BUCKLING AND POSTBUCKLING NUMERICAL ANALYSIS OF THIN-WALLED COMPOSITE BEAM WITH OPEN CROSS-SECTION

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Abstract

In the paper a numerical analysis of both critical and post-critical state of a thin-walled channel-section simply supported beam under axial compression. The beam was made of carbon-epoxy symmetrical composite prepared with a pre-preg technology using 8 layers of unidirectional band. The numerical simulations were performed with Finite Elements Method (FEM) in Abaqus and Ansys environment and an analytical-numerical method based on the Koiter theory of conservative systems stability. The value of critical load was estimated according to the Tsai-Wu stress criterion. The performed calculations enabled critical state identification, as well as deformation states and effort analysis of the laminate structure in post-critical state.

1 Introduction

Polymeric composites – the laminates - are currently one of the most developing materials group applied in contemporary thin-walled structures. This yields from an advantageous set of physical-chemical and mechanical properties, particularly high strength in relation to low mass density of the composite material [1, 2]. From a wide group of laminates mainly those having epoxy matrix reinforced with glass, carbon or kevlar fibres find applications. Exceptionally advantageous strength characteristics of a laminate can be obtained by utilization of advanced forming techniques of polymeric composites, such as autoclaving [3].

In the process of creation of complex composite materials optimization of the construction in accordance with its operational loading circumstances has an important meaning. This applies above all to the thin-walled load-carrying structures exposed to the load able to cause a loss of stability of an element or the whole structure. A modern tool in this field is numerical analysis based on Finite Elements Method (FEM) [4-9]. In this article a numerical FEM analysis of a thin-walled composite beam with open cross-section and axial compressive loading was performed. The calculations were carried out within the range of critical, as well as post-critical load for the structure supported with articulated joints. The results of the numerical analysis enabled the estimation of buckling modes and critical load values corresponding to them. Moreover, the post-critical behaviour of the structure became possible. For the purpose of the laminate effort estimation the Tsai-Wu stress criterion [10] was used. The paper contains a comparison of the numerical buckling analysis of the thin-walled open cross-section composite beam, performed both in Abaqus and Ansys software and a verification of these results by the analytical-numerical method.

2 Object and scope of research

The object of the research was the thin-walled beam made of carbon-epoxy composite formed into a channel section with the following dimensions: 80 x 40 x 1.048 mm (the cross-section) and L = 300 mm (the length). The applied material was the HexPly M12 (by Hexcel) epoxy matrix composite reinforced with carbon fibres. The 8-plies laminate was produced with autoclaving technique from unidirectional pre-preg band, in order to get a quasi-isotropic composite with a $[0/-45/45/90]_s$ ply sequence. The strength characteristics of the examined composite were estimated experimentally according to the ISO standard. The structure was supported with articulated joints and compressed axially within two load ranges: below and above critical load. This enabled observation of the structure deformation in post-critical state. In the calculations the *Layup-Ply* composite structures modelling technique, based on four-noded *Shell* type elements was employed [4, 5]. A plane-stress orthotropic material model was defined for the composite. The computations were made within the frame of linear stability analysis allowing for estimation of the critical load value and the corresponding buckling mode. A second stage of the computations was the non-linear numerical analysis, in which geometrical imperfections of the amplitude equal to 1/10 of the beam's wall thickness for the first buckling mode were introduced. The beam was loaded with a force bigger than the calculated critical one. The outcomes enabled an observation of both stress and strain field in the circumstances of post-critical deformation. The conducted numerical calculations were also a preliminary analysis of the composite materials failure. Namely, the numerical model was evaluated in respect of its ability to detect the regions in the structure, where damage occurrence was possible, as a consequence of external loading. In the conducted research for the purpose of the composite effort assessment the Tsai-Wu stress criterion [10] was used.

3 Digitized beam model

The model of the thin-walled beam was digitized with four-noded shell elements of reduced integration: S4R type in Abaqus and eight-noded shell elements Shell99 type in Ansys. In order to model the laminate structure the *Layup-Ply* technique was used – see Fig. 1.

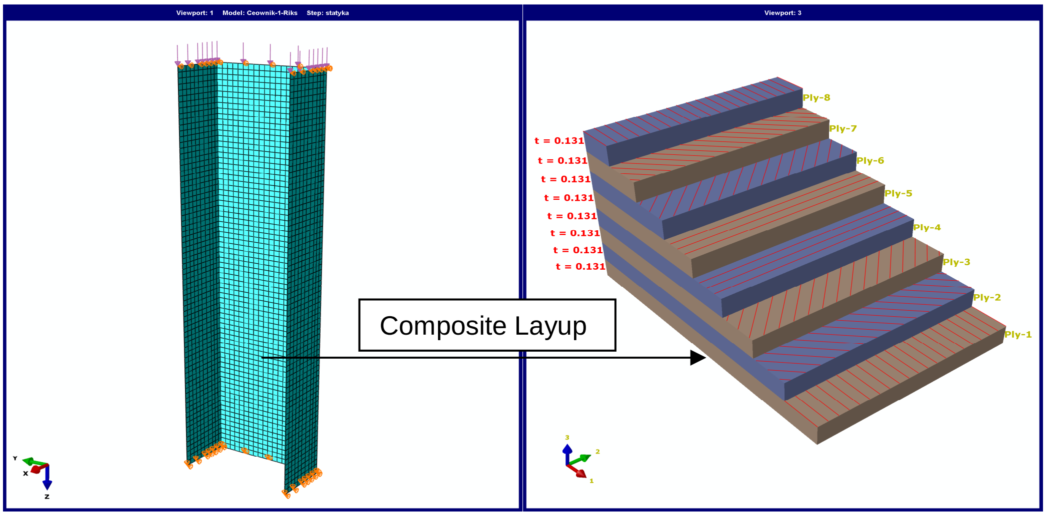

Figure 1. Digitized composite beam model

The properties of the composite were given within the definition of the plane-stress orthotropic material, what allowed for description of the laminate characteristics along particular directions, related to fibre orientation [11, 12]. The scope of the numerical calculations covered also an attempt to evaluate the possibility of the composite damage appearance in critical state according to the Tsai-Wu criterion. This needed an indication of some additional material characteristics, such as: F_{TU} – tensile strength, both in the 0^0 and in the 90^0 direction; F_{CU} – compressive strength in both directions (0^0 and 90^0) and F_{SU} – shear strength in the $\pm45^0$-ply interface. These properties, given in Table 1, were determined in experiments carried out by the authors of this article.

Tensile strength F_{TU} [MPa]		Young modulus E_T GPa]		Poisson ratio ν		Shear strength F_{SU} [MPa]	Shear modulus G [GPa]	Compressive strength F_{CU}[MPa]	
0°	90°	0°	90°	0°	90°	±45°	±45°	0°	90°
1867,2	25,97	130,71	6,36	0,32	0,02	100,15	4,18	1531	214

Table 1. Mechanical properties of the M12/35%/UD134/AS7/300 carbon-epoxy composite.

Boundary conditions of the numerical model, representing articulated support of the beam ends were realized by restraining kinematic degrees of freedom of the nodes placed at the beam ends, as shown in Fig. 2. Axial loading of the model was realized by imposing uniformly distributed concentrated forces at the upper end of the beam.

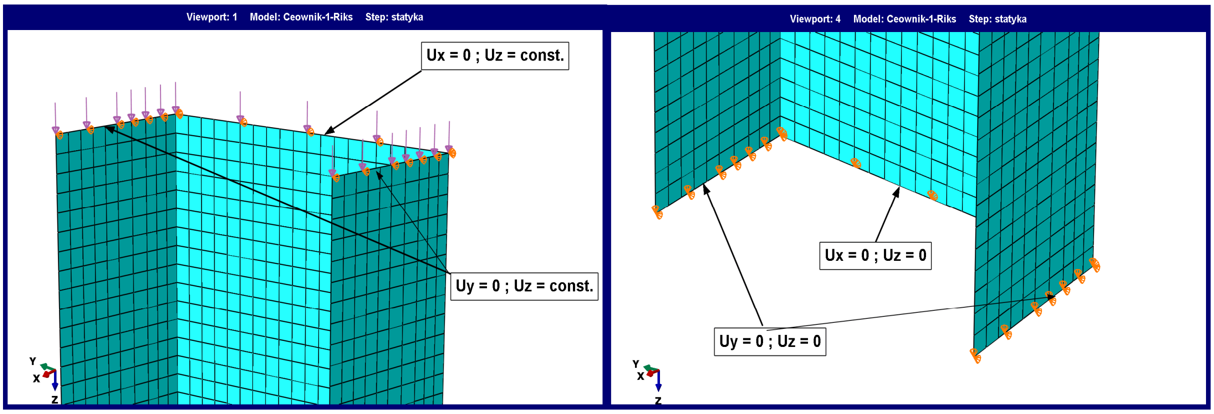

Figure 2. Boundary conditions and loading of the FE beam model

The linear *buckling analysis* procedure was exploited for simulation of the beam behaviour within the critical range. This procedure enabled determination of the critical load, as well as instability mode. Verification of the obtained results was performed by the analytical-numerical method [13-15], based on the general asymptotic Koiter theory of conservative systems stability [16]. Calculations within the post-critical state had a form of static non-linear analysis of the structure behaviour, in which the first buckling-mode imperfections were initiated. In case of the Abaqus computer program, the post-critical analysis, taking into account geometrical nonlinearity was run using the incremental-iterative Newton-Raphson method. In case of the Ansys code, the calculations were conducted in accordance with the *Arc-Length* procedure (Ricks Method). The value of failure load for the laminate material was determined with the Tsai-Wu stress criterion.

4 Results of numerical analysis – discussion

The analysis of the critical state showed a local mode of the thin-walled beam stability loss, manifesting itself by appearance of 2 half-waves on all walls of the channel section – see Fig. 3. Analogous results were obtained with all computational tools, that were used (FEM and analytical-numerical method). The computed values of the critical load for each method are given in Table 2.

Computational tool	Abaqus (FEM)	Ansys (FEM)	Analytical-numerical method
Critical load [N]	2977.2	2946.3	2848.3

Table 2. Values of the critical load for the first buckling mode.

Non-linear calculations enabled the analysis of the structure's deformed shape in post-critical state - up to the failure load (see Fig. 3).

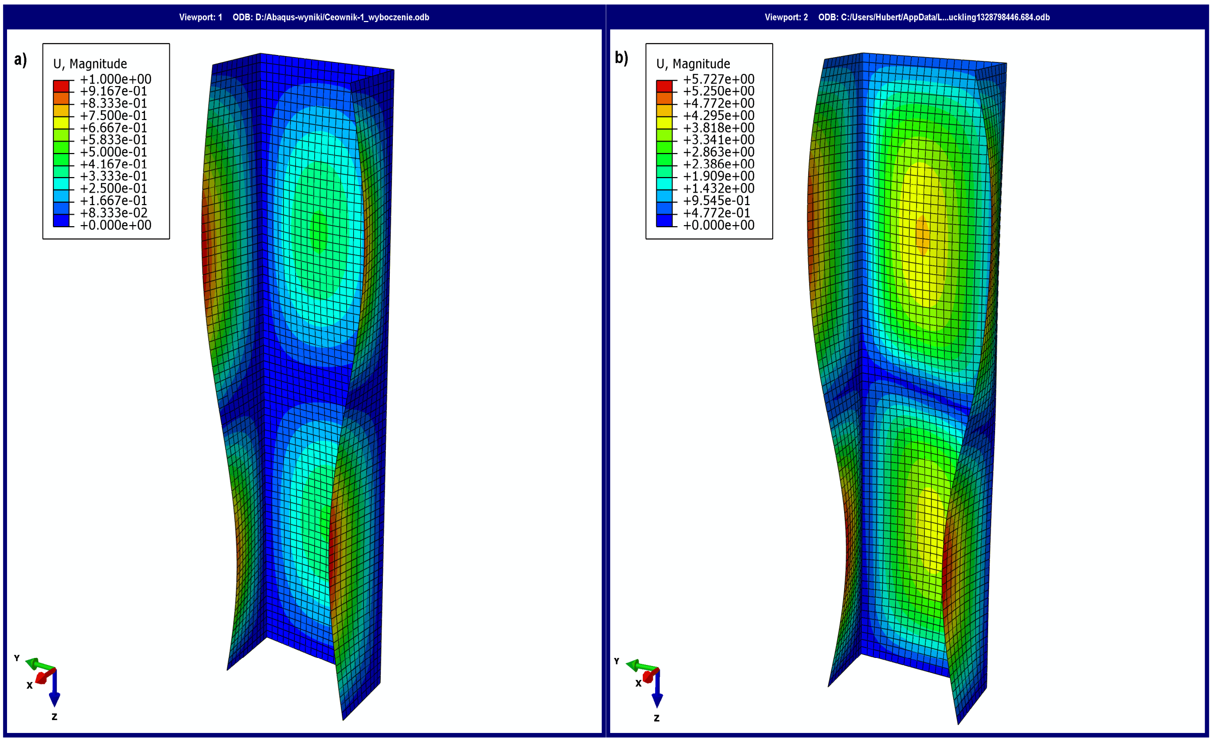

Figure 3. States of the structure's deformation: a) first buckling mode, b) post-critical deformation state

The performed calculations allow for stress analysis in particular plies of the composite under failure load. The failure load was assumed for the laminate material to be the loading force value corresponding to the damage parameter, defined within the Tsai-Wu criterion, equal to 1 (on the scale from 0 to 1) in numerical calculations made with Abaqus. Thus, the loading force causing failure was determined to be P = 10122.5 N, which is 340% of the critical force. The zones of the beam, in which the critical value of the damage parameter was reached determine the structure's regions susceptible to damage, i.e. where the probability of damage is high, at least for some plies. Exemplary stress maps in the composite layers and the Tsai-Wu-criterion map are shown in Fig. 4.

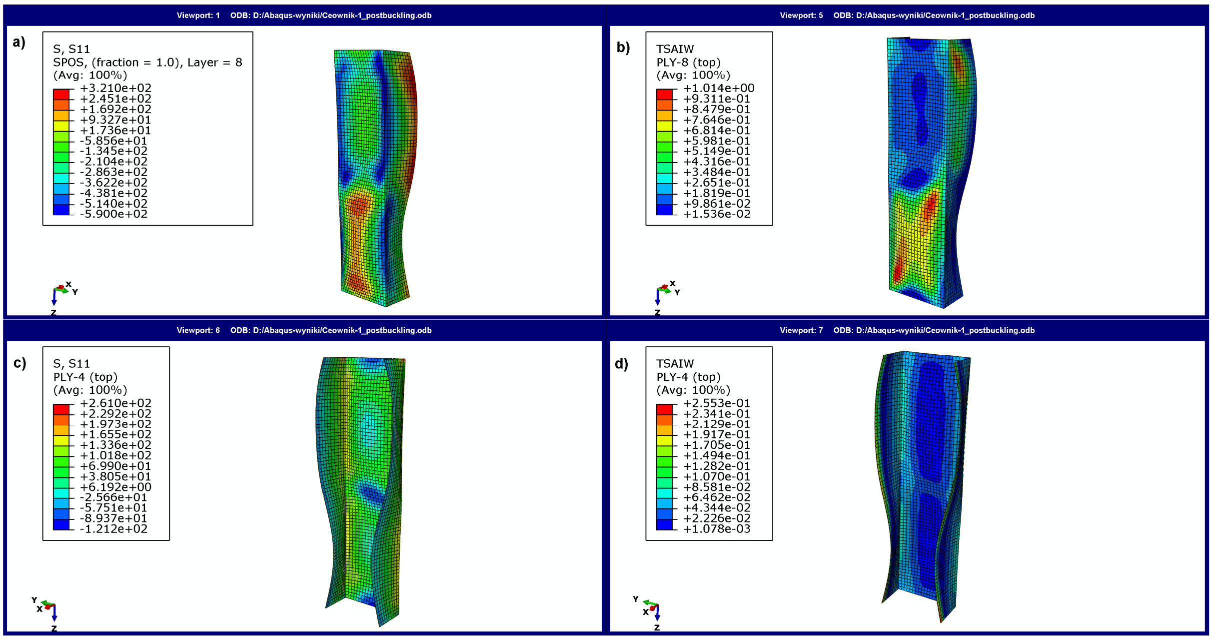

Figure 4. Effort of the composite beam: a,c) stress pattern in plies, b,d) Tsai-Wu criterion fulfilment

Analysis of the stress state and the Tsai-Wu-criterion maps allows for localization of the regions and plies experiencing the highest effort. In the analyzed case the highest stress levels, as well as the damage parameter equal to 1 were obtained in the 8-th ply (Fig. 4a, b), i.e. in the outer layer of the channel section, where the fibre orientation was 0^{o} (parallel to the beam axis). On the contrary, the lowest stress levels and the damage parameter values were observed in the middle ply (No. 4, see Fig. 4c, d), having the 90^{o} fibre orientation angle. The calculated distribution of the composite material effort confirmed the appearance of the maximal effort in the 0^{o}-plies, in which fibres were oriented along the loading direction.

The performed analysis allows for a comparison of the results obtained with different simulation methods (FEM, analytical-numerical method), as well as with different numerical tools (Abaqus, Ansys). In Table 2 values of the critical load gained with the above mentioned methods were collected. Fig. 5 presents a comparison of post-critical force-displacement equilibrium paths for the node placed in the point of the maximal amplitude of deformation of the wider channel section’s wall.

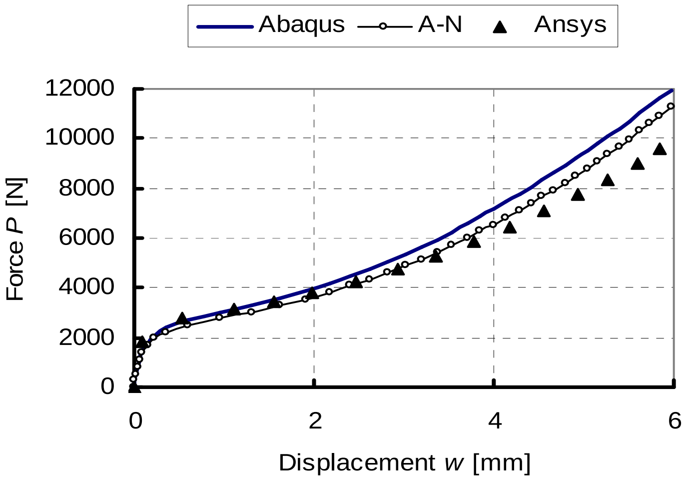

Figure 5. Post-critical force-displacement equilibrium paths of the structure

Analysis of the numerical outcomes leads to the conclusion, that in case of critical state the beam's buckling modes obtained with different simulation methods were compatible. However, the quantitative conformity of the critical force values was at the level of 95.7% (see Table 2). Non-linear analysis showed a convergence of post-critical equilibrium paths of the structure - Fig. 5.

The presented numerical analyses indicate both qualitative and quantitative compatibility of the results obtained with the exploited methods and numerical tools , what confirms the adequacy of the formulated numerical models. The acquired outcomes allow for critical, as well as post-critical state analysis of composite beams subjected to axial compression, by providing wide possibilities of deformation states and material effort observation up to the very moment of failure. This enables identification of the regions particularly jeopardized to laminate damage and lets one determine the level of failure load in connection with the critical force. The analysis of the post-critical equilibrium path makes the structure's stiffness assessment after stability loss possible, taking into account the laminate's ply sequence. Thus, the obtained results deliver a significant information in the process of forming and optimizing composite structure in the context of its loading.

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