MESH-INSENSITIVE FINITE ELEMENT MODELLING OF ELASTIC-PLASTIC COMPOSITES

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Abstract

The aim of the present study is to investigate the behaviour of high-performance polypropylene based composites, such as Dyneema® and extract the response under tension, compression and shear.

An energy-based approach to model the observed behaviour is presented. The proposed model is mesh-size independent by ensuring that the maximum element size does not exceed a computed value. Alternatively, the softening curve is adjusted such that the energy equivalence is maintained.

A damage index is formulated to degrade material stiffness in each direction and also taking into account permanent plastic deformation under tensile loading. Compressive behaviour is accounted for by a simplified elastic-plastic response while shear deformation is characterised by a cubic stress-strain response.

The proposed formulation will be implemented in a commercial finite element package, such as ABAQUS/Explicit, by means of a user-defined subroutine (VUMAT) and used to model extreme loading events on Dyneema® panels under localised and global blast loading.

1 Introduction

Materials subjected to extreme loading, such as blast, will invariably undergo damage. In the case of laminated composite materials, the initial occurrence of damage does not lead to ultimate failure, but a progressive build-up of loss of material strength and stiffness occurs as a result of various damage mechanisms which are unique to such materials, such as delamination between plies, debonding between fibre and matrix and failure of the individual constituents, viz., fibre and matrix.

The use of composites for protection against severe loads is becoming more relevant as designers and users seek lighter protection systems than conventional ones, such as steel. The range of composite materials used for the purpose has broadened from the traditional carbon and glass fibre systems to new high performance polymeric materials, such as aramids, polypropylene and polyethylene fibres.

1.3 Scope of current study

The aim of the present work is to develop the framework for a constitutive model especially for novel composites, such as those utilising high performance polyethylene fibres, which model is capable of capturing the progressive material damage under load by means of an energy-based method which is mesh-objective.

2 High performance polyethylene composites

Dyneema® is the trade name of a variety of ultra high molecular weight polyethylene spun using a process called gel spinning patented by the Dutch company DSM in 1979 and long marketed as “the world’s strongest fibre”, claimed to be 10 times stronger than steel and 50% stronger than aramid fibre, on a weight-to-weight basis [1]. It is extremely light (the specific gravity of Dyneema® is 0.97) and the tensile strength varies from 1400 to 3000 MPa [2].
Tests have been carried out on 250x25x5mm Dyneema® uni-directional composite samples [3] to determine its tensile, compressive and shear properties under monotonically increasing loading and cyclic loading.

2.1 Tensile behaviour

The tensile behaviour, as seen in Figure 1, is essentially linearly elastic with an initial modulus of almost 13 GPa until a yield strain of approximately 3.75%, after which softening occurs until a failure strain of approximately 8.5% [3].

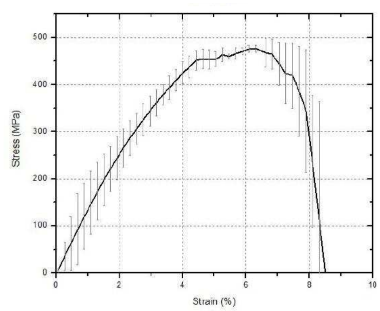

Figure 1. Tensile loading of Dyneema® sample [3]

2.2 Compressive behaviour

The compressive behaviour shown in Figure 2 has an initially linear elastic stress-strain response with a modulus of 1.25 GPa until a stress of 8 MPa, followed by a long plastic plateau [3]. The peak before the plateau is probably due to the fibre micro-kinking [3, 4].

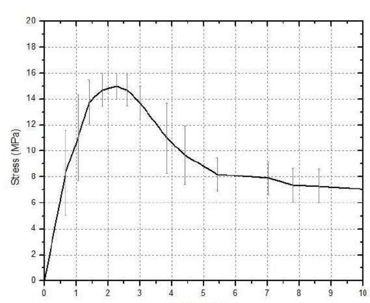

Figure 2. Compressive loading of Dyneema® sample [3]

2.3 Shear behaviour

The shear response of Dyneema® is highly non-linear as seen in Figure 3 and a cubic function is fitted to the test data of the form:

$$\tau = A\gamma^3 + B\gamma^2 + C\gamma \tag{1}$$

where A=1120.77 MPa, B=–580.42 MPa and C=109.61 MPa [3].

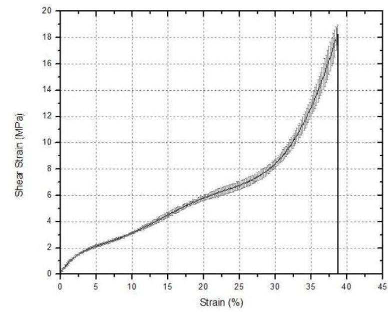

Figure 3. Shear loading of Dyneema® sample [3]

This complex combination of tensile, compressive and shear response has been successfully implemented in a finite element commercial software by Iannucci and Pope [4], who have utilised the work of Cunniff [5] which is based on fibre properties and thus links the mesoscale behaviour of the fibres with the macroscale behaviour of the laminate.

3 Damage mechanics

Modeling damage in composites can be implemented in various ways and a comprehensive review of damage mechanics theories can be found in [6]. Such theories usually make use of a damage index, whose value varies between 0 and 1 (where 0 indicates intact, undamaged material and 1 indicates a fully failed material). The damage index is multiplied with the material properties in the relevant material constitutive laws (e.g. modulii, Poisson's ratios) and one can include either a single damage index to represent multiple damage effects or a series of damage indices for each respective damage mechanism.

The progressive failure is typically done on a macro level by using a smeared crack approach. This assumes a strain-softening constitutive law intended to capture the reduction in stiffness which occurs once a specified stress (or strain) level has been attained.

The method was first developed for concrete by Bažant and Oh [7] in 1983, extending the work of Hillerborg et al. [8] of 1976. The fundamental principle is to relate the specific energy, g_f, i.e., the energy dissipated per unit volume given by the area under a stress-strain curve, with the material's fracture energy, G_f, i.e., the energy dissipated per unit area, which depends on the fracture mode being considered (mode I, II or III).

The apparent arising difficulty is that the response is highly sensitive to the mesh size. This is due to the fact that, during the softening process, as the stress reduces to zero, the constitutive model must still dissipate energy per unit volume equal to the area below the stress-strain curve, implying that, as the material degrades progressively, the softening is concentrated into a single element region [9, 10, 11].

With reference to Figure 4 below, the total specific energy, E_f, is made up of 2 parts, viz., the elastic part, E_e, and the inelastic (or propagation) part, E_p:

$$E_f = E_e + E_p \tag{2}$$

where

$$E_e = \frac{1}{2}\sigma_0\varepsilon_0 \tag{3}$$

and

$$E_p = \frac{1}{2}\sigma_0(\varepsilon_f - \varepsilon_0) \tag{4}$$

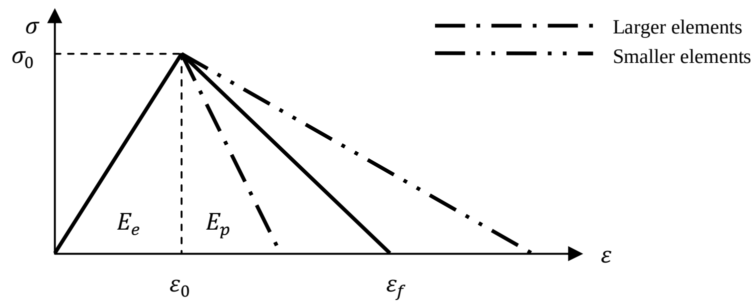

Figure 4. Bi-linear stress-strain as a function of element size (after [10])

The specific energy is given by:

$$g_f = \int_0^{\varepsilon_f} \sigma \, d\varepsilon = \frac{1}{2}\sigma_0\varepsilon_f \tag{5}$$

Defining l_c as the characteristic length defined by:

$$l_c = \frac{A}{L} \tag{6}$$

where A is the (finite) element area and L is the (finite) element's maximum length, then the total fracture energy is written in terms of the specific energy as:

$$G_f = g_f l_c = \frac{1}{2}\sigma_0\varepsilon_f l_c \tag{7}$$

From equations (3), (4) and (7), an upper limit for the characteristic length is obtained as [11]:

$$l_c < \frac{2G_f}{\sigma_0\varepsilon_0} \tag{8}$$

If A_e is the fracture area and V_e is the (finite) element volume, then:

$$(E_e + E_p)V_e = G_f A_e \tag{9}$$

The elastic portion from equation (3) is unalterable but, for energy equality with the fracture energy (which is a material property), then the softening portion of the stress-strain curve (equation 4) should be elongated or contracted according to the element size such that the energy balance is maintained:

$$E_p = \frac{G_f}{l_c} - E_e \tag{9}$$

A corollary would be to find the maximum element size which can be used such that this will ensure that the energy equivalence is not violated. For finite element meshes of square geometry, then from equation (6) and (8), it is evident that the maximum element size is equal to the characteristic length.

4 Derivation of a mesh-objective elastic-plastic constitutive model

The aim is to set up the framework for a constitutive model for a typical Dyneema® composite having fibres in 2 directions. In this constitutive model, it is assumed that there is no coupling between failure in each of these directions. The stiffness in each respective direction can be obtained on the basis of the fibre modulus only, since most Dyneema® composites are resin-poor systems and response is mostly dominated by the fibres [4]. The modulus for each direction is then obtained using a simple rule of mixtures, e.g., for a simple 0/90 UD cross-ply composite, then the modulii would be:

$$E_{11} = 0.5\, V_f\, E_f (1 - d_1) \tag{10}$$

$$E_{22} = 0.5\, V_f\, E_f (1 - d_2) \tag{11}$$

where V_f is the fibre volume fraction, E_f is the fibre modulus and d_i are the respective direction damage indices.

4.1 Tensile behaviour

For each of the 2 directions, linear elastic loading is initially assumed until a failure initiation (yield) stress, after which the response follows the softening part of the curve, as per Figure 5, until the damage parameter reaches a value of 1, in which case total failure is indicated and the (finite) element is deleted.

If unloading occurs prior to the complete failure, then the unloading curve is assumed not to be directly to the origin but offset to give a permanent (plastic) strain. The magnitude of this offset is determined by material cyclic loading tests.

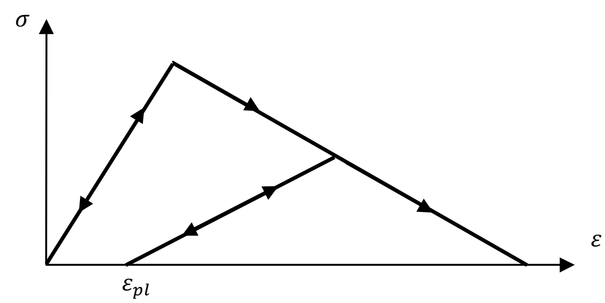

Figure 5. Tensile stress-strain behaviour

The evolution of the damage parameter from 0 to 1 is obtained by geometrical considerations of Figure 5 and is given by:

$$d_i = 1 - \left(\frac{\varepsilon_f - \varepsilon_{ii}}{\varepsilon_f - \varepsilon_0}\right)\left(\frac{\varepsilon_0}{\varepsilon_{ii} - \varepsilon_{pl}}\right) \tag{12}$$

Equation (12) can be readily implemented in a finite element package, such as ABAQUS/Explicit by means of a user-defined material subroutine (VUMAT) by expressing it in incremental form as:

$$\Delta d_i = \left(\frac{\varepsilon_f - \varepsilon_{pl}}{\varepsilon_f - \varepsilon_0}\right)\left(\frac{\varepsilon_0}{\left[\varepsilon_{pl} - \varepsilon_{ii}\right]^2}\right)\Delta\varepsilon_{ii} \tag{13}$$

4.2 Compressive behaviour

As suggested by test data and in existing literature [3, 4], the compressive response for Dyneema® can be assumed to be elastic-perfectly plastic, with an unloading curve parallel to the initial modulus to give rise to permanent (plastic) strains:

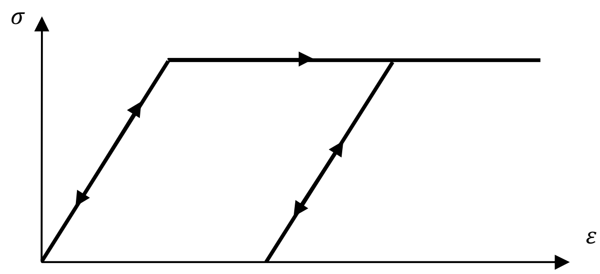

Figure 6. Compressive stress-strain behaviour

4.3 Shear behaviour

Finally, the non-linear shear behaviour is modelled with the response according to equation (1), with unloading following a damaged slope, consistent with test data, as per Figure 7:

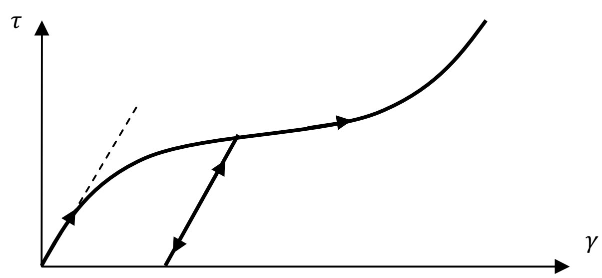

Figure 7. Shear stress-strain behaviour

5 Conclusions and future work

In this paper, an energy-based approach to model the observed behaviour of polypropylene-based composites is presented. The proposed model is mesh-size independent either by ensuring that the maximum (finite) element size does not exceed a computed value, based on a characteristic value or by adjusting the softening curve such that the energy equivalence is maintained.

Damage accumulation in tension is captured by means of a damage index for each of the composite material's directions while also taking into account permanent plastic deformation. Compressive behaviour is accounted for by a simplified elastic-plastic response while shear deformation is characterised by a cubic stress-strain response.

The formulation will now be implemented in ABAQUS/Explicit by means of a user-defined material subroutine (VUMAT), with the equations written in rate form to update the stress incrementally. The material model will then be tested by correlating the numerical results with test data on small-scale panels loaded under localised and global blast.

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