THE POSTCRITICAL DEFORMATION STAGE AND NON-LOCAL FAILURE CONDITIONS

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Abstract

The paper is devoted to study of the behavior of materials at the elastoplastic and postcritical deformation stage. Problems of implementation of the postcritical stage deformation of materials during testing where considered. Issues of limit states depending on the stiffness of the loading systems under uniaxial tension of solid cylindrical specimens experimentally were studied. Shown that are possible to obtain diagrams with advanced stage of strain softening and complete deformation curves of materials with sufficient high stiffness of loading system.

1 Introduction

Dissipative processes of inelastic deformation, including the processes of structural failure and fracturing, reflected on the deformation curve as nonlinearity. At the final stage this leads to softening materials and appearance loss of strength section on deformation curve. Study of the basic laws of this phenomenon, as well as their mathematical modeling allows for more accurately forecasting the conditions of deformed bodies destruction and analysis of the fracture process management capabilities. Issues of experimental and theoretical study of the postcritical strain laws are attract the attention of researchers in connection with the possibility of materials deformation reserves, increasing the carrying capacity and survivability of constructions [1-6].

On the postcritical stage of deformation is the formation of macro-destruction conditions. They, unlike the traditional view that defines the use of force or deformation criteria are not definitely related to the stress-strain state at the point of a deformed body. During the transition from the stage of equilibrium damage accumulation to non-equilibrium stage of destruction the key role played the interaction of a deformed body with the loading system. As a result, depending on loading conditions, each point on the descending branch of stress-strain curve can correspond to the time of the loss of bearing capacity as result a transition from stable to non-equilibrium stage of the damage accumulation process. Thus, the rigid loading system may contribute to the "adaptation" of the object in the process of destruction due to local dissipation of elastic energy.

In emergency situations, the most important property of materials is survivability. Accounting for the postcritical deformation stage in the more exact calculations is reveals reserve load-carrying capacity of structures. Completeness of the load-carrying capacity implementation of critical constructions and buildings is determined by the degree of postcritical deformation.

The study of patterns of postcritical deformation of materials and structures, the creation of appropriate mathematical models and develop methodology for integrating the strength analysis of security carrying objects contribute to the implementation of provisions of bearing capacity, increase vitality and safety of deformable systems.

2 Materials and testing methods

In a work [1] considered the specimen with reduce central part and obtained necessary condition of postcritical deformation stage realization of central part at tension. This condition is consistent with known fact of necessary sufficient of test system stiffness for registration of descending brunch in experiment. Analysis of the date shows that even when using high stiffness machines is impossible to obtain the complete strain curve by certain specimen geometry. With respect to weak zone the main volume of specimen becomes the element of loading system and increase it compliance. In tests with smooth cylindrical specimens the realization of strain softening stage associated with strain localization in small volume were elastic energy of system is spent.

Tests carried out on universal biaxial servohydraulic test system Instron 8850 [6,7]. Strain in test part registered by dynamic extensometer Instron 2620-601 with gauge length 12.5 mm and travel ± 5 mm.

In the first test group used the specimens with additional compliance elements and reduce central part with smaller diameter (figure 1). In this case reduce central part is the test part of specimen.

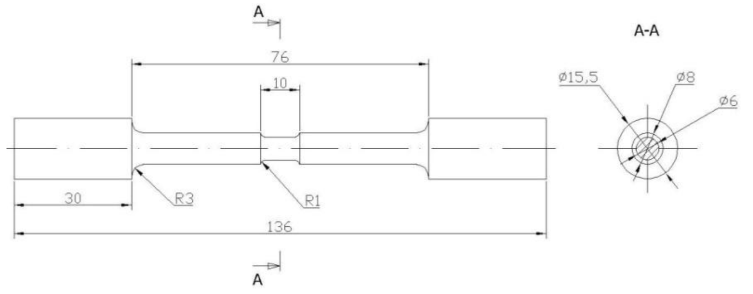


Figure 1 – Sketch of specimen with additional elements and reduce test part

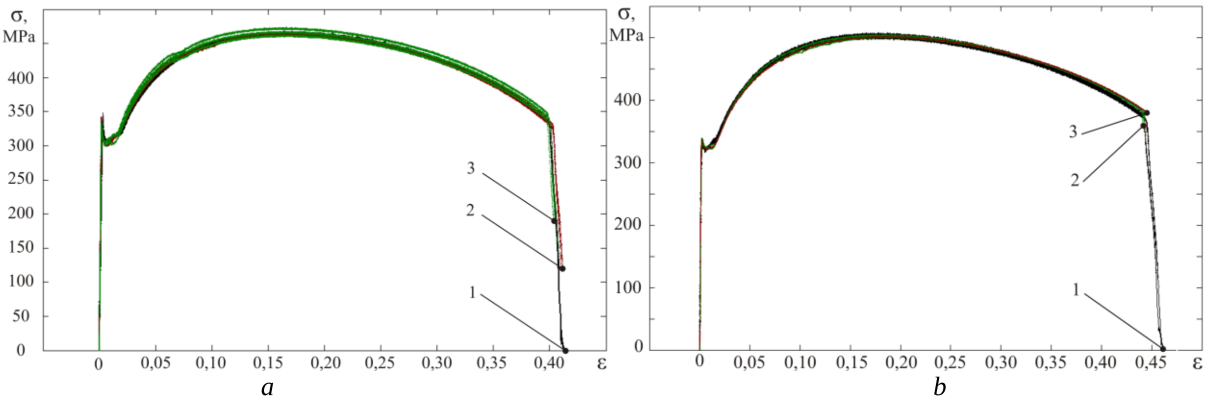
Change in the rate of loading system stiffness and stiffness of specimen test part was achieved by changing of additional elements length. The specimens with additional element length of 30 mm, 60 mm and without additional elements were used. The specimens made of steel 20 as-received condition bar.

In the second test group change in the rate of loading system stiffness and stiffness of specimen test part was achieved by changing of specimen stiffness. For solid cylindrical specimen the stiffness is

$$R_0 = D\pi d^2 / 4l \qquad (1)$$

where R_o — specimens stiffness on strain softening stage, D – softening modulus of material ($D = -d\sigma/d\varepsilon$ — current value of tangent softening modulus), d – diameter of specimen test part, l – length of specimen test part. Specimens with constant rate of length and diameter of test part (l/d=1) were used. Proportional change of specimen length and diameter leads to a stiffness change (equation 1).

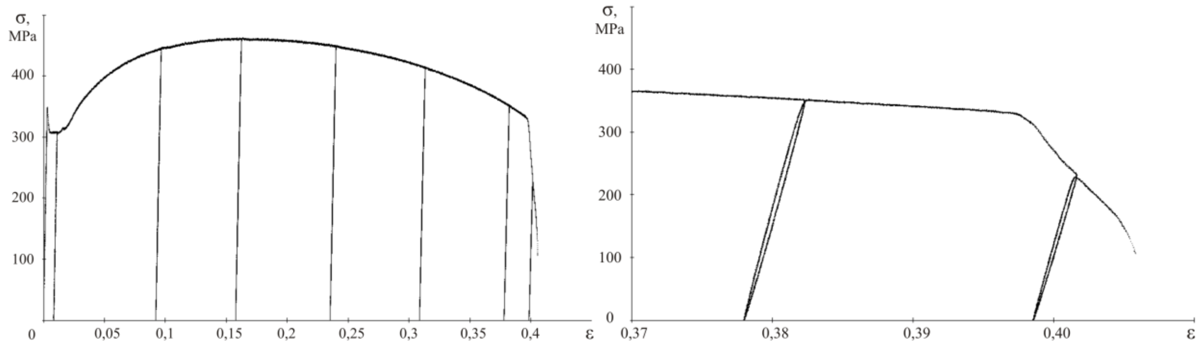
In results of tests obtained deformation curves of steel 20 with advanced softening section and complete deformation curves (figure 2) which break in different points (*1*,*2* and *3*) on postcritical deformation stage. For complete deformation curves (figure 2, *a – 1* and *b – 1*) break moment correspond to achievement of the zero load.



a *b*

Figure 2 – Deformation curves of steel 20 and destruction moments: *a* – test results of specimens with additional elements (*1* – specimen without additional elements, *2* and *3* – with additional elements total length 30 and 60 mm), *b* – test results of different stiffness specimens (*1, 2* and *3* – specimens with diameter 5, 10 and 13,5 mm)

Shown that with the sufficient high stiffness of the loading system to obtain curves with advanced stage of softening and complete deformation curves are possible. On figure 3 is presented the strain curve of steel 20 by uniaxial tension with unloading on different stage of elastoplastic and postcritical deformation (*a*) and the final section of the deformation curve with unloading (*b*). Strain rate during tension and unloading is 0.02 min^{-1}. This date confirm that equilibrium deformation of material on softening stage is possible.



a *b*

Figure 3 – Deformation curve of steel 20 by uniaxial tension: *a* – with unloading on different stages of elastoplastic and postcritical deformation, *b* – final section of the postcritical stage of deformation

On figures 4 and 5 are shown the strain curves of steel 40X at different types of stress state. Uniaxial tension tests carried out on solid cylindrical specimens with diameter of test part is 5 mm, length of test part is 8 mm. Torsion tests and proportional tension with torsion tests carried out on thin-walled tube specimens with length of test part is 14 mm, outside diameter is 10 mm, wall thickness is 1 mm.

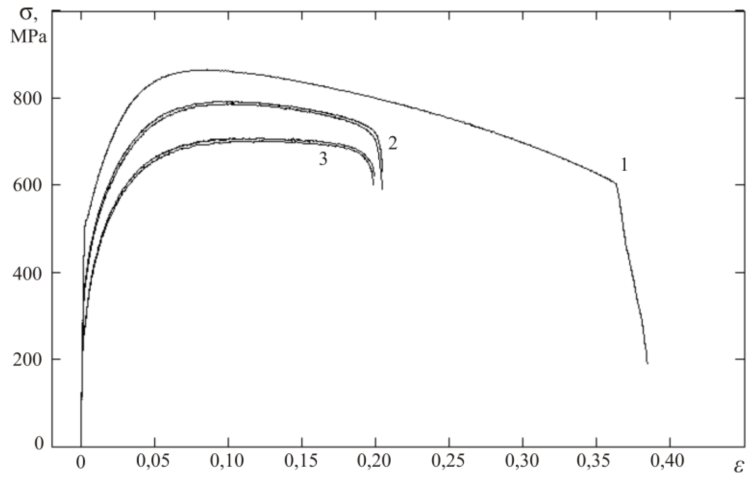


Figure 4 – Axial stress – axial strain curves of steel 40X: *1* – uniaxial tension, *2* and *3* – proportional tension with torsion with rate ε/γ=1.6 and ε/γ=0.8

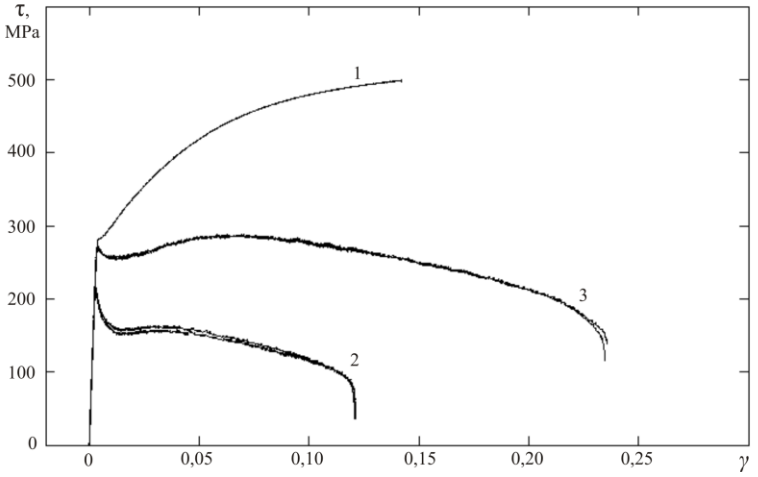


Figure 5 – Shear stress – shear angle curves of steel 40X: *1* – torsion test, *2* and *3* – proportional tension with torsion with rate ε/γ=1.6 and ε/γ=0.8

During tension of plastic materials we could observe shoulders effect especially at the postcritical deformation stage. And if we using specimens with different length of test part we obtain the set of curves which are different on postcritical section, as shown on fig.4. In order to analyze the effect of loading system on the formation the fracture conditions at the softening stage, the analysis of issues of data processing under the strain localization tested the different specimens. The specimens of steel 20 and 40X with different rate of test part length to diameter were used. Strain rate during tests is 0.02 min^{-1} up to specimen destruction. Obtained the deformation curves of steel 20 by specimens with rate of test part length to diameter 1.68 (figure 4, *a – 1*) and 1.16 (figure 4, *a – 2*) and steel 40X by specimens with rate of test part length to diameter 2.50 (figure 4, *b – 1*) and 1.60 (figure 4, *b – 2*). As shown on figure 4 this curves differ on strain softening stage.

The difference between the deformation curves obtained on samples with different ratio of length to diameter of the test part can be explained by the presence of strain localization and the elastic unloading region on the final deformation stages, when the homogeneous deformation conditions is violated. Thus in [8] noted essential influence of elastic deformed zones of loading body on deformation and destruction processes.

For test data processing in strain localization conditions on postcritical deformation stage under uniaxial tension of solid cylindrical specimens can offer the following model. The test part of solid cylindrical specimen with length is *l* regards as the compound bar which consist of strain localization zone δ and elastic unloading zone (l-δ). Under deformation in strain localization zone occurs the material softening, so that parts of specimen length (l-δ) not included in localization change zone is undergo elastic unloading. As a first approximation we can assume that when the maximum value of the efforts, the portion of material that has entered the localization zone with a further deformation does not change.

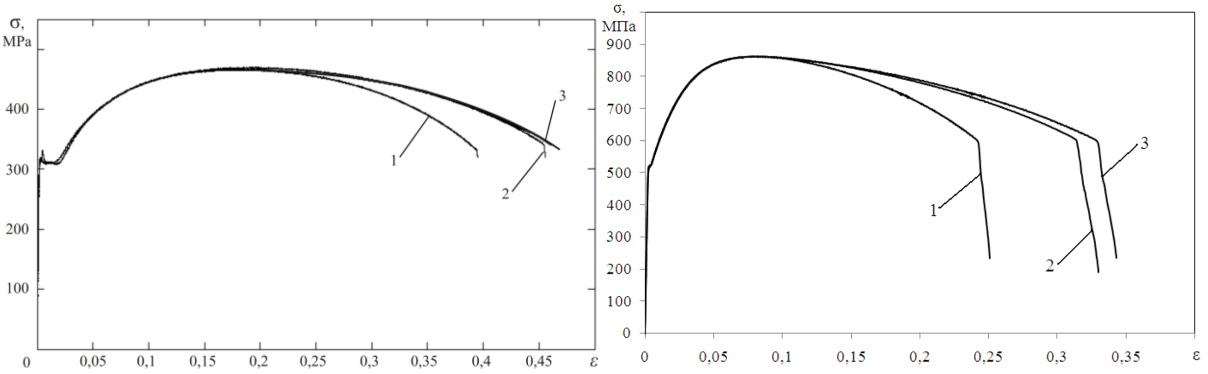
The increment of strain in localization zone due to increment of average strain over the specimen using the following equation:

$$\Delta\varepsilon_{\delta} = \Delta\varepsilon \frac{l}{l_{\delta}} - \frac{(l - l_{\delta})}{\mathrm{E} l_{\delta}} \Delta\sigma \tag{2}$$

and increment of strain of certain test part reduced length l_{red} due to increment of average strain over the specimen using the following equation:

$$\Delta\varepsilon_{red} = \Delta\varepsilon \frac{l}{l_{red}} - \frac{(l - l_{red})}{E l_{red}} \Delta\sigma \tag{3}$$

Based on experimental data obtained in uniaxial tension specimens of difference lengths and assuming that the volume of the localization zone is not changed, we can calculate the increment of the average deformation of the material on a common basis – reduced length (figure 6).



a *b*

Figure 6 – Deformation curves of specimens with different rates of test part length to diameter: *a* – steel 20, rate of test part length to diameter 1,68 – *1*; 1,16 – *2* and calculated curve – *3*; *b* – steel 40X, rate of test part length to diameter 2,50 – *1*, 1,60 – *2* and calculated curve – *3*

In a result obtained the calculated drop parts of strain curves *3* for steel 20 (figure 6, *a*) and steel 40X (figure 6, *b*) obtained by recalculating of test date (curves *1*) on reduce length. Reduce length is test part length of specimens which correspond curve *2*.

Experimental study of postcritical deformation stage than the need to ensure sufficient stiffness of loading system requires overcoming a number of methodological difficulties associated with the interpretation of test results in strain localization conditions [9].

3 Modeling

Accurate computation of constructions subject to mechanisms and kinetics of destruction make it possibly to describe of equilibrium crack propagation stages and predict of object load-carrying ability and survivability on the basis of loss of stability conditions of damage progress processes.

The postcritical deformation mechanisms are correspond with modeling results of destruction processes of holed plates obtained by numerical solution of physically-nonlinear boundary problem (Finite Element Method) [10]. Damage progress process of plate with stress concentrator under horizontal direction deformation is shown on figure 7. In the case of rigid loading a load-displacement curve had drop section under crack propagation.

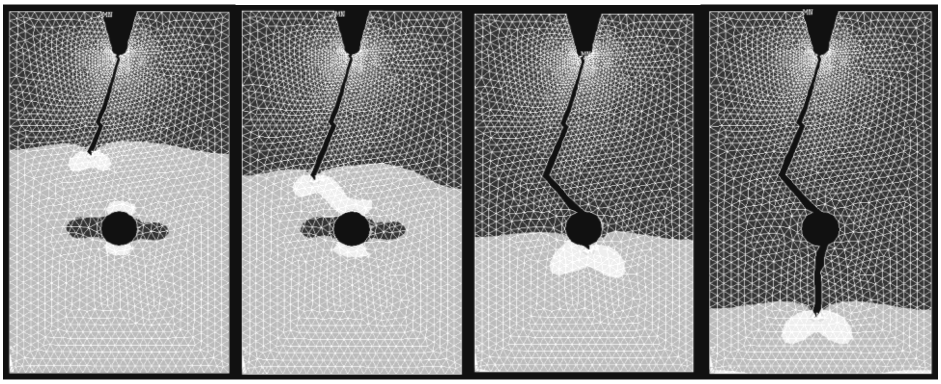
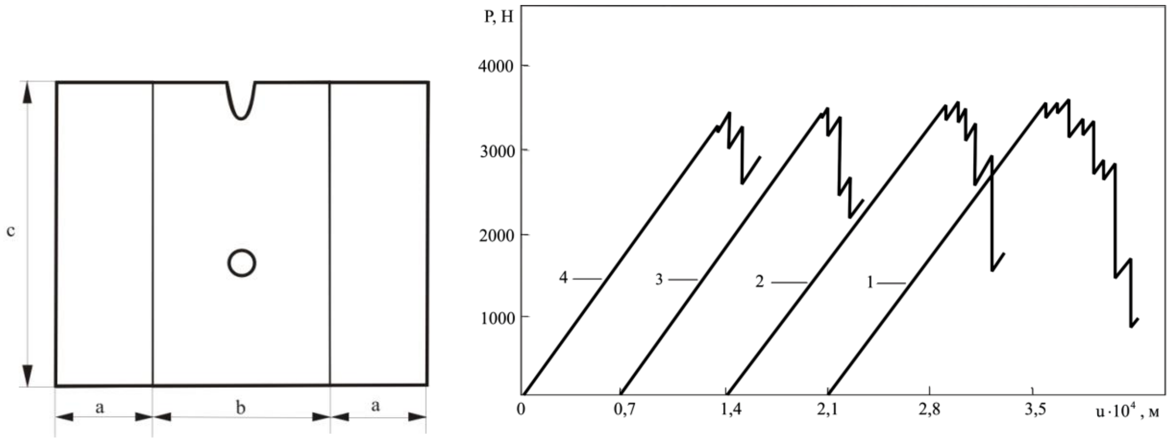


Figure 7 – Destruction kinetics of holed plate under rigid loading

Within the bounds of this problem is considered the role of loading system. In this case the loading system is additional part of elastic material (a on figure 8, *a*).



a *b*

Figure 8 – Plate with additional parts – *a* and calculation curves of plates tension – *b* (u – extension of area b: 1– a=0, 2– a=0.5b, 3– a=b, 4– a=2b)

As shown on figure 7, *b* destruction point on curve is depended of additional parts length that is loading system stiffness. It is confirm that limiting damaged state is depend of area size were controlled of kinematic boundary conditions. The difference of loading system properties with respect to invariable damage formation area is the reason for this effect.

Summary

The questions of implementation of the postcritical strain stage of materials in experiments were considered. After experimental investigations with using the specimens with additional compliance elements got results are confirming to influence of the loading system on behavior of materials on postcritical deformation stage. This is reflected in the presence of different points on the deformation curve that corresponds to destruction moments of the specimen. Shown that material is deforming equilibrium up to destruction of the specimen on the part with sufficient of the loading system stiffness. The analysis and account of postcritical deformation mechanisms and application of control methods of processes of damage accumulation on postcritical deformation stage with a view to generation the foundations for increasing strain resources of new materials is very important for assurance the safety constructions and structures.

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