DISCREET DAMAGE MODELING IN OPEN HOLE POLYMER MATRIX COMPOSITE LAMINATES

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Abstract
The present paper addresses the issue of direct simulation of complex local failure patterns in laminated polymer matrix composites including matrix cracking, delamination, and fiber failure. The analytical technique uses the eXtended Finitie Element Method for the simulation of matrix crack initiation and propagation at initially unknown locations and uses a cohesive interface model for delamination and continuum damage mechanics model for fiber failure. An important feature of the technique is that it uses independently measured standard ply-level mechanical properties of the unidirectional composite (stiffness, strength, fracture toughness). Failure simulations of composites containing open holes are presented. Mesh and parameter sensitivity of the strength values is investigated.

1 Introduction
Strength prediction in laminated composite materials is a formidable challenge, even more than four decades after the publication of pioneering works by Rosen [1] and Tsai and Wu [2]. Detailed modeling of damage initiation and propagation is particularly difficult in the regime where various damage modes, such as delamination and matrix cracking, interact. Availability and rapid increase of computer power has enabled recent successes in the development of the discrete damage modeling (DDM) technique, which is based on the direct simulation of displacement discontinuities associated with individual instances of matrix cracking occurring inside the composite plies, and delaminations at the interfaces between the plies. The Extended Finite Element Method (X-FEM) [3] is one method used to represent displacement discontinuities in arbitrary locations. In the regularized X-FEM formulation [4-7], the Heaviside step function is approximated using the displacement approximation shape functions. As a result, the integration domain for the displacements on either side of the crack coincides with the original element integration domain, and can be performed using standard Gaussian quadrature, which makes the task of connecting plies with arbitrary cracking directions trivial. In the present paper, we will also simulate the fiber failure mode by using several approaches ranging from statistical fiber failure criterion to progressive fiber failure simulation.

On the other hand, the continuum damage mechanics (CDM) methods offer a framework to perform progressive failure simulation in the fiber direction, albeit not including the statistical nature of fiber failure mechanism in the consideration at all. The constitutive model [8,9] includes the initiation stage, defined by stress failure criterion, and the propagation stage, defined by fracture toughness. It is expected that the initiation strength may have a significant effect on the failure load when the propagation stage is short. It is thereby desirable to replace the failure initiation load, which is inherently dependent on the volume of the highly stressed material, with the statistically scaled value. This is accomplished in the present manuscript by using the CFV method.

The results of the analysis are compared with experimental data presented in a series of papers by Wisnom and Hallett [10] and Hallett et al. [11] devoted to studying the scaling effects of strength in quasi-isotropic laminates with open holes. The mesh size effects, as well as the initial strength parameter effects on the open hole strength prediction accuracy is investigated.

2 Mesh Independent Crack (MIC) Modeling by Regularized X-FEM Approach

The DDM approach consists of mesh-independent modeling of matrix cracks in each ply of the laminate, and modeling the delamination between the plies by using a cohesive formulation at the ply interface. The simulation begins without any initial matrix cracks, which then are inserted based on a failure criterion during the simulation. In the present paper, we utilize the LaRC03 failure criterion [12]. The matrix cracks are modeled by using the regularized MIC formulation [4-7]. This formulation is a derivative of the x-FEM proposed in [3], where the cracked element is enriched by additional degrees of freedom to ensure the displacement jump. A critical aspect of the regularized formulation is the constitutive behavior in the gradient zone. The approach taken below directly incorporates the surface discontinuity-based cohesive law developed by Camanho et al. [13] in the formulation of the fracture energy of the gradient zone. According to [13], the cohesive force $\boldsymbol{\tau}$ resisting the opening displacement jump Δu at an arbitrary crack surface point is:

$$\boldsymbol{\tau} = (1-d)K\Delta\mathbf{u} + dK\langle\Delta u_n\rangle\mathbf{n} \tag{1}$$

where K is a high initial stiffness and d is the damage parameter. The first term in Eqn. (1) represents the crack cohesive force, and the second term prevents interpenetration of the crack surfaces. The brackets $\langle x\rangle={}^1/_2(x+|x|)$ represent the McAuley operator and vector $\boldsymbol{n}$ is the unit normal vector to the crack surface. The damage parameter controls the crack opening, and will be used below to display the length of the cracks. A bilinear relationship between the absolute values of $\boldsymbol{\tau}$ and Δu is assumed and defined by the initial and final values of the displacement gap Δ_0 and Δ_f. The initial value of the gap corresponds to the onset of bond softening, so that d=0 if $\Delta u \le \Delta_0$ and the final value Δ_f of the displacement gap corresponds to complete separation, i.e. d=1 if $\Delta u \ge \Delta_f$. The typical ratio of $\Delta_f / \Delta_0 = 2KG_c/X^2$ is on the order of magnitude of 10^4 due to very high values of the initial bond stiffness K, where G_c and X are the critical value of the energy release rate (ERR) and the initial strength respectively (such that Δ_0=X/K). The value of the damage variable d=0.5 corresponds to a very small displacement gap of approximately $2\Delta_0$ whereas the displacement gap of approximately $0.5\Delta_f$, which is indicative of interface separation, corresponds to a damage variable value of $d=1-\Delta_0/\Delta_f$, which is very close to 1. In the subsequent figures we will fix the crack display at d=0.995. The same cohesive law is also used to model the delaminations between plies. Mathematical details of the formulation are described elsewhere [5,7] , and will be omitted here.

3 Fiber Failure Simulation

3.1 Continuum Damage Mechanics (CDM)

Fiber failure is the only damage mode simulated by using CDM in the present work. Determination of the numerical values of the parameters defining the cohesive curve in Figure 1 for IM7/8552 material system used in the present analysis including the fracture toughness, G_{XT}=81.5 N/mm and coefficients f_{XT}=0.2, f_{GT}=0.4 as well as the characteristic length, has been addressed in [14] The initiation stress value, X_t, represents the fiber direction strength measured on standard unidirectional coupon type tests. According to the experimental data obtained by Wisnom et al. [15], this value depends on the volume of the coupon and for IM7/8552 follows the Weibull distribution with the modulus of α=40. Thus the specific value used for simulation with CDM is not entirely defined. In our simulations, we will use two values of the failure initiation strength: (i) X_t=2400N/mm^2 obtained for a typical 8-ply unidirectional coupon with total volume of V_0=34300mm^3 and X_t=3116N/mm^3 scaled strength for a 1mm^3 of stressed volume.

3.2 Critical Failure Volume Method

Volumetric scaling of the fiber direction strength is experimentally verified [15] to follow the Weibull distribution as following:

$$f(\sigma,V) = 1 - e^{-\frac{V}{V_0}\left(\frac{\sigma}{\beta}\right)^{\alpha}} \qquad (2)$$

where α and β are the Weibull parameters defining the scatter and mean of the distribution, V and V_0 are the specimen volume and the control volume respectively, and σ is the fiber direction stress. The idea of the CFV method is to apply scaling (2) to various subregions of the composite with nonuniform stress field and find the subregion with the maximum failure probability. Two considerations are required to achieve this goal. First, one needs to define the probability of failure for a volume with nonuniform stress distribution, and second, develop a procedure for the systematic search of the subvolumes. The assumption, which we will use to evaluate the probability of failure in the nonuniformly loaded regions, states that (2) provides a lower bound of probability of failure of a specimen with nonuniform stress distribution, if the stress in each point is higher or equal to σ.

We refer the reader for more details to references [16,17]. In the original CFV concept, it was assumed that the probability of failure of the entire specimen because if a finite subvolume suffers fiber failure; then it was assumed that the entire specimen cannot sustain anymore load. The remaining question is selecting the value for probability of failure for computing the failure load. A common approach [11,16,17] in this case is not assigning a specific value of the probability of failure, but instead computing the value of f_c so that the applied load is equal to the average failure load for the Weibull distribution, i.e.

$$\left(-\ln(1-f_c)\right)^{-1/\alpha}\Gamma\left(1+\frac{1}{\alpha}\right) = 1 \qquad (3)$$

Thus the failure load is defined as the applied load at which the failure probability f_c of CFV satisfies Eqn. (3).

3.3 CDM with the Initiation Stress Determined by CFV

In the present paper we will relax the assumption that the failure of CFV is equivalent to the failure of the entire specimen, and allow this volume to carry load in the framework of CDM, described in section 3.1, replacing the initiation stress value X_t with σ_c. This is a logical approach to define a statistically scaled initiation value, which is important in the situations when the final failure follows the failure initiation in the fiber direction very closely.

4 Results and Discussion

Tensile strength scaling effects in quasi-isotropic laminates $[45_m/90_m/\text{-}45_m/0_m]_{ns}$ was studied experimentally and analytically in [8,9] for different combinations of the number of blocked plies *m* and ply groups (parameter *n*-number of sublaminates), along with the in-plane dimension scaling. The plate width *W* to the hole diameter *D* ratio was kept constant *W*/*D*=5, as was the ratio of length *L* (distance between the grips) to the hole diameter, *L*/*D*=20. With increasing numbers of blocked plies, a change of failure mode from fiber failure to delamination failure was observed. In the case of single ply blocks, *m*=1, the fiber failure mode determines the laminate's strength, and the well-known tendency of strength decrease with increasing hole diameter was observed [16]. In the present paper, we will address the subset of data exhibiting fiber failure, i.e. m=1.

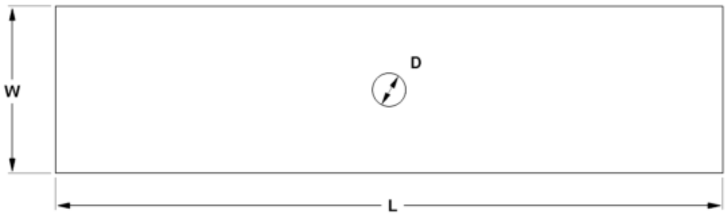

Figure 1. Scaled Geometry of Hole-Size Effect Study

4.1 Problem Statement and Boundary Conditions

An open hole coupon shown in Figure 2 is considered. Tensile loading in the x-direction will be applied by incrementing the displacement u_x at the edges *x=0,L*, so that

$$\boldsymbol{u_x^i}(\boldsymbol{0},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{u_x^{i-1}}(\boldsymbol{0},\boldsymbol{y},\boldsymbol{z}) - \Delta^{\boldsymbol{i}} \text{ and } \boldsymbol{u_x^i}(\boldsymbol{L},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{u_x^{i-1}}(\boldsymbol{L},\boldsymbol{y},\boldsymbol{z}) + \Delta^{\boldsymbol{i}} \tag{4}$$

Where $\Delta^{\boldsymbol{i}}$ is a constant and *i* is the loading step number. Such incremental formulation is required to properly account for the thermal curing stresses prior to the mechanical loading. The displacement field $\boldsymbol{u_x^0}$ appearing in equation (4) is computed by solving a thermal-mechanical expansion problem under boundary conditions which simulate free expansion and only restrict rigid body motion, i.e.

$$\boldsymbol{u_x^0}(\boldsymbol{0},\boldsymbol{0},\boldsymbol{0}) = \boldsymbol{0}, \boldsymbol{u_y^0}(\boldsymbol{0},\boldsymbol{W},\boldsymbol{0}) = \boldsymbol{0} \text{ and } \boldsymbol{u_z^0}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{0}) = \boldsymbol{0}. \tag{5}$$

The incremental loading boundary conditions (8) are supplemented with constraint conditions on the other displacement components at the lateral edges *x=0* and *L*, so that

$$\boldsymbol{u_y^i}(\boldsymbol{0},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{u_y^0}(\boldsymbol{0},\boldsymbol{y},\boldsymbol{z}) \text{ and } \boldsymbol{u_y^i}(\boldsymbol{L},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{u_y^0}(\boldsymbol{L},\boldsymbol{y},\boldsymbol{z}),$$

$$\boldsymbol{u_z^i}(\boldsymbol{0},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{u_z^0}(\boldsymbol{0},\boldsymbol{y},\boldsymbol{z}) \text{ and } \boldsymbol{u_z^i}(\boldsymbol{L},\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{u_z^0}(\boldsymbol{L},\boldsymbol{y},\boldsymbol{z}). \tag{6}$$

Unidirectional ply properties used for analysis are shown in Table 1. Unidirectional Stiffness and Strength Properties for IM7/8552and include the stiffness as well as strength properties.

E_1	161	GPa
E_2	11.38	GPa
G_{12}	5.17	GPa
G_{23}	4.00	GPa
υ_{12}	0.32	
υ_{23}	0.436	
Y_t	76	MPa
S	90	MPa
Y_c	275	MPa

Table 1. Unidirectional Stiffness and Strength Properties for IM7/8552

The ply level strength properties are used only for MIC initiation. Crack initiation is determined by checking the ply level failure criterion, LaRC0310, at each integration point. If the criteria is met or exceeded, a closed MIC is inserted into the model. Propagation of MICs is governed by the cohesive law (1), where all input parameters are listed in Table 2 and the initial strength values are equal to that of the unidirectional ply.

K	2.71E+08	N/mm^3
G_{Ic}	0.224	N/mm
G_{IIc}	0.911	N/mm

Table 2. Properties Used in Cohesive Law

4.2 Comparison with Experimental Stress and Inverse Hole Size Effect

Two FE meshes were created, a coarse unstructured mesh with 64 elements around the hole and a fine parametric mesh with 144 elements around the hole, as shown in Figure 1. The software allowed these meshes to be scaled in any dimension, thus allowing to create two mesh configurations for each specimen in the study of scaling effects on tensile strength of open hole laminates. Table 3 lists the summary of the results of the simulations and the experimental results reported by Wisnom and Hallett [11].

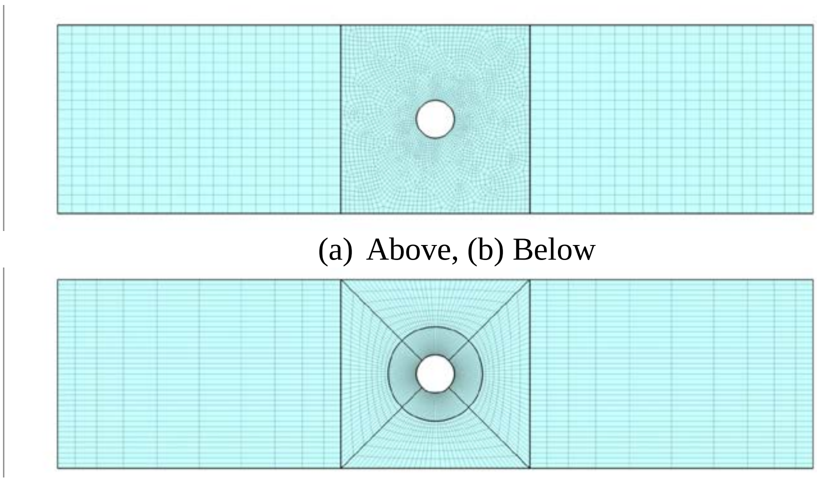

Figure 2. Mesh Configuration for Tensile Strength Scaling Studies (a) Coarse, (b) Fine

n	3.175mm	6.35mm	12.7mm	25.4mm
1	570 ± 43.8	---	---	---
2	500 ± 19.8	438 ± 10.7	---	---
4	478 ± 14.8	433 ± 8.8	374 ± 3.8	331 ± 9.9
8	476 ± 24.1	---	---	332 ± 4.3

Table 3. Failure Stress for Various Laminates $[45/90/\text{-}45/0]_{ns}$. Subject to Fiber Failure [16]

In our study, only a subset of data presented in Table 3 was considered. Mesh sensitivity and parameter sensitivity studies were conducted for the case *n=1* with the D=3.175mm hole. Further studies are being conducted to evaluate these influences for different hole diameters and sublaminate numbers. Three different fiber failure criteria described above were applied. These were CDM, CFV, and CDM+CFV – which stands for CDM with CFV scaled initiation stress. Two failure criteria, namely the CDM and CFV were applied by using the coarse and fine mesh shown in Figure 1. The CDM failure criteria was applied by using two different values of the initiation stress, corresponding to different scaled volume sizes of the test specimens for fiber direction strength testing. Lastly, the CFV criterion was applied with and without simulation of matrix damage. Note that a similar experiment with the simplified CDM criterion cannot be performed because the specimen will be held together by off-axis plies even when the 0° ply fails, since no matrix damage of any kind is accounted for. Table 4 shows the predicted strength vaules for all cases in MPa, as well as the percent difference from the experimental average value of 570 MPa. Note that the coefficient of the variation of the experimental data is 7.6%, which envelopes practically all data in Table 4.

Fiber Failure Criterion	Coarse Mesh	Fine Mesh
CFV, No Matrix Damage	523.7 (-8.21%)	525.1 (-7.8%)
CFV, MIC and Delaminations	555 (-2.3%)	559 (-1.9%)
CDM, X_t = 2400	531 (-6.8%)	---
CDM, X_t = 3116	549 (-3.6%)	560 (-1.75%)

Table 4. Predicted Strength in MPa and Percent Deviation from Experimental Value

However, several observations can be made. Firstly, all predictions are below the average experimental value, which is consistent with the known difficulties in accounting for several effects, contributing to reduced notch sensitivity of composites, namely fiber stress relaxation due to matrix damage and volumetric effect of fiber direction strength. The first row in Table 4 can be considered as a quick screening baseline result, where a statistical failure criterion is applied to elastic stress field in pristine composite without any matrix damage. In this case we only account for one of the two factors responsible for apparent notched strength. These predictions give the most conservative and least mesh dependent values because of simplicity of the approach. The second line in Table 4 shows the predictions by using the exact same statistical failure criterion, but in the presence of matrix cracking and delaminations shown in Figure 2. The increased predicted strength values are the results of significant fiber stress relaxation due to matrix cracking and delamination. The mesh dependence of the results is also within acceptable limits. In the case of progressive fiber failure modeling by using CDM, the results are also acceptably mesh independent, however the influence of the initial strength value X_t is more significant. Numerical studies of combined application of CDM and statistical failure criterion are continuing.

5 Conclusions

A finite element method and software implementing Rx-FEM approach and allowing modeling of complex interactive networks of matrix cracks and delaminations are developed. The regularized formulation preserves the element Gauss integration schema for arbitrary cracking direction, and allows straight forward connections between neighboring plies with different orientations of properties and cracks. A cohesive zone method is used to predict the matrix cracking and delamination evolution. Ply level stiffness, strength, and fracture toughness properties measured in independent experiments are the only properties required for analysis.

Mesh dependence of CFV and CDM methods applied to the prediction of the fiber failure mode is studied and found to be acceptable, i.e. within limits comparable with natural scatter of the experimental data. The influence of initial strength values in the CDM approach was found more significant. Application of statistical scaling to initial strength values in CDM is proposed to mitigate this problem.

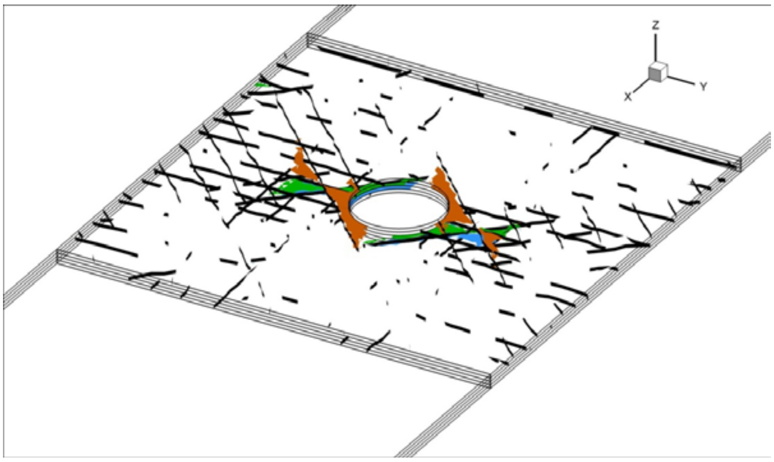

Figure 3. Matrix Cracking and Delamination prior to Final Fiber Failure in all Plies for n=1 and D=3.175mm

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