A MULTISCALE HYBRID DAMAGE AND FAILURE APPROACH FOR STRENGTH PREDICTIONS OF COMPOSITE STRUCTURES

F. Laurin[1*], C. Carrere[1,2], C. Huchette[1], J.-F. Maire[1]

[1]*Onera – The French Aerospace Lab, BP 72 – 29 avenue de la Division Leclerc, FR-92322 CHATILLON cedex FRANCE*
[2] *LBMS EA 4325 - ENSTA Bretagne, 2 rue François VERNY, FR-29806 BREST Cedex 9 France*
**frederic.laurin@onera.fr*

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Abstract
Although the use of fiber-reinforced composites has spread increasingly in the design of primary structures, the prediction of the behavior and damage until the final failure of structure subjected to complex multiaxial loadings still remains a major problem in the design of composite parts. A multiscale hybrid damage and failure approach which permits to predict accurately the damage and failure of composite structures subjected to complex multiaxial 3D loadings has been proposed. This approach has been compared successfully with test results on laminates subjected to multiaxial in-plane or triaxial loadings extracted from the literature or performed at Onera. After implementation in a finite element code, simulations on open-hole plates subjected to tensile or compressive loadings have also been performed in order to ensure the relevance of the proposed modeling.

1 Introduction
Due to their high specific properties, the use of fiber-reinforced composites has spread increasingly, over the past few years, for the conception of high performance structures in a large range of industrial applications. Nevertheless, due to the specific nature of composite materials, complex failure mechanisms occur and lead to a lack of confidence into the existing strength design tools and to high security margins that induce a loss of competitiveness of composite solutions. The successive World Wide Failure Exercises (WWFE) have aimed to make a state of the art on the predictive capabilities of existing approaches to predict (i) the behavior and failure of different laminated composites subjected to in-plane multiaxial loadings (WWFE–I [1]), (ii) the strength analysis of structures subjected to triaxial loadings (WWFE-II [2]) and (iii) to predict accurately the damage (evolution of the crack density) until the final failure of composite structures (WWFE-III).
In the framework of the three World Wide Failure Exercises, a Multiscale Hybrid damage and progressive failure Model [3,4] has been developed to predict in a correct manner the evolution of the crack density and the final failure of laminated composite structures subjected to 3D complex loading.
This kind of approach permits to predict the final failure of laminate from the knowledge of the thermo-mechanical properties of the unidirectional (UD) ply and also describes accurately the effect of the damaged ply on the macroscopic behavior and on the final failure. Finally, this approach has been implemented into a finite element code in order to predict the strength

of composite structures and especially the strength of open-hole plates subjected to tensile or compressive loadings. The main ideas of the proposed modeling, which permits to predict accurately the damage and the failure of laminates subjected to 3D complex loading, are reminded in the section 2. The section 3 is devoted to the comparisons between the predictions of the proposed approach - evolution of crack density and final failure of laminates – and tests results available in literature [5] or performed at Onera [4]. Finally, the damage and strength predictions obtained through finite element simulations on perforated laminated plates are also presented and discussed.

2 Multiscale hybrid damage and failure approach

The proposed hybrid multiscale damage and failure approach can be divided into five main parts: (i) the method of change of scale which permits to determine from the 3D applied loading to the laminate, the strain field within the constitutive plies, (ii) the determination of micro-damages and their effects on the behavior and strengths of the UD ply, (iii) the constitutive equations to determine the mesoscopic stresses and strains, (iv) the failure criterion to predict the failure of the different plies within the laminate and (v) the damage model to estimate the evolution of the crack density and the associated local delamination until the failure of the specimen. The prediction of the evolution of transverse crack until the final failure is the main improvement as compared to the previous version of the model [3] and is detailed in section 2.4. The other parts of the approach, already presented, will be briefly reminded in the following sections.

2.1 Modeling of micro-damages within the UD ply

One of the main ideas of the proposed hybrid multiscale failure approach consists in introducing, at the mesoscopic scale, the effects of failure occurring at the microscopic scale (which could be fiber/matrix debondings or cracks within the matrix) on the non linear behavior and the strengths of the UD ply.

In order to model these micro-damages (δ_2, δ_3) observed within the ply, the proposed damage law depends on the elastic strain within the matrix (ε_m^e), obtained through a simple method of localization. This scalar damage law assumes that the orientations of the cracks are imposed by the architecture of the material. The proposed micro-damage modeling is thermodynamically consistent, takes into account the unilateral aspect and distinguishes the effects of opened micro-cracks from closed ones. The micro-damage variable (δ_f) represents the premature fiber failures, due to a statistical effect in the fiber strength, leading to fiber/matrix debondings which facilitate the emergence of the first transverse cracks within the UD ply.

2.2 Mesoscopic non linear behavior

In order to predict accurately the final failure of laminates, it is necessary to describe in a correct manner the non linear behavior of the UD plies. A non linear thermo-viscoelastic behavior, defined at the ply scale, is presented in Eq. 1.

$$\sigma = \tilde{C} : (\varepsilon - \varepsilon^{th} - \varepsilon^{ve} - \varepsilon^{p}) \text{ with } \tilde{C} = \tilde{S}^{-1} \quad (1)$$

where $\tilde{C}$ is the apparent stiffness which depends on the mesoscopic damages, σ the stress, ε the total strain, ε^{th} the thermal strain, ε^{ve} the viscoelastic strain and ε^{p} the permanent strain. It is essential to take into account the thermal residual stresses in order to estimate accurately the first ply failure in a laminate. The viscoelastic model permits to predict accurately the failure of $[\pm\theta°]_s$ laminates, to take into account the effects of the loading rate and to estimate the failure during creep or relaxation tests. The proposed viscoelastic law presents two different sources of non linearity: (i) the first non linearity is inherent to the viscosity of the matrix and is described through a non linear function depending on an equivalent stress which distinguishes a deviatoric and a hydrostatic part in order to take into account the effect of a hydrostatic pressure on the mesoscopic behavior and (ii) the second non linearity is due to the coupling between the matrix micro-damages (δ_2, δ_3) and the viscous compliance. Moreover, a permanent strain, modeling the residual strain after unloading, has been introduced in order to describe more accurately the cyclic behavior. This permanent strain is a linear function of the evolution of the matrix micro-damages within the ply.

2.3 Mesoscopic failure criteria

The predictions of the ply failure within a laminate is performed using a multi-criteria, based on Hashin's hypotheses, which distinguishes the ply failure in fiber mode ($f_1^{\pm}$), in in-plane interfiber mode ($f_2^{\pm}$) and in out-of-plane interfiber mode ($f_3^{\pm}$).

2.3.1 Fiber failure criteria

In the fiber failure mode, ply failure in tension and in compression are treated separately. The *tensile fiber failure criterion* (f_1^+) is a maximal strain criterion expressed in Eq. 2, depending on the effective longitudinal tensile strain at failure of the UD ply ($\tilde{X}_{\varepsilon t}$), which is a function of the effective micro-damages (δ_2, δ_3). In fact, it has been experimentally demonstrated that the effective longitudinal tensile strain at failure of the UD ply depends on the state of degradation of the matrix. This coupling permits to obtain conservative final failure predictions, especially for Eglass/epoxy composite materials.

$$f_1^+ = \eta_1 \; \frac{\varepsilon_{11}}{\tilde{X}_{\varepsilon t}\left(\delta_2, \delta_3\right)} = 1 \quad \text{with} \quad \eta_1 = \begin{cases} 1 \;\; \text{si}\; \sigma_{11} \geq 0 \\ 0 \;\; \text{si}\; \sigma_{11} < 0 \end{cases} \tag{2}$$

The longitudinal compressive ply failure is due to fiber kinking. The *compressive fiber failure criterion* (f_1^-) could be considered as an energy criterion (reported in Eq. 3) expressed in the fracture planes (1, 2) or (1, 3). The variable $\tilde{Y}_{1-}^0$ represents the onset of the compressive fiber failure criterion (initially equal to 1) which depends on the micro-damages (δ_f, δ_2, δ_3) within the ply. Again, the state of degradation of the matrix has a strong influence on the apparent compressive strength. The g_n^{1-}, g_t^{1-} are material parameters which have to be identified. It is worth mentioning that the coupling between the different failure mechanisms is due to the effects of the micro-damages on the effective strengths of the UD ply.

$$f_1^- = \frac{\underset{i=\{2,3\}}{\text{Max}}\left(\sqrt{g_n^{1-}\left(\sigma_{11} - \sigma_{ii}\right)^2 + g_t^{1-}\tau_{1i}^2}\right).\varepsilon_{11}^-}{\tilde{Y}_{1-}^0(\delta_f, \delta_2, \delta_3)} = 1 \tag{3}$$

2.3.2 Interfiber failure criteria

In the interfiber failure modes, the in-plane failure mechanisms are distinguished from the out-of-plane ones. Moreover, ply failure in tension and in compression are treated separately. The failure criteria for the *in-plane tensile interfiber ply failure* (f_2^+) and for *the out-of-plane tensile interfiber ply failure* (f_3^+) are stress criteria expressed in Eq. 4.

$$f_i^+ = \frac{Y_{i+}^{eq}}{\tilde{Y}_{i+}^0} = 1 \text{ with } \begin{cases} Y_{i+}^{eq} = {}^t\sigma : F^{i+} : \sigma \\ \tilde{Y}_{i+}^0 = Y_{i+}^0 (1-\delta_f) \end{cases} \text{ for } i = (2,3) \quad (4)$$

$$\begin{cases} F_{22}^{2+} = \eta_2 / \tilde{Y}_t(\delta_2)^2, \, F_{23}^{2+} = 1/\tilde{S}_{23}(\delta_2)^2 \text{ and } F_{12}^{2+} = 1/\tilde{S}_{12}(\delta_2)^2 \\ F_{33}^{3+} = \eta_3 / \tilde{Z}_t(\delta_3)^2, \, F_{23}^{3+} = 1/\tilde{S}_{23}(\delta_3)^2 \text{ and } F_{13}^{3+} = 1/\tilde{S}_{13}(\delta_3)^2 \\ \text{The other components of the tensors } F^{2+} \text{ and } F^{3+} \text{ are null} \end{cases} \quad (5)$$

where F_i^+ are the associated failure tensors (Eq. 5) and depend on the effective in-plane and out-of-plane tensile strengths ($\tilde{Y}_t, \tilde{Z}_t$), and on the in-plane and out-of-plane shear strengths ($\tilde{S}_{12}, \tilde{S}_{23}, \tilde{S}_{13}$). Again, the matrix micro-dommages (δ_2, δ_3) tend to decrease the apparent mesoscopic strengths. The onsets of the fiber failure criterion noted $\tilde{Y}_{i+}^0$ for i=(2,3) are also function of the premature fiber failure micro-damage (δ_f). The activation indexes η_i for i=(2,3) permit to distinguish the failure in tension from the one in compression.

The failure criteria for the in-*plane compressive interfiber ply failure* (f_2^- in Eq. 6) and for the *out-of-plane compressive interfiber ply failure* (f_3^- in Eq. 7) are energy criteria, expressed respectively in the fracture planes (3,2) and (2,3) making an angle ±α with respect to the axis 2 and 3.

$$f_2^- = \frac{\underset{\theta=\{-\alpha,+\alpha\}}{\mathrm{Max}} \left(g_n^{2-} \left| \tau_n^{(3,2)\theta} \gamma_n^{(3,2)\theta} \right| + g_t^{2-} \left| \tau_t^{(3,2)\theta} \gamma_t^{(3,2)\theta} \right| \right)}{\tilde{Y}_{2-}(\delta_f, \delta_2, \delta_3)} = 1 \quad \text{if} \quad \sigma_2 < 0 \quad (6)$$

$$f_3^- = \frac{\underset{\theta=\{-\alpha,+\alpha\}}{\mathrm{Max}} \left(g_n^{3-} \left| \tau_n^{(2,3)\theta} \gamma_n^{(2,3)\theta} \right| + g_t^{3-} \left| \tau_t^{(2,3)\theta} \gamma_t^{(2,3)\theta} \right| \right)}{\tilde{Y}_{3-}(\delta_f, \delta_2, \delta_3)} = 1 \quad \text{if} \quad \sigma_3 < 0 \quad (7)$$

where $\tau_n^{(2,3)\theta}$, $\tau_t^{(2,3)\theta}$, $\gamma_n^{(2,3)\theta}$, $\gamma_t^{(2,3)\theta}$ and $\tau_n^{(3,2)\theta}$, $\tau_t^{(3,2)\theta}$, $\gamma_n^{(3,2)\theta}$, $\gamma_t^{(3,2)\theta}$. are respectively the normal and tangential shear stresses and strains in the fracture planes (3,2) and (2,3) making an angle ±α with respect to the axis 2 and 3. The onsets of the failure compressive criteria $(\tilde{Y}_{2-}, \tilde{Y}_{3-})$ depend on the micro-damages (δ_f, δ_2, δ_3). $g_n^{2-}, g_t^{2-}, g_n^{3-}, g_t^{3-}$ are material parameters. The angles of the fracture planes in in-plane and out-of-plane interfiber compression are assumed to be equal to α=45° for the sake of simplicity.

2.4 Transverse crack modeling

The proposition of a damage law able to predict accurately the crack density and the associated local delamination rate is the major improvement introduced into the present hybrid multiscale damage and failure approach as compared to the model proposed in the WWFE-II. These local delaminations have an important effect on the saturation of transverse cracking. This is the reason why, two damage variables are taken into account into the present model: $\bar{\rho}$ which is the normalized crack density (i.e. the crack density multiplied by the thickness of the considered ply) and $\bar{\mu}$ which is the delamination rate (i.e. the total length of local delaminations divided by the total length of the interface).

In order to develop a mesoscopic damage law, it is first necessary to identify the effect of the damage on the stiffness of the damaged ply. The identification of the effect of the damage (transverse cracks and associated local-delamination) is usually determined through finite element simulations performed on unit cells. The effects of damage on the effective compliance tensor ($\Delta \underset{\approx}{\tilde{S}}(\bar{\rho},\bar{\mu})$) are defined in the present model as reported in Eq. 8, where the effect tensors $\underset{\approx}{H}^{a,b,c,d,e}$ are diagonal and identified through the FE simulations.

$$\Delta\tilde{S}(\bar{\rho},\bar{\mu}) = \bar{\rho}.H^{a} + \frac{\bar{\mu}}{1-\bar{\mu}}.H^{b} + \bar{\rho}^{2}.H^{c} + \bar{\rho}.\frac{\bar{\mu}.}{1-\bar{\mu}}.H^{d} + \frac{\bar{\mu}}{(1+\bar{\rho}-\bar{\mu})^{2}}.H^{e} \qquad (8)$$

The damage kinetics are given by the Eq. 9, where *h* is the thickness of the ply, (y_I, y_{II}, y_{III}) are the thermodynamic forces depend on the effective compliance $\Delta \underset{\approx}{\tilde{S}}(\bar{\rho},\bar{\mu})$, ($y_I^0, y_{II}^0, y_{III}^0$) are the thresholds of the damage and (α_I, α_{II}, α_{III}, n) and (a,b) are material parameters. $<>_+$ are the Macauley brackets.

$$\begin{cases} \bar{\rho} = h\ (1-\bar{\mu})\left[\alpha_I \left\langle y_I - y_I^0 \right\rangle_+^n + \alpha_{II} \left\langle y_{II} - y_{II}^0 \right\rangle_+^n + \alpha_{III} \left\langle y_{III} - y_{III}^0 \right\rangle_+^n \right] \\ \bar{\mu} = \left\langle a_h\ \bar{\rho}^2 + h\ b_h\ \bar{\rho} \right\rangle_+ \end{cases} \qquad (9)$$

It is worth mentioning that the local delamination is an explicit function of the crack density. Moreover, the local delamination tends to slow down the kinetics of the transverse cracks until the saturation and permits to describe accurately the available experimental results, especially for the Eglass/epoxy materials. It is worth mentioning that the kinetics of the transverse cracks and of the associated local delamination are linked to the thickness of the ply. It has been experimentally demonstrated that the threshold of the damage and the evolution of the crack density are a function of the thickness of the ply. Experimental observations lead us to consider that the onset of transverse cracking needs to fulfill a mixed criterion based on a competition between a “stress” criterion and an “energy” criterion. Both conditions are complementary and necessary and are reported in Eq. 10.

$$y_I^o = \max\left[\frac{y_I^{oE}}{h}, y_I^{o\sigma}\right],\ y_{II}^o = \max\left[\frac{y_{II}^{oE}}{h}, y_{II}^{o\sigma}\right] \text{ and } y_{III}^o = \max\left[\frac{y_{III}^{oE}}{h}, y_{III}^{o\sigma}\right] \qquad (10)$$

where $(y_I^{oE}, y_{II}^{oE}, y_{III}^{oE})$ are the thresholds associated to the “energy” criterion which have to be identified (linked to the toughness of the material) and $(y_I^{o\sigma}, y_{II}^{o\sigma}, y_{III}^{o\sigma})$ are the thresholds of the stress criterion which are the thermodynamical forces calculated when the in-plane interfiber criterion (f_2^+) is first fulfilled.

Finally, the degradation of the mechanical properties associated to the catastrophic failure modes (it means fiber failure and interfiber compressive failure) are function of the value of the different failure criteria (detailed in section 2.3) and the kinetics of these damage variables lead to softening behaviors (sudden and huge decrease of the mechanical properties of the failed ply).

2.5 Strength predictions of composite structures

This hybrid damage and failure approach has been implemented into the finite element code ZeBuLoN, in order to predict the strength of composite structures subjected to bending or to predict the failure of open-hole plates subjected to tensile or compressive loadings with different diameters of the hole and different thicknesses of the constitutive plies. Between the plies, cohesive zone elements have been introduced to describe the delamination. This model, based on [6], is defined with two entities: a strength (σ_{max}) and a toughness (G_c). The original point of the proposed approach consists in the introduction of a coupling between the apparent strength of the interface ($\tilde{\sigma}_{max}$) and the local delamination rate $\bar{\mu}$ observed at the tips of the transverse cracks.

It is worth mentioning that in order to avoid localization problems, two different numerical methods have been selected among the existing approaches found in the literature: (i) the reformulation of the problem in a non-local framework associated with (ii) the introduction of a delay effect in the mesoscopic damage.

3 Comparisons with available experimental data

The objective of the present study consists in proposing an approach able to predict the damage and the failure of laminated composite structures for the 39 test cases proposed in the framework of the three “World Wide Failure Exercises” for the 10 different materials (Eglass/Epoxy and Carbon/Epoxy). The proposed hybrid multiscale failure approach necessitates the identification of an important number of coefficients. Nevertheless, the number of parameters to be identified could be drastically reduced thanks to mechanical considerations, such as the assumption of transverse isotropy of the UD ply. A procedure of identification of the proposed hybrid failure approach has been established, by using few simple tests at the different scales of the problem. The following comparisons with the available experimental data have permitted to validate both the proposed approach and the associated identification procedure.

The first exercise WWFE-I was dedicated to the predictions of the failure of UD ply and laminates subjected to in-plane multiaxial loading. The proposed approach has already been validated through the comparisons with the experimental data.

The second exercise WWFE-II was dedicated to the predictions of the failure of UD ply and laminates subjected to (i) in-plane loading combined to hydrostatic pressure or to (ii) triaxial complex loading.

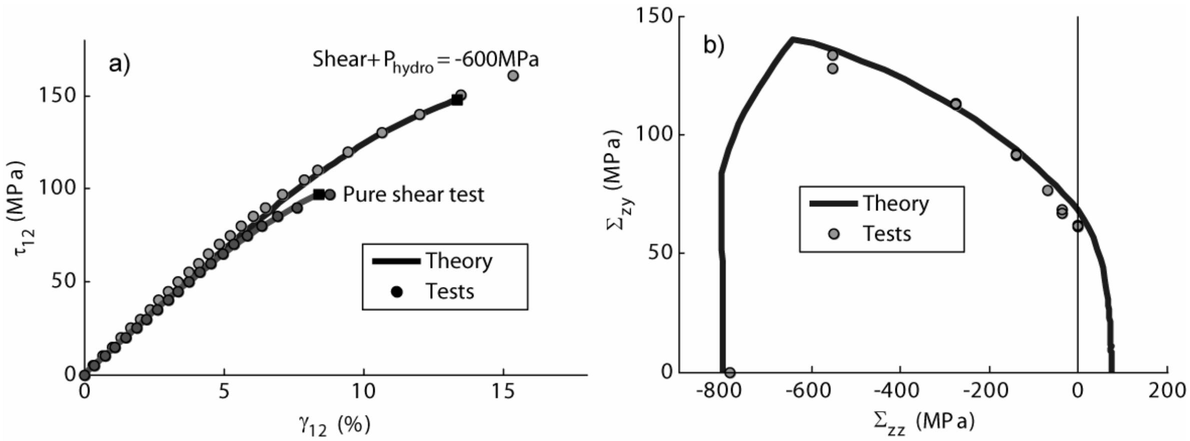

Figure 1. Comparisons between the test results and the simulations for (a) the behavior of a T300/PR319 UD ply subjected to combined in-plane shear loading and hydrostatic pressure and for (b) the macroscopic failure envelope of a IM7/8551-7 [0°/90°]$_s$ laminate in the macroscopic stress plane (Σ_{zz}, Σ_{yz}).

The Figure 1.a presents the predicted influence of the hydrostatic pressure on the in-plane shear behavior up to the rupture of a T300/PR319 UD ply. It is observed that (i) the hydrostatic pressure induces a decrease of the non linearity of the mesoscopic behavior which tends to the elastic behavior and (ii) that the effective in-plane shear strength increases with hydrostatic pressure which tends to delay the onset of micro-damages. The Figure 1.b presents the predicted macroscopic failure envelope of a IM7/8551-7 $[0°/90°]_s$ laminate in the stress plane (Σ_{zz}, Σ_{yz}). An important reinforcement of the apparent out-of-plane shear strength is observed for combined out-of-plane compression and shear loading because the out-of-plane compression tends to delay the onset of the micro-damages within the UD plies.
The third exercise WWFE-III was dedicated to (i) the prediction of the evolution of the transverse cracks in laminate and their effects on the macroscopic behavior until the final failure and (ii) to the strength prediction of composites structures (laminates plates subjected to four-point bending loading, or open-hole plate subjected to uniaxial tension and compression tests). The Figure 2 presents a) the evolution of the normalized crack density as a function of the applied loading and b) the evolution of the length of the local delamination associated to the crack density on different T700GC/M21 laminates $[0_2°/90_n°/0_2°]$ with n={1,2,4,6} subjected to uniaxial tensile loading. The test results performed at Onera [4], reported in Figure 2, are in good agreement with predictions of the present approach.

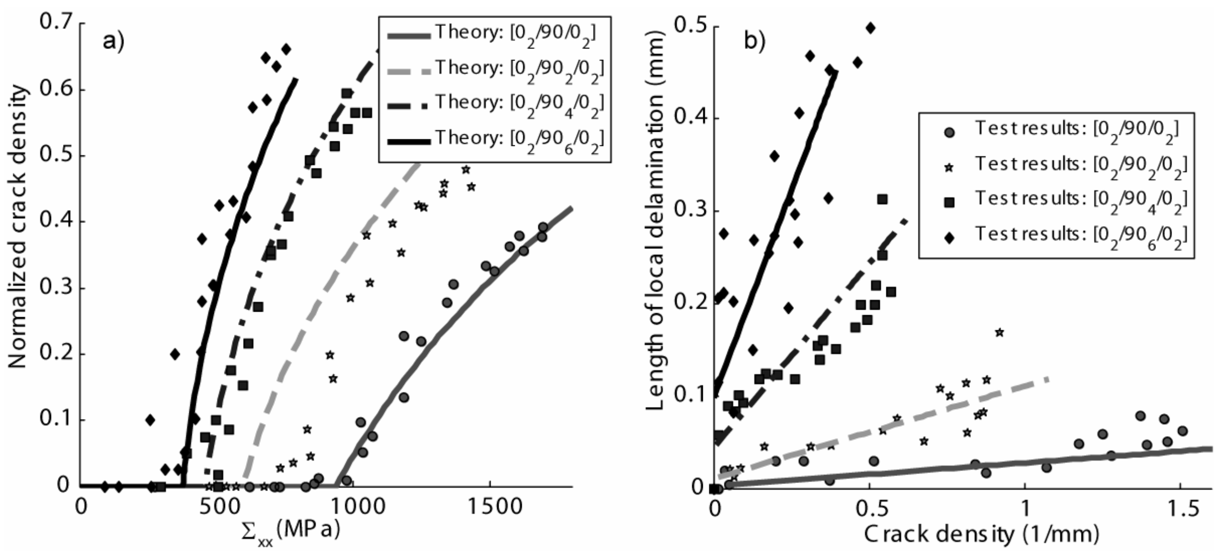

Figure 2. Uniaxial tensile tests on $[0°/90_n°]_s$ T700GC/M21 laminates with n=(0.5, 1, 2, 3) a) measured and predicted normalized crack density in 90° ply as a function of the applied macroscopic stress and b) measured and predicted length of local delamination as a function of the crack density in 90° ply.

The predictions of the damage patterns and of the failure loads of IM7/8552 quasi-isotropic $[45_4°/90_4°/-45_4°/0_4°]_s$ open-hole plates with different diameters of hole have also been studied in the framework of the WWFE-III. The Figure 3 presents the damage pattern (transverse cracks and delamination) just prior the failure of the open-hole plate with a diameter of hole equal to 3.15mm and subjected to uniaxial tension loading. A coupling between the transverse cracks and the delamination, experimentally observed, is accurately predicted with the proposed approach.

4 Conclusions

A hybrid multiscale damage and failure approach, which permits to predict accurately both damage and the failure of laminates subjected to complex 3D loadings, has been proposed.

The obtained predictions has been compared successfully with test results on laminates subjected to complex 3D loadings extracted from the literature or performed at Onera. Finally, this approach has been implemented into a FE code to predict the strength of composite structures. The damage and failure predictions are promising. The comparisons with experimental data, which should be performed soon in the framework of the WWFE-III, should permit to evaluate the predictive capabilities of the proposed approach.

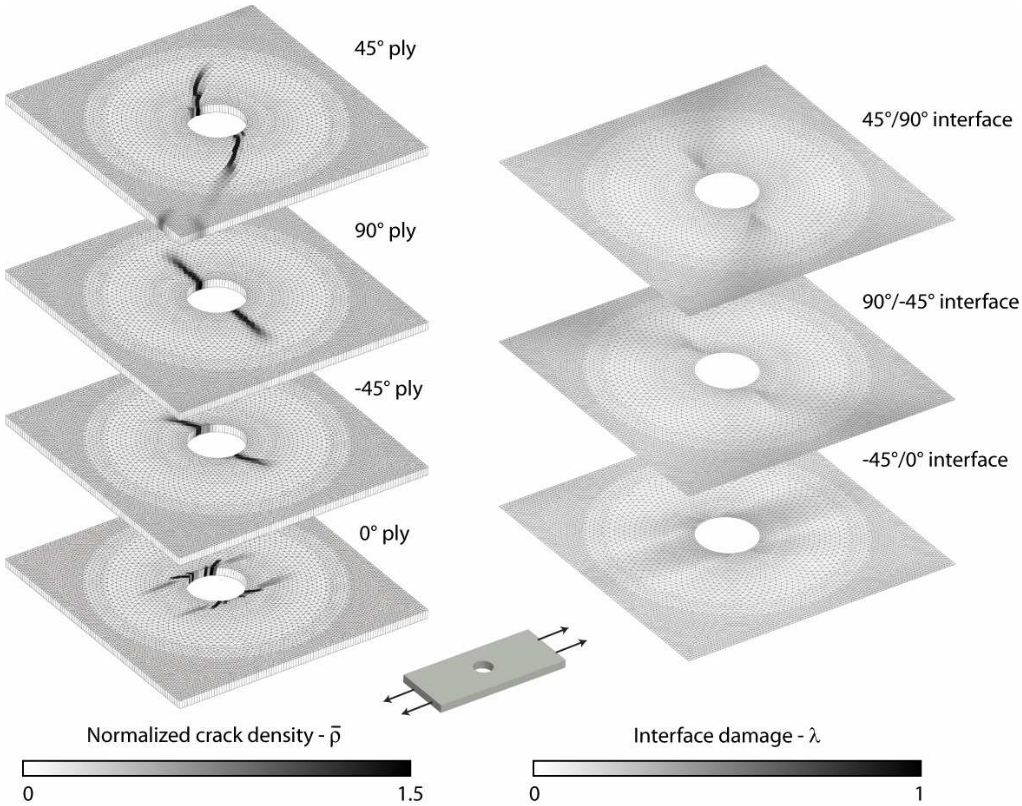

Figure 3. Patterns at failure load of in-plane mesoscopic damage in each ply and delamination at each interface in the $[45°/90°/-45°/0°]_s$ IM7/8552 open-hole plate with d=3.175mm.

References

[1] Soden P.D., Hinton M.J. Kaddour A.S. A comparison of the predictive capabilities of current failure theories for composite laminates. *Composites Science and Technology*, **58**, pp. 1225-1254, (1998).

[2] Kaddour A. S., Hinton M. J. 3D properties of unidirectional composite laminae and test problems used in the WWFEII. *Journal of Composite Materials*, in press (2012).

[3] Carrere N., Laurin F., Maire J.-F. Micromechanical based hybrid mesoscopic 3D approach for non-linear progressive failure analysis of composite structures. *Journal of Composite Materials*, accepted (2012).

[4] Laurin F., Carrere N., Huchette C., Maire J.-F. A multiscale hybrid damage and failure approach for strength predictions of composite structures. *Journal of Composite Materials*, accepted (2012).

[5] Carrere N., Laurin F., Maire J.-F. Micromechanical based hybrid mesoscopic 3D approach for non-linear progressive failure analysis of composite structures - part B: comparison with experimental data. *Journal of Composite Materials*, accepted (2012).

[6] Tvergaard V. Effect of fibre debonding in a whisker-reinforced metal. *Materials Science and Engineering A*, **125**, pp. 203-213, (1990).