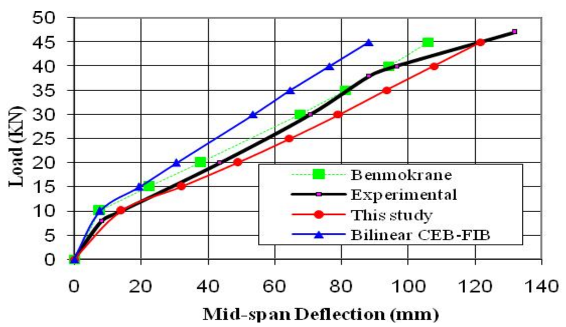
Figure 1. Cross section and loading of beams studied.

Beam N°	L [m]	a[mm]	b[mm]	h[mm]	d[mm]	I_g[mm^4]
1	3.4	1200	500	180	145	2.43E+8
2	2.1	700	154	254	222	2.10E+8
3	3.05	1067	127	304	273	3.00E+8
4	1.55	625	150	300	250	3.38E+8

Table 1. Geometrical properties of beams.

Beam N°	f'_c [MPa]	E_c [MPa]	E_{frp} [MPa]	f_{frp} [MPa]	A_{fr} [mm²]	ρ (%)
1	30	24650	42000	886	887	1.22
2	42	29160	34000	586	530	1.55
3	29	27240	26220	724	724.5	2.09
4	34	29000	100000	1200	390	1.04

Table 2. Mechanical properties of beams.



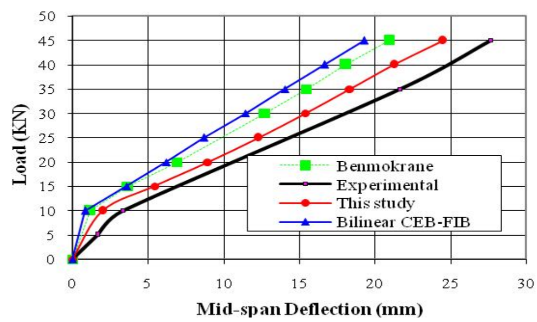


Figure 2. Load-mid span deflections for beam 1. **Figure 3.** Load-mid span deflections for beam 1and beam 2.

The figure 2 shows in general an acceptable agreement with the experimental curve. Particularly in this case, analytical predictions overestimate the stiffness of the beam beyond the applied load equal to 10 kN. But, figures 3-5 show in general a good agreement between experimental and analytical load-deflection curves. The Benmokrame's approach for the beam 1 presents an accurate solution compared to experimental deflections. Curves compared using this approach show stiffer post-cracking of the beams. The analytical load-deflection curves were presented a well concordance up about 70% of the ultimate load. It is evident from figures 3-5 that deflections of this procedure are in excellent agreement with the experimental results during the cracking range.
The CEB-FIP model and Benmokrane's approach show an under prediction of experimental results of FRP reinforced concrete beams (Fig. 3), particularly at the level of application of limit load, which occur a problem for the structural integrity and safety. As the service load is increased the accuracy of the deflection predicted becomes very significant. The figure 4 reveals that CEB-FIP model and Benmokrane's method become consistent basing on the predicting deflection values. The model presented predicts the response of the beam 3 up to 67%. The CEB-FIP model and Benmokrane's equation are seen to slightly overestimate the stiffness resulting with less deflection (Fig. 4).

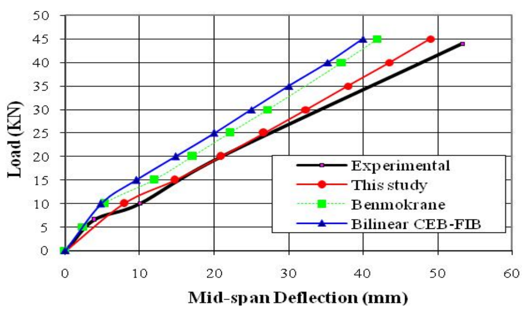


Figure 4. The load-mid span deflection for beam 3.

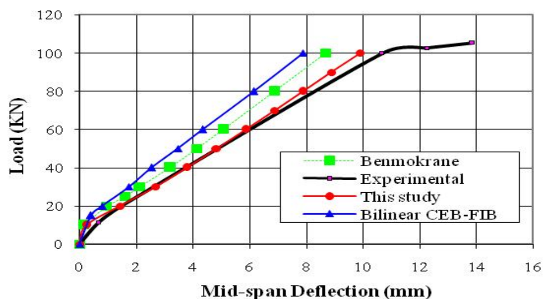


Figure 5. The load-mid span deflection for beam 4.

In this case, analytical deflections are predicted up to 80% and an improvement is widely observed compared to results obtained by CEB-FIP model and by that Benmokrane's approach (Fig. 5).
In this case, this approach shows an excellent concordance by the comparison made on the convergence between results obtained by this procedure and experimental data. Particularly, the following table regroups the ultimate load for the four beams studied.

Beam N°	1	2	3	4
CEB-FIP model	72.50	69.82	75.84	73.75
Benmokrane approach	87.00	75.86	78.56	81.25
This study	98.00	88.56	91.98	92.64

Table 3. Comparison in % of calculated /experimental load.

5. Summary and conclusions

In this study, a simple analytical approach is derived to predict load-deflection response of FRP bars reinforced concrete beams. Obtained results of the four beams with different mechanical properties revealed that the current models for predicting the deflection of beams reinforced by FRP bars under-estimate deflections of beams. The formulation introduces correction factors in Benmokrane's equation reducing the tension stiffening effect to reasonable level by decreasing of the (I_g/I_{cr}) ratio. The developed model can be applied to

both RC and FRP reinforced beams, indifferently and a wide range of analytical load-deflection curves are compared, the present model, the CEB-FIP, Benmokrane et al. equations with the experimental data. The comparison showed that this approach can lead to a favorable agreement with experimental results.
The response established by this procedure is overestimated experimental results for the beam 1. Probably, it's due to geometrical properties that having the rapport h/b <0.36 but the rest of beams introduce an admirable concordance between analytical results and those experimental ones. Two equations, largely used in literature, are compared and shown a greatly difference between them. This variation is well modeled by considering the tension stiffening effect and the cracking effect, respectively. These parameters are directly included in the novel relationship for predicting beam deflections and analytical results have confirmed the accuracy of the model and estimates better deflections of beams reinforced by FRP bars opposite to test data.

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