

# INFLUENCE OF THE REINFORCEMENT GEOMETRY ON THE ELASTIC BEHAVIOR OF PARTICLE-REINFORCED COMPOSITES

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## Abstract

*The present contribution describes a computational analysis to determine the influence of reinforcement geometry on the elastic behavior of particle-reinforced composites. To this end, the elastic response of elementary cells of such composite materials with different particle shapes (used as reinforcement units) are numerically analyzed by the finite element method, thus demonstrating the influence of constraint (provided by the reinforcement) on the elastic behavior of the composite material, and the important role of the angles defining the reinforcement in the case of polygonal shape. In addition, elementary cells containing two reinforcement particles are analyzed to study the interaction between two reinforcement units.*

## 1 Introduction

The influence of reinforcement geometry (materialographic structure) on the effective properties of a composite material is a topic of the highest interest in materials science and engineering, because it is a fundamental question related to the relationship between material microstructure (or structural arrangement) and mechanical properties (macroscopic response) of the composite.

Previous analyses deal with the role of particle reinforcement shape in metal matrix composites [1-2], analyzing the influence of particle angle on material behaviour [1] or the role of shape and orientation of the reinforcement [2]. After comparing the results with the predictions of theoretical models, it is seen that the size and distribution of reinforcement particles inside the matrix have significant effects on the effective elastic modulus of the composite material [3].

The final aim of micromechanical models is obtaining the general elastic properties of the composite material. The main procedures may be classified as: (i) homogenization models, (ii) perturbation models, (iii) self-consistent models and (iv) differential scheme. Shen and Li [4] propose an analytical method to calculate the elastic modulus of a composite on the basis of energetic methods, providing good results due to the fact that the volume fraction (0.20 in their analysis) is lower than 0.60 [5]. Paper [6] gives an expression for the effective elastic modulus of the composite material with ellipsoidal heterogeneities.

This paper studies the influence of the materialographic structure of a composite (i.e., of the sizes, shapes and distributions of reinforcement particles) on the elastic response of the material. To this end, the elastic response of unit cells of composite material (with different shapes for the particle reinforcement) is numerically analyzed by the finite element method.

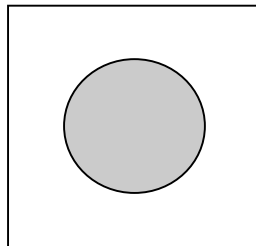
## 2 Modeling of elastic behavior of particle-reinforced composites

The present paper analyses five composite materials with different phases: MC<sub>1</sub>, Al-matrix composite reinforced with alumina particles; MC<sub>2</sub>, epoxy-matrix (I) composite reinforced with fibre-glass particles (I); MC<sub>3</sub>, epoxy-matrix (II) composite reinforced with fibre-glass particles (II); MC<sub>5</sub>, composite material with concrete matrix reinforced with ferrite particles. Material constants of the phases are given in Table 1.

Material type	Young's modulus E (GPa)	Poisson's coefficient $\nu$
2124-T6 Al alloy	70,0	0,30
Aluminium oxide (Al <sub>2</sub> O <sub>3</sub> )	350,0	0,30
Fibre glass (I)	66,2	0,20
Fibre glass (II)	69,0	0,20
Epoxy (I)	3,7	0,34
Epoxy (II)	3,0	0,39
Epoxy (III)	3,4	0,34
Vitreous particle	76,0	0,23
Concrete	20,1	0,20
Ferrite	205,0	0,29
Cementite (Fe <sub>3</sub> C)	211,9	0,46

**Table 1.** Elastic constants of the analyzed materials

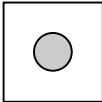
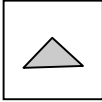
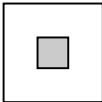
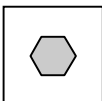
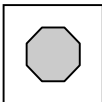

To model the elastic behaviour of the different composite materials, a homogeneous solid (representing the matrix) with inclusions (representing the reinforcement particles) is considered. A unit cell composed by a matrix and a unique particle acting as reinforcement (Fig. 1) is used in the analysis.



**Figure 1.** Unit cell with a single reinforcement particle.

Different particle shapes were considered in the numerical analysis, in order to elucidate the influence of reinforcement constraint (in the form of number of sides) of the particles or the angularity (angles between sides of the particle). After determining the relative situation between matrix and reinforcement unit, different particle shapes (surrounded by the matrix) will be analyzed by the finite element method. The shapes were chosen on the basis of two basic criteria: (i) to analyze the number of sides of a reinforcement particle of polygonal shape on the elastic parameters of the composite material, and considering the limit case of a circular shape (i.e., a 2D circle representing a cylindrical 3D particle) as a polygon with an infinite number of sides; (ii) to study the evolution of the elastic response of the composite with the angularity of the polygonal reinforcement.

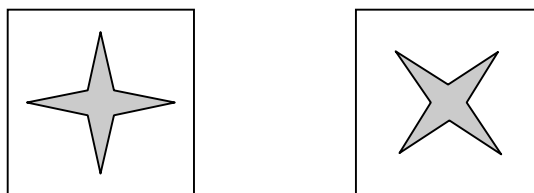
On the basis of the first criterion, five geometric shapes are considered in the analysis: triangular, square, hexagonal, octagonal and circular. Considering the second criterion: a star-shaped particle of eight sides is chosen, varying the angle between its sides. Table 2 shows the relevant parameters to represent the shape of the reinforcement particle used in the numerical calculations.

Geometric shape	Representation	Parameters of study
Spherical		Number of sides ( $\infty$ )
Triangular		Number of sides (3) Orientation effect
Square		Number of sides (4) Orientation effect
Hexagonal		Number of sides (6) Orientation effect
Octagonal		Number of sides (8) Orientation effect Angularity effect
Star shape		Orientation effect Angularity effect

**Table 2.** General aspects of the composite system.

In the case of the study of the influence of the angularity (second criterion), a star shape was chosen (i.e., a particle with an internal angle between sides higher than  $180^\circ$ ), and two basic orientations of the particle are considered, as sketched in Fig. 2, while Fig. 3 offers the geometric parameters defining the shape of the star.

Numerical computations were performed by the finite element method using a commercial code. Load was applied on one side of the unit cell, cf. Fig. 4. Material was considered as homogeneous and isotropic, and linear-elastic behaviour was assumed. A plane strain state was considered in all cases. Fig. 5 shows the unit cell with an octagonal particle placed in two different orientations and Fig. 6 shows the finite element meshes of the two particles.



**Figure 2.** Sketch of the cells of composite material reinforced with a star-shaped particle: two basic orientations: a star and the same star inclined 45.

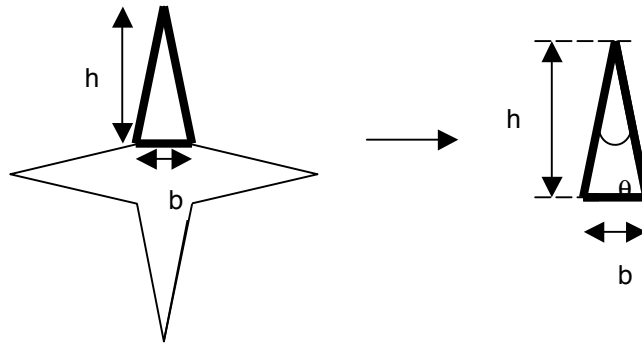


Figure 3. Geometric parameters defining the star.

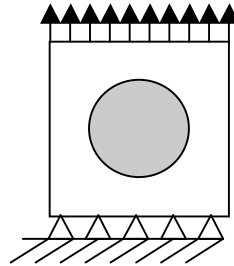


Figure 4. Load applied to the unit cell.

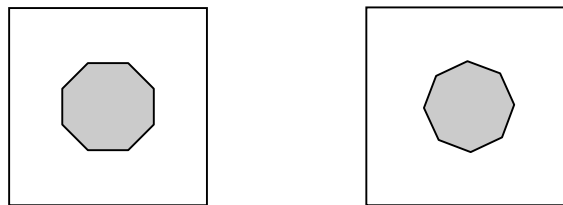


Figure 5. Unit cells with an octagonal particle placed in two different orientations: with sides parallel to the loading direction (left) and inclined 22.5° (right).

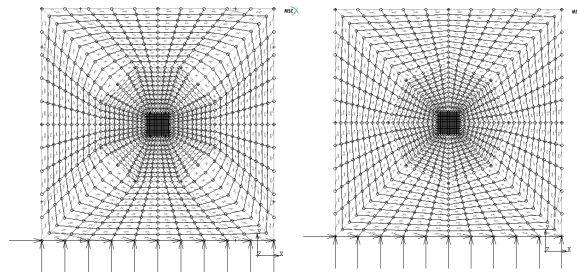


Figure 6. Finite element meshes of the two afore-said octagonal particles.

Fig. 7 gives the distribution of longitudinal stress (parallel to the loading direction). A clear stress concentration is observed in the corners of the particle. Such a concentration effect is higher in triangular reinforcements and decreases as the number of sides of the reinforcement particle increases, and reaching its minimum value in the case of the circular particle.

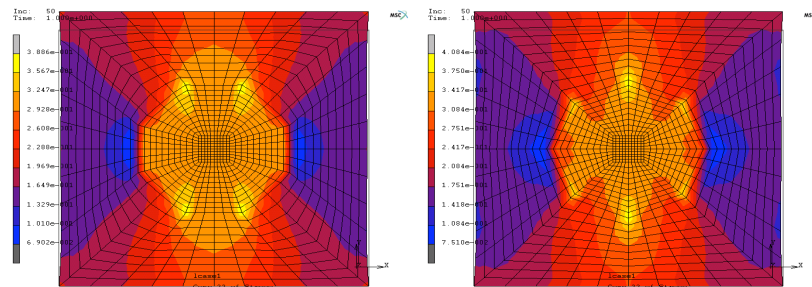


Figure 7. Distribution of longitudinal stresses (octagonal particles in two different orientations).

### 3 Elastic response of the composite

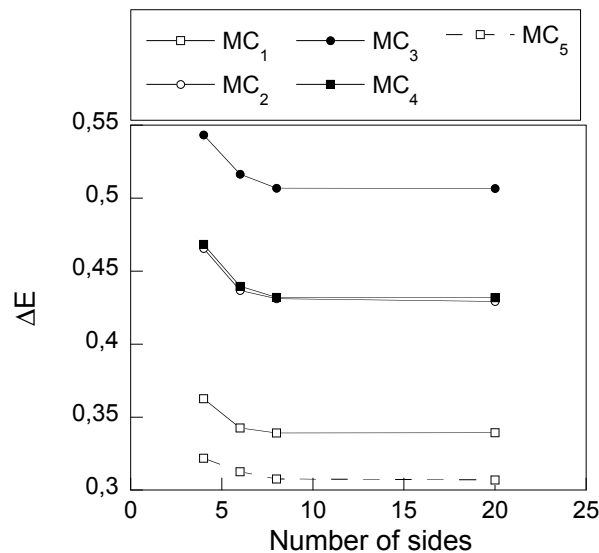
The increase of the elastic modulus of the composite with the reinforcement is calculated as:

$$\Delta E = \frac{E_c - E_1}{E_c} \quad (1)$$

where  $E_c$  is the elastic modulus of the composite and  $E_1$  that of the matrix phase.

#### 3.1 Influence of the number of sides of the reinforcement particle

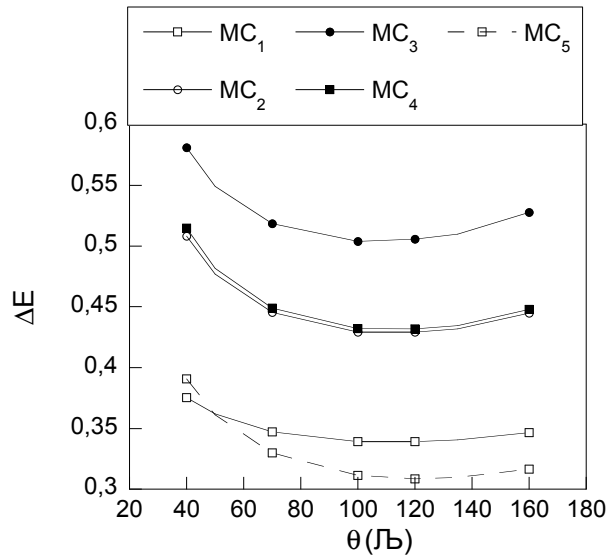
Fig. 8 shows the influence of the number of sides of the reinforcement on the increase of the elastic modulus of the composite for all the analysed composite systems ( $MC_1$  to  $MC_5$ ), a volume fraction of 0.2 and oriented particle, i.e., a side parallel to the loading direction. It is seen that the elastic modulus of the composite material decreases as the number of sides of the reinforcement increases. The trend approaches the value corresponding to the reinforcement of circular shape (limit case as a polygon of infinite sides). The decrease of the elastic modulus with the number of sides can be explained by several reasons: (i) the loss of constraint in the material as the periphery of the particle becomes smoother (less angled); (ii) the decrease on the length of the interphase with the number of sides (the circular shape minimizes the periphery length for a given area).



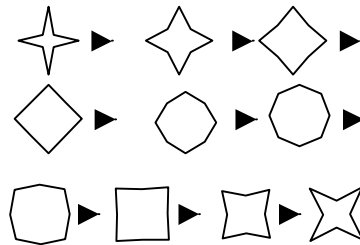
**Figure 8.** Influence of the number of sides of the reinforcement on the increase of the elastic modulus of the composite.

#### 3.2 Influence of the internal angle between sides of the reinforcement particle

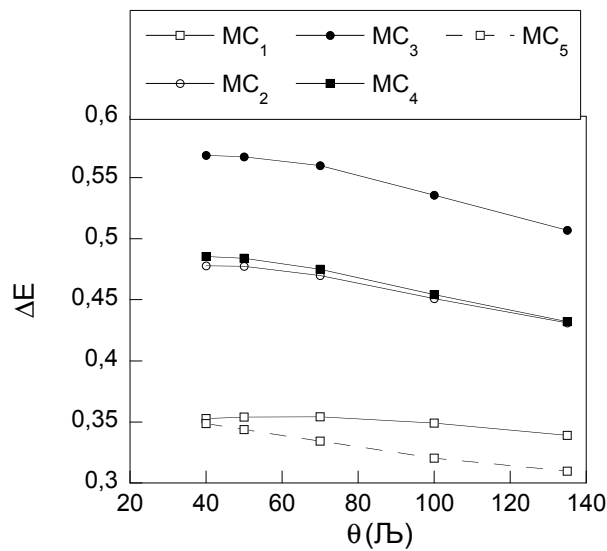
To analyse the evolution of the elastic modulus with the angularity of the reinforcement, a star-shaped particle was analysed. Fig. 9 plots the increase  $\Delta E$  of the elastic modulus of the composite material with the angularity of the reinforcement (evaluated by the angle  $\theta$ , cf. Fig. 3) for the five considered composite materials, in the case of reinforcement parallel to the loading direction. The plot reflects how, as the angle  $\theta$  increases up to a value next to  $90^\circ$ , the elastic modulus of the composite decreases. This may be explained because the increase of angle (decrease of angularity of the particle) produces a smaller interphase between the phases of the composite, with the associated decrease of the elastic modulus. When the angle is  $90^\circ$ , it is a square particle, and when the angle reaches  $135^\circ$ , it is a regular polygon of eight sides. Once the angle of  $135^\circ$  is exceeded, the star is again formed, but in this case it is perpendicularly oriented (see Fig. 10). When the star-geometry is inclined  $45^\circ$ , the plot of Fig. 11 is obtained, indicating a decrease of the elastic modulus with the increase of the angle.



**Figure 9.** Increase  $\Delta E$  of the elastic modulus of the composite material with the angularity of the reinforcement, angle  $\theta$  in Fig. 3 (reinforcement parallel to the loading direction).



**Figure 10.** Evolution of the star geometry with the increase of the angle  $\theta$ .



**Figure 11.** Increase  $\Delta E$  of the elastic modulus of the composite material with the angularity of the reinforcement, angle  $\theta$  in Fig. 3 (reinforcement inclined  $45^\circ$  in relation to the loading direction).

## 4 Discussion

### 4.1 Influence of the elastic constants of the phases

To analyse the influence of the elastic constants of the constitutive phases on the effective elastic modulus of the composite material, the relationship between the elastic modulus of reinforcement and matrix is calculated. Classifying the composite materials used in this work considering the relationship between their elastic modulus of the reinforcement and the

matrix, the order is MC<sub>3</sub>, MC<sub>4</sub>, MC<sub>2</sub>, MC<sub>5</sub> y MC<sub>1</sub> (relationships 25, 20, 15, 10 and 5). Generally speaking, a higher relationship between the elastic moduli of the constitutive phases implies a higher increment of the elastic modulus of the composite. The scarce exceptions are the two composites with lower relationship between phases.

#### 4.2 Comparison with results from analytical models

Numerical results obtained by the finite element method in this paper are compared with analytical results given by the model developed by Shen and Li [4]. The deviation between the values of the elastic modulus numerically obtained in this paper and those analytically calculated by the Shen-Li model is given by the following expression:

$$\partial\epsilon_E = \frac{E_{num} - E_{an}}{E_{num}} \quad (2)$$

where  $E_{num}$  and  $E_{an}$  are respectively the elastic modulus numerically and analytically obtained. This variable is plotted in Fig. 12 against the number of sides of the reinforcement particle, showing an evolution similar to that represented in Fig. 8, a fact that could be expected, since the analytical models do not account for the specific shape of the reinforcement in the composite material. Results from the analytical model present a maximum deviation of 30% in relation to results from the finite element analysis, the deviation being maximum in the case of materials MC<sub>3</sub> and MC<sub>5</sub> presenting the maximum increments of elastic modulus.

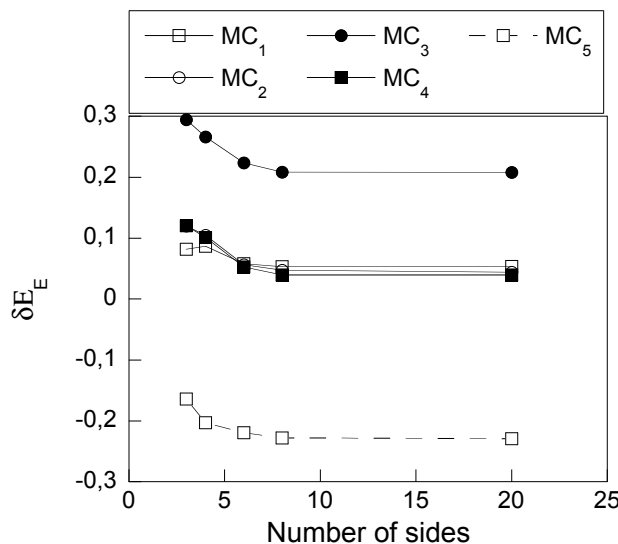


Figure 12. Deviation of elastic modulus values with the number of sides of the particle.

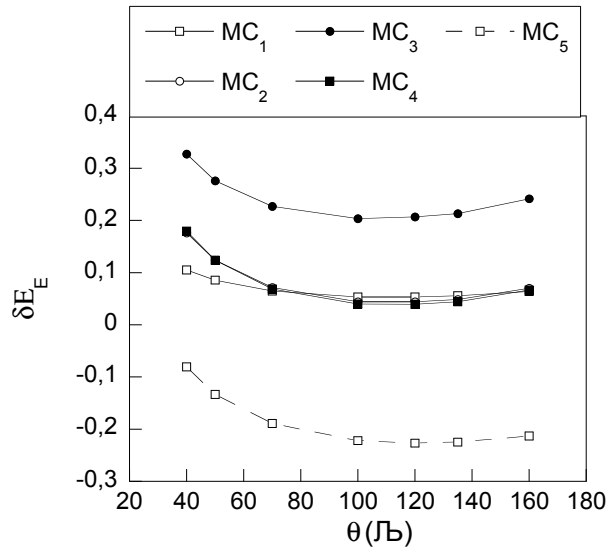
Figs. 13 and 14 plot the deviation of elastic modulus with the angularity of reinforcement. As happened with the number of sides, the higher deviation between both calculation procedures is  $\pm 30\%$ . The evolution of this deviation (Figs. 13 and 14) is similar to the variation of the elastic modulus, cf. Figs. 9 and 11.

## 5 Conclusions

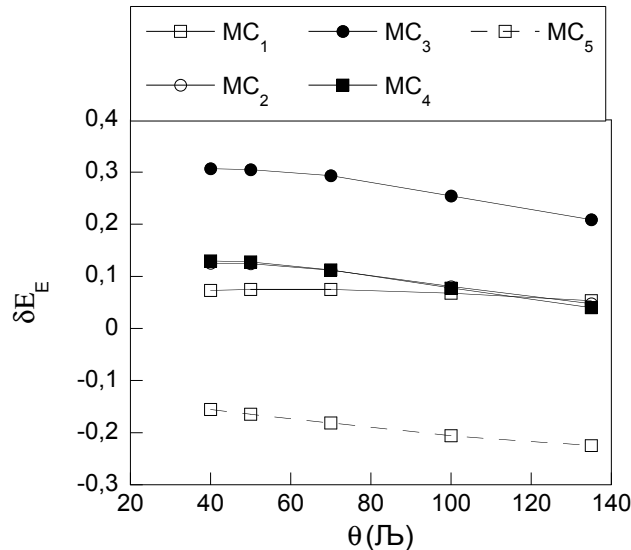
When polygonal reinforcement particles with different number of sides are considered, the elastic modulus increases by both diminishing the number of sides and the internal angle between sides due to the constraint increase associated with the reinforcement angularity.

These effects, obtained by numerical simulations by using the finite element method, are not considered by the analytical models.

A higher relationship between the elastic moduli of the constitutive phases implies a higher increment of the elastic modulus of the composite.



**Figure 13.** Deviation of elastic modulus values with the angularity of the reinforcement (evaluated by the angle  $\theta$ , cf. Fig. 3). Case of reinforcement parallel to the loading direction.



**Figure 14.** Deviation of elastic modulus values with the angularity of the reinforcement (evaluated by the angle  $\theta$ , cf. Fig. 3). Case of reinforcement inclined  $45^\circ$  in relation to the loading direction.

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