ON CONSTITUTIVE MODELLING OF RATE-DEPENDENCE IN ORTHOTROPIC ELASTO-PLASTIC MEDIA

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Abstract

The aim of the present study is to develop a constitutive model which is able to capture full orthotropic behaviour of a laminated polymeric elasto-plastic composite as well as taking into account material strain-rate sensitivity, making it particularly suitable for investigation of dynamic response under high rate blast and impact loads. The formulation is an extension of the work by Ogihara and Reifsnider and including strain-rate effects using the method proposed by Thiruppukuzhi and Sun. The proposed model is implemented in incremental form as a VUMAT in ABAQUS/Explicit and a set of parametric studies assessing dependence on strain-rate and loading direction are carried out. From this study, it is found that the proposed model is suitable for characterisation of composites subjected to dynamic loads and by using appropriate values of material parameters, the model can also be used for various composite architectures, including woven and uni-directional.

1 Introduction

1.1 Motivation

Engineering materials which exhibit ductility, such as steel, typically have significant reserve strength beyond their initial yield stress and such strength is used in practical applications to estimate the capacity up to failure when such a material is subjected to load. Thus, it is meaningful to investigate the behaviour of materials which are subjected to a load of sufficient intensity such that plastic deformation occurs. This includes fibre-reinforced laminated polymeric composites.

In the case of structures subjected to dynamic loading, the increased rate of loading will affect the response of the material, with increase in both strength and stress levels [1]. Thus, it is necessary to consider the material response beyond the linear-elastic regime and for different loading rates.

Composites are, in general, non-isotropic materials and tend to exhibit highly non-linear behaviour under various kinds of loading. In addition, the occurrence of plastic behaviour leads to progressive damage of the material in its response to load.

Thus, it is necessary to develop a constitutive relationship which can predict non-linear, large strain, strain-rate dependent behaviour to failure of anisotropic composites of various architectures, viz., uni-directional (UD) and woven composites, which are subjected to loads of a dynamic nature.

1.2 Constitutive models for composites

The constitutive model of a material can be defined as the relationship used to characterise its physical properties and such a model will be used to describe the material response of the material when subjected to loading.

The non-linear response of composites has long been recognised to be an important consideration in constitutive modelling. It is common in non-linear constitutive theory to develop the concept of effective stress, $\bar{\sigma}$, and effective plastic strain, $\bar{\varepsilon}^p$.

In 1973, Hahn and Tsai [2] used a complementary elastic energy density function to derive a stress-strain relation which is linear in the longitudinal and transverse directions but non-linear in the in-plane shear.

A non-linear plastic theory was first applied to composites by Sun and Chen [3], who suggested that a plastic potential function could be applied to UD composites. Using a simple 1 parameter plastic flow rule, the potential function adopted was a derivative of Hill's generalised plastic potential for a 3D orthotropic material [4]. Sun and Chen assumed there is no plastic strain in the fibre direction and limiting plasticity only to the matrix. This assumption is unacceptable in the case of woven composites, since experimental data [5] has shown that non-linearity did occur when material was loaded in the 0° and 90° directions.

In fact, Ogihara and Reifsnider [5] expanded the work of Sun and Chen by using a more general plastic potential with 4 unknown parameters. These parameters were determined by a number of simple tension experiments for different specimen angles. The effective stress and effective plastic strain were found for each test angle and the parameter combinations resulted in all effective stress-effective plastic strain curves to converge into a single master curve.

The concept of using a master curve has been shown to be valid for various materials by Sun et al. [3, 6, 7, 8, 9]. Non-linearity is expressed by a function representing the master curve. Sun and Chen [3] proposed a power law relating effective stress to effective strain:

$$\bar{\varepsilon}^p = A\,\bar{\sigma}^n \tag{1}$$

Weeks and Sun [6] found that the exponent n in the above relation was appropriate at all tested strain-rates and thus rate independent. Thus, the rate sensitivity is described solely by the parameter A. Thiruppukuzhi and Sun [7] proposed a developed power law describing the strain-rate dependence of A:

$$A = \chi\,(\dot{\bar{\varepsilon}}^p)^m \tag{2}$$

Thus, substituting (2) in (1),

$$\bar{\varepsilon}^p = \chi\,(\dot{\bar{\varepsilon}}^p)^m\,\bar{\sigma}^n \tag{3}$$

More recently, Hufner and Accorsi [10] have extended the 4 parameter plastic potential function of Ogihara and Reifsnider to include strain-rate effects using the strain-dependent power law of Thiruppukuzhi and Sun.

However, this model cannot take into account the possibility of full material orthotropy.

1.3 Scope of current study

The present work will extend the formulation of Ogihara and Reifsnider from a model with 4 parameters to consider all possible parameters, i.e., 9, and taking into account the strain-rate effects as in the work of Thiruppukuzhi and Sun, thus having a rate-dependent constitutive model which is adequate for all materials with complexity up to the level of orthotropy.

The model will be implemented in ABAQUS/Explicit by means of a user-defined material subroutine (VUMAT) and a series of numerical models carried out to investigate the effect of strain-rate on a typical UD composite with various fibre orientations.

2 Derivation of an orthotropic visco-plastic constitutive model

2.1 Plastic potential

A plastic potential function which has the most general form with 9 parameters is proposed:

$$2f(\sigma_{ij}) = a_{11}{\sigma_{11}}^2 + a_{22}{\sigma_{22}}^2 + a_{33}{\sigma_{33}}^2 + 2a_{44}{\sigma_{23}}^2 + 2a_{55}{\sigma_{13}}^2 + 2a_{66}{\sigma_{12}}^2 + 2a_{12}\sigma_{11}\sigma_{22} + 2a_{13}\sigma_{11}\sigma_{33} + 2a_{23}\sigma_{22}\sigma_{33} \tag{4}$$

Using appropriate values for the various parameters, the function could be used to describe a range of material architecture systems, e.g., substituting $a_{11} = a_{12} = 0$ and $a_{22} = 1$ reduces the above potential function to the one parameter potential of Sun and Chen [3] for UD composites and by setting $a_{11} = a_{12} = a_{22} = 0$ gives the function used by Thiruppukuzhi and Sun [7, 8] for woven composites. The generalised anisotropic constitutive equations, in rate form, are expressed as follows:

$$[\dot{\varepsilon}] = [C][\dot{\sigma}] \tag{5}$$

The strain-rate tensor is decomposed into two components, viz., the elastic and plastic strain-rate components:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \tag{6}$$

Thus, the compliance matrix is expressed as the sum of the elastic and plastic components:

$$[\dot{\varepsilon}] = [C][\dot{\sigma}] = [C^e + C^p][\dot{\sigma}] \tag{7}$$

2.2 Elastic compliance matrix

The elastic compliance matrix for an orthotropic material is obtained from standard materials textbooks (e.g. [11]) and this includes 9 unknown material parameters (E_{11}, E_{22}, E_{33}, G_{23}, G_{31}, G_{21}, ν_{12}, ν_{13}, ν_{23}):

$$[C^e] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{8}$$

It is assumed that there is no strain-rate dependency in the elastic region, which is consistent with experimental observation [7, 8].

2.3 Plastic compliance matrix

The plastic compliance matrix is assembled by first computing the plastic strain-rate components from the potential function in equation (4) using an associated flow rule:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda} \frac{\partial f\left(\sigma_{ij}\right)}{\partial \sigma_{ij}} \tag{9}$$

where $\dot{\lambda}$ is the proportionality factor rate. Applying this flow rule to the plastic potential, the 6 plastic strain-rate components are derived. The proportionality factor rate, $\dot{\lambda}$, is derived using the equivalence of the rate of plastic work, $\dot{W}^{p}$, namely:

$$\dot{W}^{p} = \bar{\sigma}\dot{\bar{\varepsilon}}^{p} = \sigma_{ij}\dot{\varepsilon}_{ij}^{p} \tag{10}$$

Defining the effective stress, $\bar{\sigma}$, as:

$$\bar{\sigma} = \sqrt{3f} \tag{11}$$

And the hardening modulus, H_p, as:

$$H_p = \frac{\dot{\bar{\sigma}}}{\dot{\bar{\varepsilon}}^{p}} = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^{p}} \tag{12}$$

It is implied that the proportionality factor rate is expressed as:

$$\dot{\lambda} = \frac{3\,\dot{\bar{\sigma}}}{2\,H_p\,\bar{\sigma}} \tag{13}$$

Using the definition of $\bar{\varepsilon}^{p}$ proposed by Thiruppukuzhi and Sun [7, 8] given in equation (3) earlier and differentiating it with respect to $\bar{\sigma}$, gives the hardening modulus as:

$$H_p = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^{p}} = \frac{1}{n\,\chi\,(\dot{\bar{\varepsilon}}^{p})^{m}\,\bar{\sigma}^{n-1}} \tag{14}$$

The effective stress rate is defined as:

$$\dot{\bar{\sigma}} = \frac{3}{2\,\bar{\sigma}}[(a_{11}\sigma_{11} + a_{12}\sigma_{22} + a_{13}\sigma_{33})\dot{\sigma}_{11} + (a_{22}\sigma_{22} + a_{23}\sigma_{33} + a_{12}\sigma_{11})\dot{\sigma}_{22} + (a_{33}\sigma_{33} + a_{23}\sigma_{22} + a_{13}\sigma_{11})\dot{\sigma}_{33} + (2a_{44}\sigma_{23})\dot{\sigma}_{12} + (2a_{55}\sigma_{13})\dot{\sigma}_{13} + (2a_{66}\sigma_{12})\dot{\sigma}_{23}] \tag{15}$$

This gives the final expression for $\dot{\lambda}$ as:

$$\dot{\lambda} = \frac{9}{4\,H_p\,\bar{\sigma}^{2}}[(a_{11}\sigma_{11} + a_{12}\sigma_{22} + a_{13}\sigma_{33})\dot{\sigma}_{11} + (a_{22}\sigma_{22} + a_{23}\sigma_{33} + a_{12}\sigma_{11})\dot{\sigma}_{22} + (a_{33}\sigma_{33} + a_{23}\sigma_{22} + a_{13}\sigma_{11})\dot{\sigma}_{33} + (2a_{44}\sigma_{23})\dot{\sigma}_{12} + (2a_{55}\sigma_{13})\dot{\sigma}_{13} + (2a_{66}\sigma_{12})\dot{\sigma}_{23}] \tag{16}$$

By substituting this expression for $\dot{\lambda}$ into the plastic strain-rate components using equation (9), it is possible to find the individual terms of the plastic compliance matrix. The full set of the elasto-plastic compliances is presented elsewhere [12, 13].

3 Correlation with experimental data

The developed constitutive model was introduced into ABAQUS/Explicit by means of the VUMAT function, whereby the stress-strain relationships are defined through a user-defined subroutine. Results from the numerical model were compared with uni-axial test data from literature [7] and excellent correlation is obtained, within 10% accuracy.

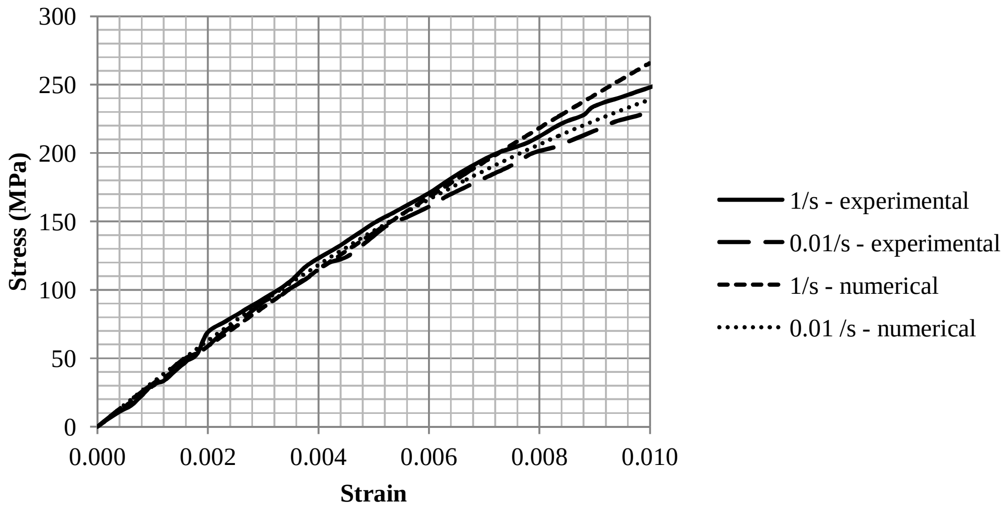

Figure 1. Comparison with test data (after [7]) for UD S2-glass/8553-40 at 15° off-axis orientation

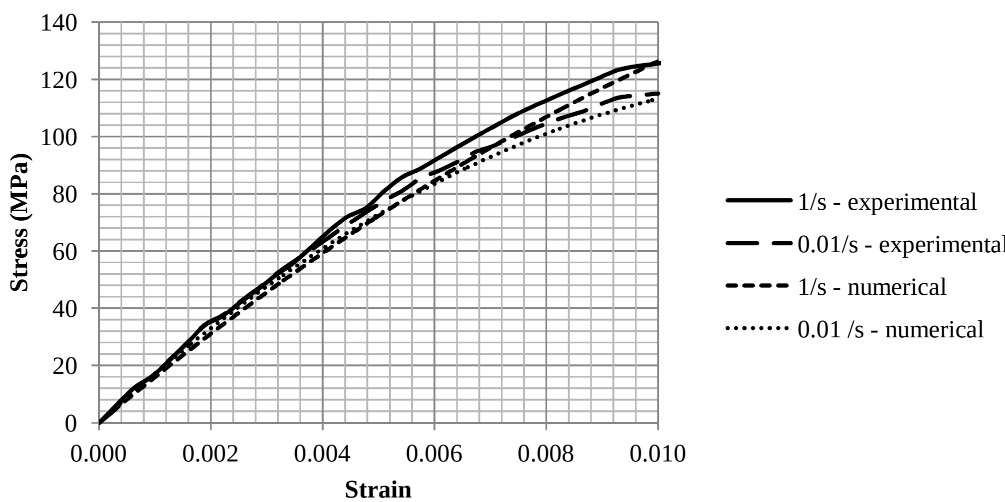

Figure 2. Comparison with test data (after [7]) for UD S2-glass/8553-40 at 30° off-axis orientation

4 Parametric studies

4.1 Problem definition and material data

A number of numerical models of various UD composite laminates with 5 different fibre orientation angles (0°, 30°, 45°, 60° and 90°) and subjected to a uni-axial load at 3 strain-rates (1 /s, 10/s and 1000 /s). The material used in this study is a UD S2-glass/8553-40 composite, also studied by Thiruppukuzhi and Sun [7, 8]. Damage initiation is also monitored by means of Hashin's quadratic failure criteria for uni-directional composites [14].

4.2 Specimen geometry and finite element model

The laminate considered is a 50x20mm specimen having a central 5mm diameter circular hole and with a thickness of unity.

The laminate was modelled in ABAQUS/Explicit v. 6.9-1 using 8-noded linear brick elements (C3D8) with no reduced integration to eliminate shear locking and hour glassing effects. A uniform mesh size of 0.5mm was used and results were obtained for various elements around the central circular hole.

Each of the angle/rate combination was carried out using 2 versions of the VUMAT subroutine, viz., with the inclusion of visco-plastic effects and without.

4.3 Results

4.3.1 Non visco case

A selection of results is shown below:

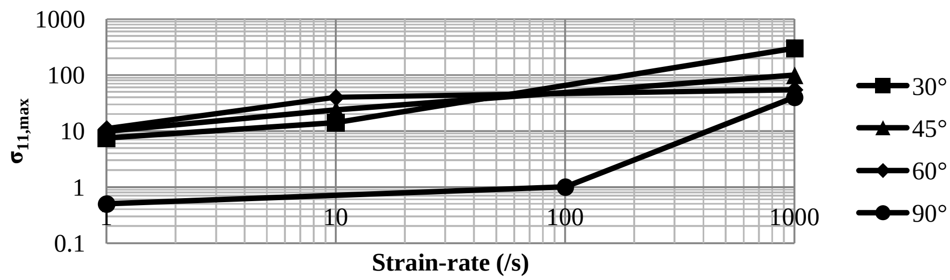

Figure 3. Variation of $\sigma_{11,max}$ with strain-rate, non-visco case

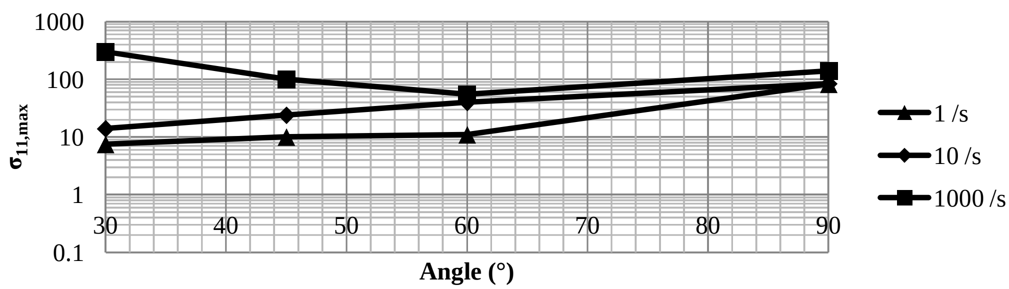

Figure 4. Variation of $\sigma_{11,max}$ with off-axis angle, non-visco case

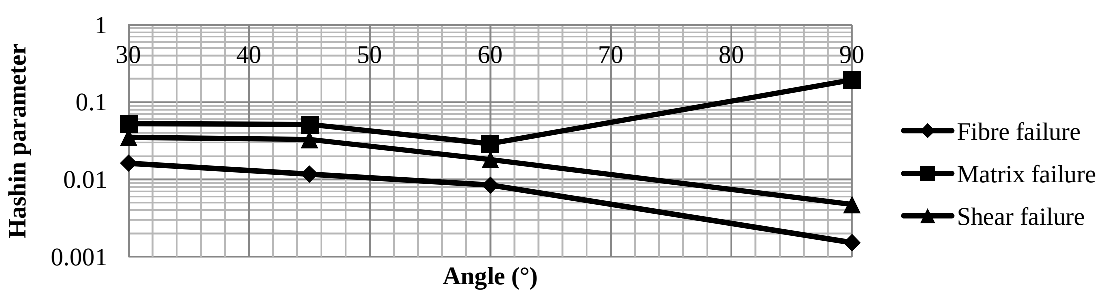

Figure 5. Variation of failure incidence with off-axis angle, 1000 /s rate, non-visco case

4.3.2 Visco case

Similar results but with consideration of visco-plastic effects were obtained, e.g.:

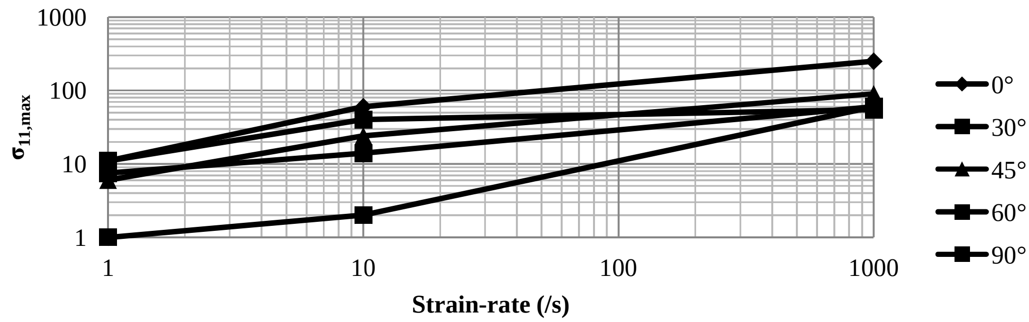

Figure 6. Variation of $\sigma_{11,max}$ with strain-rate, visco case

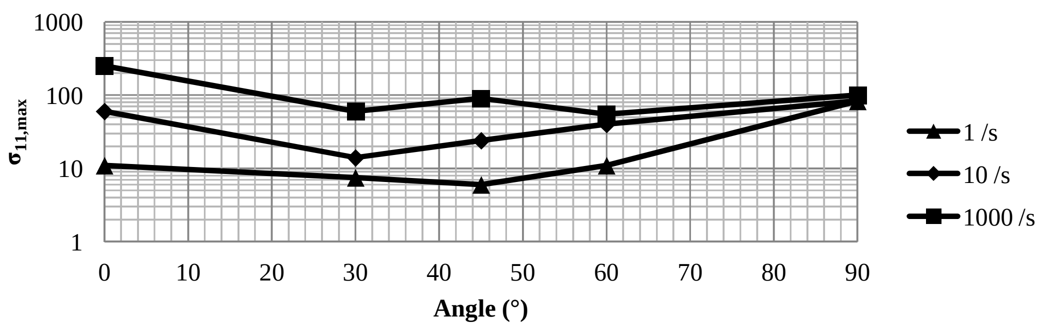

Figure 7. Variation of $\sigma_{11,max}$ with off-axis angle, visco case

The failure incidence variation for each failure mode was also obtained as a function of both fibre orientation angle and also the element position around the central opening, e.g.:

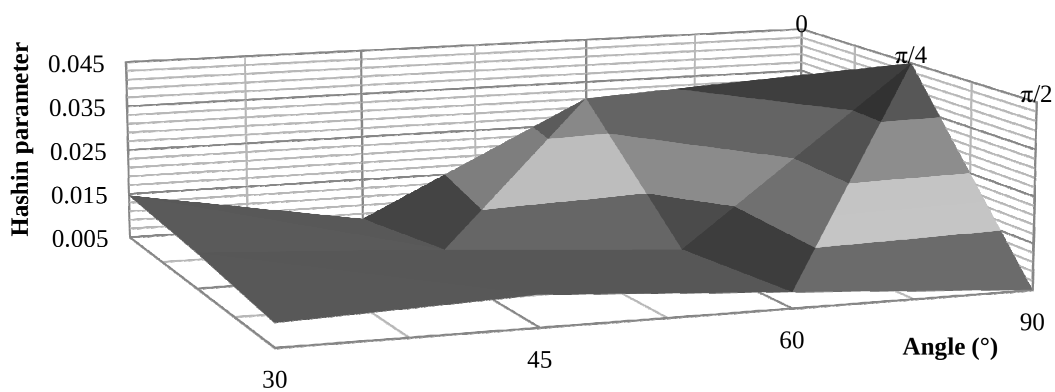

Figure 8. Variation of fibre failure incidence with off-axis angle and element position, 1000 /s rate, visco case

5 Conclusions

In this paper, the framework for a constitutive model which captures full orthotropic behaviour of an elastic-visco-plastic composite lamina (ply) has been presented.

The devised model was implemented in a user-defined material subroutine VUMAT in the finite element program ABAQUS/Explicit and the model was verified by comparison with experimental data available in literature.
A comprehensive parametric study was conducted on a punctured plate (a 2D medium with a central circular hole) of the same properties, investigating the variation with strain-rate (from medium to high rate of loading) and the fibre orientation angle.
It is observed that visco-plastic effects are especially significant for the high strain-rates, with the increase in stress being up to 25% and even 50% in some cases. The influence of fibre alignment angle was found to be less critical in most cases, except for the 90° orientation, in which case the composite's performance was limited by the matrix strength and stiffness.
Due to the various interactions between the stress components, the main failure criteria associated with composited, viz., fibre damage, matrix cracking and in-plane shear, and their initiation were also investigated. The onset of failure was also monitored using Hashin's failure criteria for the different failure modes mentioned above and it was found that visco-plasticity does not significantly affect fibre performance whilst matrix and, in particular, shear failure is severely affected by the inclusion of visco-plastic effects.

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