# DEVELOPMENT OF A FATIGUE MODEL FOR 3D WOVEN POLYMER MATRIX COMPOSITES BASED ON A DAMAGE MODEL

C. Rakotoarisoa[1,2*], F. Laurin[1], M. Hirsekorn[1], J.-F. Maire[1], L. Olivier[2]

[1]*Onera – The French Aerospace Lab, F – 92322 Châtillon*
[2]*Snecma Villaroche, F – 77550 Moissy-Cramayel*
** carole.rakotoarisoa@onera.fr*

**Keywords:** fatigue, damage model, woven, cumulative law

## Abstract

*A new fatigue model for woven polymer matrix composites (PMC) is presented, based on a combination of a macroscopic static damage model and a cumulative matrix damage law. The proposed damage model allows for predicting fatigue lifetime for a large range of complex load cases, except for spectral loadings. Tensile tests at 0°, 90° and 45° with respect to the warp direction performed on carbon/epoxy woven interlock materials are used to identify the static and fatigue parameters of the model. The predictions of the fatigue lifetime for different configurations agree well with available test results. The aim is to apply the present approach to predict the lifetime of industrial woven PMC structures.*

## 1 Introduction

Because of their very good mechanical properties – improved through-thickness elastic properties, resistance to delamination and to impact damage – and the simplicity of the manufacturing process compared to laminates, composites with 3D interlock woven reinforcements are more and more used in industrial applications, and are exposed to increasingly severe conditions and for longer lifetimes. This is the case for some aircraft engine components, such as fan blades, which are exposed to a very high number of engine cycles (start, take-off, flight, landing and shut down). However, modeling the behavior of interlock woven composites is still a scientific challenge, and consequently large safety factors are currently used in the design aeronautical structures. This difficulty comes from the complexity and the multiplicity of the damage mechanisms involved during both static and cyclic loadings. Moreover, the study of fatigue behavior and fatigue lifetime of these woven composites is still a key point in the design of engine components. Special attention has been paid to minimizing the computational costs in order to transfer this kind of approach to engineering design offices.

Fatigue models have been developed for metal structures since the 1840s. However, there are several fundamental differences between PMC and metals: for example, there is no visible crack for almost the complete lifetime in metals, whereas cracks (at micro and mesoscopic scale) appear very early in the life of composite structures. For that reason, the fatigue methodologies developed and validated for metal structures are not suitable for composite structures.

Several fatigue models for composite structures already exist and can be found in the literature. The review of Degrieck and Van Paepegem [1] proposes a classification of the fatigue models in three major categories: *(i)* fatigue life models, which do not take into account damage mechanisms and are empirically related to experimental data; *(ii)* phenomenological models developed to predict residual stiffness and strength; and finally *(iii)* progressive damage models, which describe the material behavior, predict residual stiffness and strength, and can be used in Finite Element simulations to study industrial structures. Since our aim is the application to industrial composite structures, a progressive damage approach has been chosen.

Lemaitre *et al.* [2] developed an incremental fatigue model designed for PMC with a detailed description of damage mechanisms. However, every single loading cycle is modeled, and therefore computation costs are very high. Moreover, this model has been developed for unidirectional laminates and not for woven composites. Hochard *et al.* [3] developed a fatigue damage approach as a combination of a static damage model and a cumulative damage evolution law. With their model, there is no need of modeling every single cycle. Thus, computational costs are lower than for the incremental model, although to some extent, damage mechanisms are still taken into account. However, this model is designed for 2D woven laminates and not for 3D woven interlocks. Marcin *et al.* [4] studied the behavior of interlock woven PMC under static loadings. A damage model (Onera Damage Model, ODM) has been developed specifically for these woven composites, taking into account their particular damage mechanisms.

In this paper, a damage model able to predict fatigue lifetime of components made of interlock woven PMC is presented. It is based upon ODM, but extended to fatigue load cases following the methodology proposed in [3]. The fatigue model is finally compared to available experimental data (static and fatigue test cases).

## 2 Modeling

### *2.1 Static model review*

Modeling interlock woven PMC can be done at different scales of the composite. Some use models at the mesoscale [5-7], which allows for a precise description of the damage mechanisms. However, this kind of approach is computationally expensive and can thus only be used to simulate elementary specimens. One of the main advantages of macroscopic models [3, 8] consists in being able to model complex industrial structures.

The Onera Damage Model (ODM) has been developed at Onera in order to model damage in woven interlock composites. It is able to describe macroscopic static behavior taking into account the effect of damage and to predict final failure of the specimen (see Figure 1). ODM has been developed for both ceramic and polymer matrix composites [4, 8, 9]. The present study is dedicated to PMC. In this kind of material, damage mechanisms are complex and multiple. Moreover, due to the high contrast between fiber and matrix properties, damage is mainly oriented by the microstructure. In the model based on continuum damage mechanics, these damages are described using damage variables which describe the effects of damage on the behavior in the three main directions of the woven composite. Experimental studies have demonstrated that out-of-plane cracks appear also during in-plane tensile tests. This phenomenon is specific to woven composite because of their architecture and is called in the following the “in-plane/out-of-plane” coupling. Furthermore, the model takes into account the unilateral character of damage, *i.e.*, it distinguishes an active state of the damage, when cracks are opened, from a passive state of the damage, when they are closed due to local compression loading. The change from one state to the other is not instantaneous, because not all cracks (which are not perfectly parallel) are closed at the same time. Therefore, the deactivation index evolves continuously upon crack closure [8].

Another cause of the non-linear behavior of PMC is due to the viscosity of the matrix. In order to take into account the time-dependence of the matrix behavior, a spectral model of viscoelasticity is used [9]. Matrix viscosity also plays an important role in predicting the behavior and the failure of PMC during creep/relaxation tests.

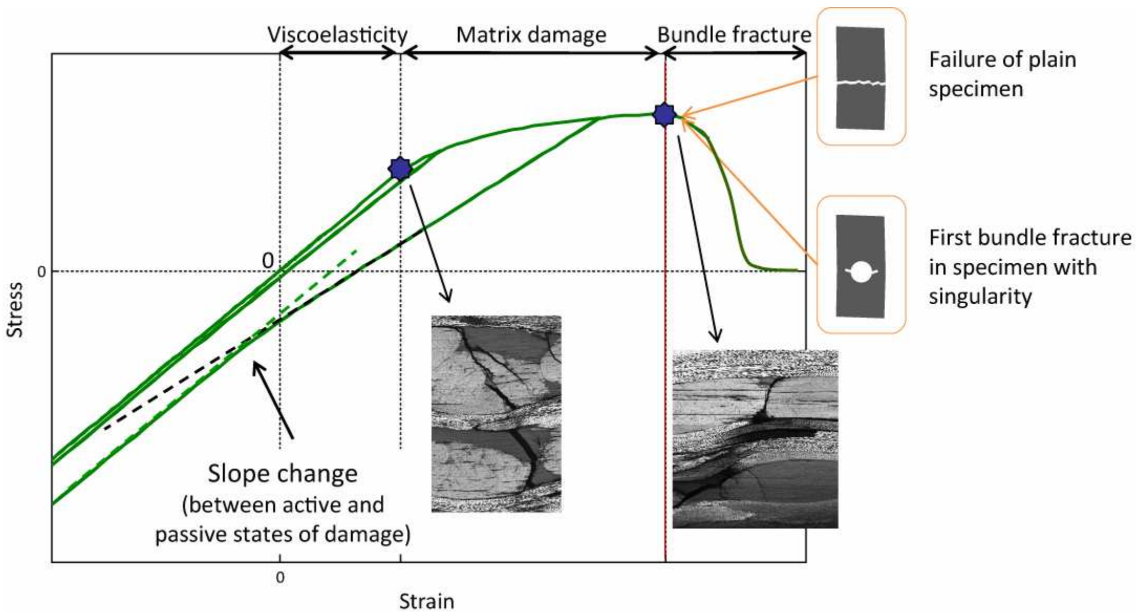


**Figure 1.** Macroscopic behavior of interlock woven PMC

In the case of a plain specimen with homogeneous stress and strain fields, fiber bundle fractures are catastrophic. However, in a structure presenting geometrical singularities (such as a hole), bundle fractures occur due to stress concentrations, but do not induce the final failure of the structure. Progressive bundle fracture is thus described by a softening law.
In this model, there are two kinds of damage variables: *(i)* those linked to non-softening behavior, called matrix damage variables ($d_1^m$, $d_2^m$ and $d_3^m$), which include both matrix cracks and isolated fiber fracture; and *(ii)* those linked to softening behavior, called bundle fracture variables, which include both bundle fracture (variables $d_1^f$, $d_2^f$) and coalescence of debonded zones between bundles and matrix (variable $d_3^f$), similar to delaminations in laminates. The evolution laws of these different damage variables are written as:

$$d_i = d_{c(i)} \cdot (1 - \exp\left( - \left( \frac{\left\langle \sqrt{y_i} - \sqrt{y_{0(i)}} \right\rangle_+}{\sqrt{y_{c(i)}}} \right)^{p_i} \right)) \tag{1}$$

where $d_{c(i)}$ are the saturation points of damages and $y_i$ the corresponding thermodynamic forces. $y_{0(i)}$ are the damage thresholds. $\langle\rangle_+$ are the Macaulay brackets and $p_i$ and $y_{c(i)}$ are model parameters linked to the kinetics of damage.

### *2.2 Behavior under fatigue loads*

ODM describes accurately the static behavior of woven PMC but it needs to be extended to fatigue loading. This extension requires a modification of the previous version of static ODM [9], consisting in *(i)* a new formulation dissociating the residual strain caused by damage from the mechanical strain driving the damage, and *(ii)* a new description of the evolution of matrix damage and fiber bundle fracture. The formulation of the proposed static and fatigue model becomes more logical from a physical point of view and presents also the advantages to be easier to identify than the previous version.

Experimental analysis has shown that the same damage mechanisms occur in woven interlock PMC during static and fatigue loads [10, 11]. The only difference between static and fatigue mechanisms is the damage kinetics. Thus, the proposed fatigue law includes the description of the evolution of matrix damage during cyclic loading into ODM. Nevertheless, damage does not directly depend on the number of cycles. The presented law describes the evolution of damage ensuring the continuity of the damage kinetics. As in the works of Hochard and Thollon [3], the damage evolution law is divided into three main blocks. The first block permits to introduce a saturation value of the damage, usually observed during fatigue loading. The second block takes into account the effect of the load amplitude on the kinetics of damage. Finally, a third block describes the influence of the maximum load applied during a cycle. With these two last blocks, all kinds of fatigue loading can be described, even creep load cases. The only exception is spectral loading, in which all cycles have a different load evolution. The evolution under fatigue loading of the three matrix damage variables is thus given by:

$$\frac{\partial d^{(m)}_{i\,total}}{\partial N} = \left(d^{Fatigue}_{c(i)} - d^{(m)}_{i\,total}\right)^{\gamma_i} \left(1 + \left(\frac{\Delta y_i}{\Delta y^{Fatigue}_{0(i)}}\right)^{\beta_i}\right) \left(\frac{\left\langle y^{(m)}_{(i)max} - y^{Fatigue}_{0(i)} \right\rangle_+}{y^{Fatigue}_{c(i)}}\right)^{\delta_i} \tag{2}$$

$$\Delta y_i = y^{(m)}_{(i)max} - y^{(m)}_{(i)min} \quad \text{with} \quad i = \{1,2,3\} \tag{3}$$

where $d^{(m)}_{total}$ is the total cumulated matrix damage (static and fatigue), N the number of cycles, $y_{min}$ and $y_{max}$ respectively the minimum and maximum driving forces. $d_c^{Fatigue}$ is the saturation point of matrix damage for fatigue loads (usually assumed to be equal to the saturation point in the static case), $y_0^{Fatigue}$ is the fatigue damage threshold under which no damage can appear. $\gamma$, $\beta$, $\delta$, $\Delta y_0^{Fatigue}$ and $y_c^{Fatigue}$ are model parameters.

The damage evolution depends on driving forces [3, 12] (also called thermodynamic forces) instead of the stress tensor. This leads to a scalar (instead of a tensor) formulation of fatigue load, which is easier to analyze and to generalize to multiaxial loading. The matrix damage driving forces for static loads are also assumed to drive the matrix damage during fatigue loads.

The fibers are assumed to be insensitive to fatigue loads [13]. However, we can easily understand that the higher the cumulated matrix damage, the higher the load transfers to the fiber bundles. Consequently, fiber bundle fracture variables have the same kinetics as in the case of static loading, but an influence of the matrix damage on bundle fracture is introduced (this influence is available for both static and fatigue loadings).

*2.3 Model strategy*

The strategy of using the fatigue model to calculate lifetime and residual strength of woven PMC components is presented in Figure 2. The input parameters are the material properties, the maximum fatigue load, the loading ratio, the maximum number of cycles and, as fatigue loadings are divided in groups of cycles, the number of cycles in one group of cycles.

A first loading up to maximum load is simulated using the quasi-static model. If the material has not failed, the first group of cycles is applied. The resulting matrix damage variables are calculated by means of the cumulative damage law. Then, in order to check the fiber bundle failure criterion and to be able to perform next fatigue calculation, strain fields, fiber bundle fracture variables, and matrix damage driving forces are updated.

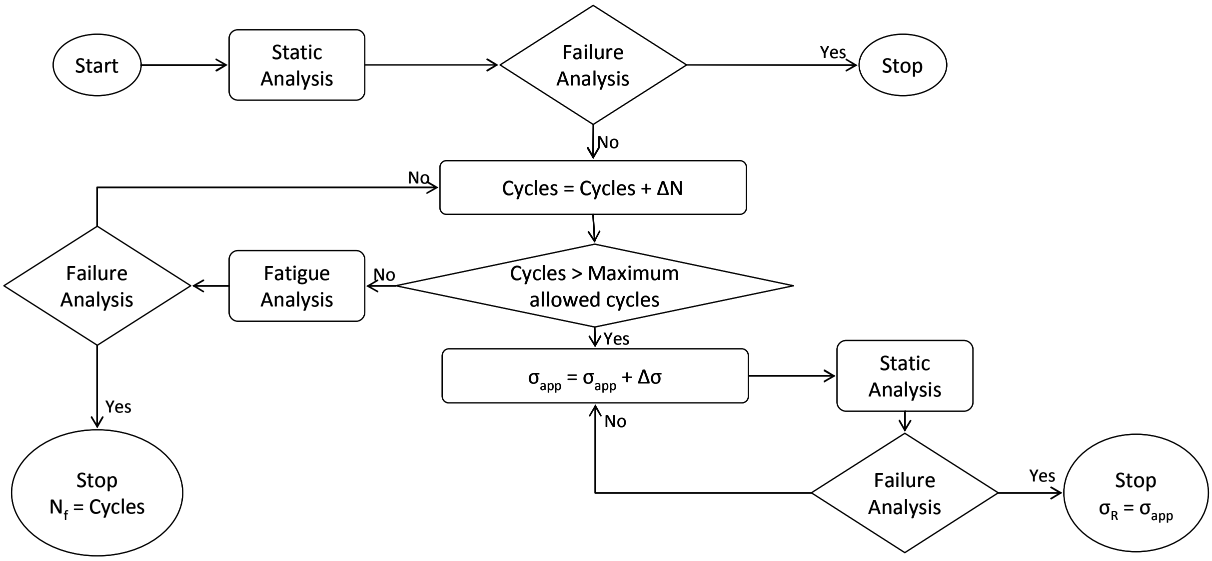


**Figure 2.** Modeling strategy for lifetime and residual strength prediction

These two steps (fatigue calculation and fields updating) constitute the Fatigue Analysis reported in Figure 2. As long as the maximum number of cycles, or the failure criterion, has not been reached, a loop on this Fatigue Analysis is carried out.

If the maximum number of cycles allowed by the user is reached without failure of the component, a residual strength analysis can be performed. To do so, an increasing load is simulated using the static damage model until failure occurs. It is important to note that in the case of a plain specimen, the first bundle fracture is catastrophic. Hence, a simple fiber bundle fracture criterion is employed:

$$y_i^{(f)} - y_{o(i)}^{(f)} \geq 0 \tag{4}$$

where $y_i^{(f)}$ is fiber bundle fracture driving force (influenced by matrix damage) and $y_{0(i)}^{(f)}$ the corresponding fiber bundle failure threshold.

After the calculation of the matrix damage accumulated during a group of cycles, the strain fields, the bundle fracture variables and the matrix damage driving forces can be updated by simulating an entire cycle with the quasi-static model. In this work, in order to decrease the computational costs of the model, the updating is performed at three characteristic load levels only: Maximum and minimum load were chosen in order to calculate parameters needed for the failure analysis and the next fatigue analysis. The mean load is chosen in order to avoid overestimation of viscous strain.

## 3 Results

### *3.1 Comparisons between experimental and modeling results*

Static material properties are identified by means of static incremental tensile loading/unloading tests in warp and weft direction of carbon/epoxy woven interlocks materials. Viscous parameters are identified by means of creep test at different stress levels on an interlock carbon/epoxy material oriented at 45 degrees with respect to the warp direction. The results of the model are compared to experimental data in Figures 3 and 4. The stress and strain axes are normalized for confidential reasons.

In order to identify the parameters of the fatigue matrix damage evolution law, the carbon/epoxy interlocks have been tested under tensile fatigue loading in warp and weft direction. Note that in the simulations the $\beta$ and $\Delta y_0^{Fatigue}$ parameters have been fixed in order to eliminate the influence of the load amplitude, since all the available experimental data have been obtained at the same loading ratio. The fatigue saturation value and damage threshold are set equal to those defined for static loading. The predicted and experimental curves of stiffness degradation are shown in Figure 5. The influence of matrix damage on the bundle failure criterion is identified from only one fatigue test in warp and weft direction (one data point on Figures 6 and 7).

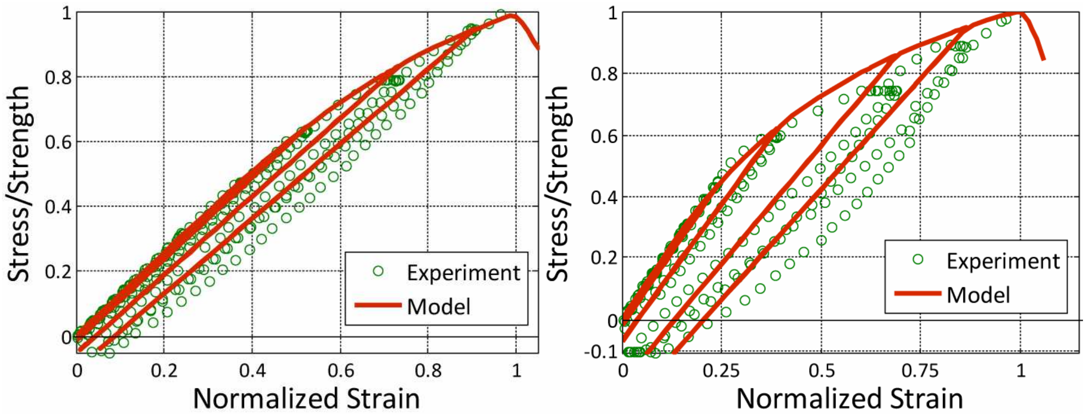


**Figure 3.** Experimental data and modeling results for interlock woven PMC in warp direction (left) and in weft direction (right)

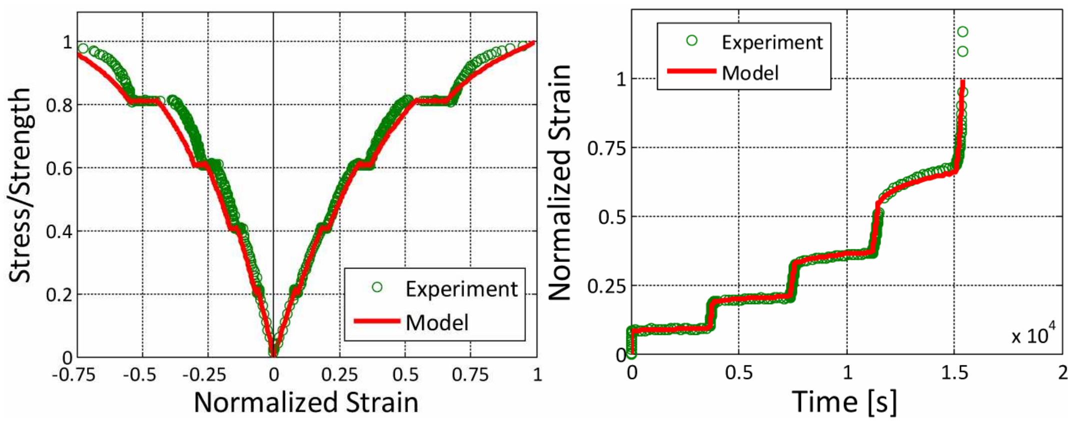


**Figure 4.** Experimental data and modeling results for woven interlock PMC under creep load in 45° direction

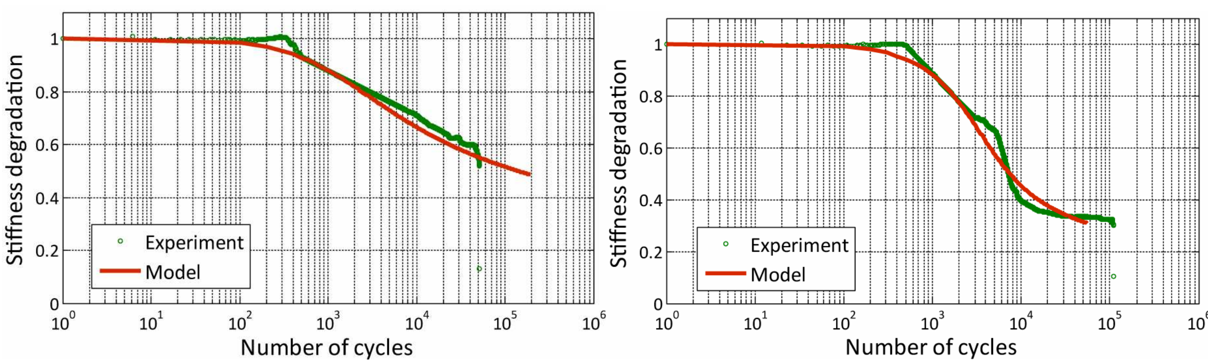


**Figure 5.** Stiffness degradation during fatigue loads: on the left, in warp direction (68% of the strength); on the right, in weft direction (71% of the strength)

The experimental data, obtained on a woven carbon/epoxy interlock material, have been provided by Snecma. All the tests have been performed with the same loading ratio but with different levels of maximum loads. The S-N curves predicted with the proposed fatigue model in the warp and weft direction are shown in Figures 6 and 7, respectively. The results are in good agreement with the available experimental data. The present model can also be applied to complex multi-axial fatigue loading, with the only exception of spectral loadings.

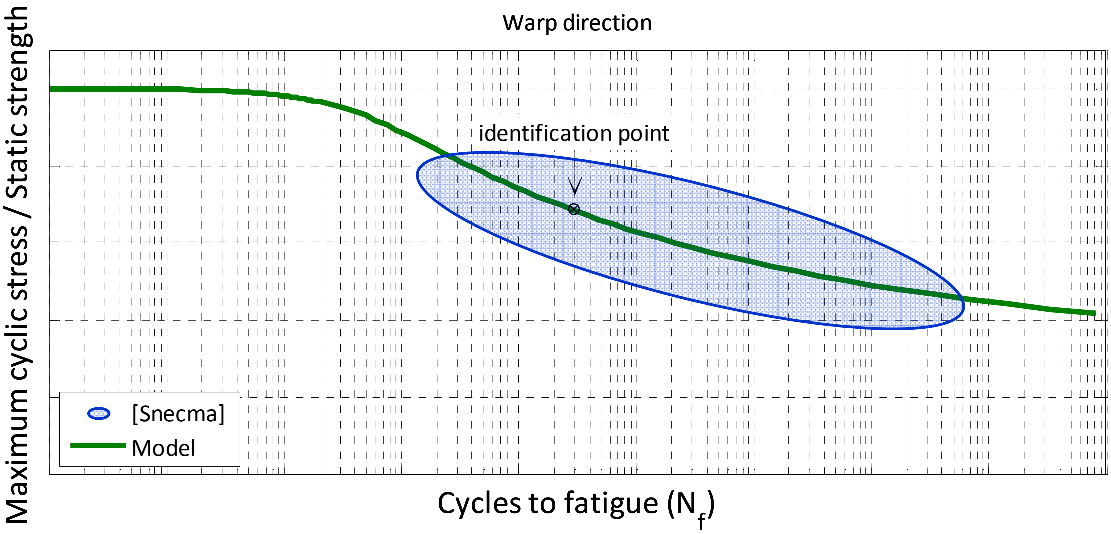


**Figure 6.** S-N curve predicted with ODM_Fatigue compared to experimental data in warp direction

*3.2 Other predictive capabilities of the present approach*

A residual strength ($\sigma_R$) analysis can be performed combining static and fatigue loadings as shown in Figure 2. For a given maximum cycle stress $\sigma_{max}$, the residual strength curve represents the static load that a specimen can still handle after having been exposed to a certain number of cycles $N \leq N_R(\sigma_{max})$. This ensures that the initial strength is equal to the static strength, and that after constant amplitude fatigue loading, failure occurs when the applied stress reaches the current value of strength. The simulated residual strength curve corresponding to $\sigma_{max}$ = 66% of the static strength is shown in Figure 7.

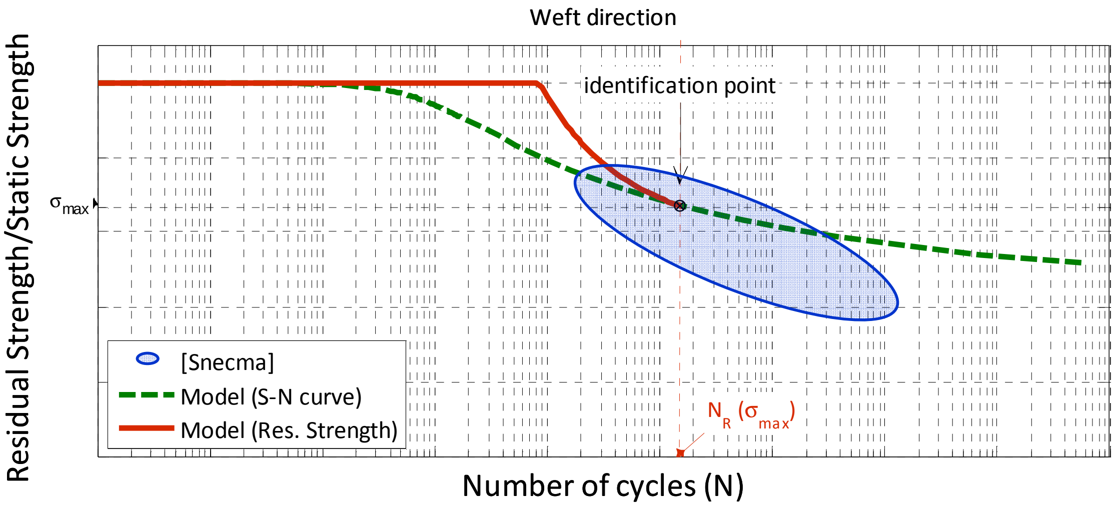


**Figure 7.** S-N curve predicted with ODM_Fatigue compared to experimental data in weft direction and predicted residual strength for a maximum load equal to 66% of the static strength

## 3 Conclusions

This paper presents a fatigue damage model for woven PMC, based on a damage model developed for interlock woven PMC under static loadings (ODM). In order to consider fatigue load cases, a cumulative matrix damage law has been introduced, and this matrix damage leads to load transfer to fiber bundles, influencing the failure of fiber bundles. The proposed macroscopic fatigue model permits to simulate complex load cases with different stress levels and/or different amplitude, except for spectral loading. The model shows a good agreement with experimental results in terms of fatigue lifetime, while the computational costs are much lower than those of incremental fatigue models. Moreover, it is possible to combine static and fatigue loadings and also to consider multiaxial fatigue load cases. Future work consists in the application of the present model to predict the lifetime of industrial woven PMC structures.

## Acknowledgement

This work has been carried out in collaboration with Snecma (Safran Group) and the Université de Technologie (UTC) de Compiègne. This study is closely connected to the PRC-Composites research project between Onera – The French Aerospace Lab and Safran Group.

## References

[1] Degrieck J., Van Paepegem W.: *Fatigue damage modeling of fibre-reinforced composite materials: Review*. Applied Mechanics Reviews, **54**, pp. 279-300 (2001).

[2] Lemaitre J., Desmorat R., Sauzay M.: *Anisotropic damage law of evolution*. European Journal of Mechanics, **19**, pp. 187-208 (2000).

[3] Hochard C., Thollon Y.: *A generalized damage model for woven ply laminates under static and fatigue loading conditions*. Int. Journal of Fatigue, **32**, pp. 158-165 (2010).

[4] Marcin L., Maire J.-F., Carrère N.: *Development of a Macroscopic Damage Model for Woven Ceramic Matrix Composites*. Int. J. Damage Mechanics, **20**, pp. 939-957 (2011).

[5] Grail G., Hirsekorn M., Carrère C., Hivet G., Hambli R.: *Lien entre paramètres matériaux/procédés et performances d'une structure composite à renfort tissé: approche multimodèle et multiéchelle*. Proceeding of *JNC17*, Poitiers, France, (2011).

[6] Couegnat G.: *Approche multiéchelle du comportement mécanique de matériaux composites à renfort tissé*. PhD Thesis of University of Bordeaux 1. (2008).

[7] Scida D., Aboura Z., Benzeggagh M.L., Bocherens E.: *A micromechanics model for 3D elasticity and failure of woven-fibre composite materials*. Composites Science and Technology, **59**, pp. 505-517 (1999).

[8] Maire J.-F., Chaboche J.-L. : *A new formulation of continuum damage mechanics (CDM) for composite materials*. Aerospace Science and Technology, **1**, pp. 247-257 (1997).

[9] Marcin L. : *Modélisation du comportement, de l'endommagement et de la rupture de matériaux composites à renforts tissés pour le dimensionnement robuste de structures*. PhD Thesis of University of Bordeaux 1 / Onera – The French Aerospace Lab (2010).

[10] Schneider J.: *Mécanismes d'endommagement dans les composites multicouches à renforts interlock*. PhD Thesis of Université de Technologie de Compiègne UTC (2011).

[11] Henry J., Aboura Z., Khellil K., Otin S.: *Suivi de l'endommagement en fatigue d'un composite à renfort Interlock Carbone/Epoxy par Emission Acoustique*. Proceedings of *JNC 17*, Poitiers, France (2011).

[12] Gornet L., Ijaz H.: *High Cycle Fatigue Damage Model for Delamination Crack Growth in CF/Epoxy Composite Laminates*. Int. J. Damage Mechanics, **20**, pp. 783-807 (2011).

[13] Song DY., Nobuo O.: *Approximate estimation of fatigue strength of polymer matrix composites by material properties*. Materials Science and Engineering A, **254**, pp. 200-206 (1998).