PREDICTION OF DEFORMATION AND BIAXIAL STRENGTH OF FIBER REINFORCED LAMINATES FOR WWFE BY USING MICRO DAMAGE MECHANICS

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ABSTRACT

After recently finishing of well-known the World Wide Failure Exercise we presented results of comparison of WWFE experiments and our predictions of biaxial strength and deformation the set of laminates under a range of specified loading conditions in accordance with our mathematical model. This paper describes the details of our model based on so called 'dispersed micro damage mechanics', secant modules of lamina and the absolutely stable algorithm to calculate nonlinear stress-strain curve up to failure load including 'scissoring' effect. Our predictions of strength are much better than best predictions of WWFE in the recent past (Zinoviev, Bogetti, Puck, Cuntze and Tsai).

1. INTRODUCTION

Current prediction of deformation and biaxial strength of fiber reinforced laminates is the main aim for all researchers and engineers involved in developing of high effective structures in aerospace, automotive, ship-building and civil industries of all countries during last sixty ears. In the USSR we were working with wide range of plane and rocket structures made of different FRP's. Therefore in the border of 80th and 90th we have developed our mathematical model [1,2] published rather before wide spreading of WWFE data. The WWFE was organized by M.J. Hinton, A.S. Kaddour and P.D. Soden [3] to compare known theories with detailed experimental data for predictions of biaxial strength and deformation the set of laminates under a range of specified loading conditions.

The objective of this study was to estimate the quality of our theory in accordance with the rules of WWFE.

2. CURRENT THEORY

Here we use the well-known 2D theory of laminate elasticity modified by the incremental (step-by-step) and proportional loading and the secant modulus (always positive) of lamina elasticity, Eq.1:

$$\bar{N}_i = A^s_{ij}\bar{\varepsilon}_j; \quad i,j,r,l = 1,2,3; \quad N_i^{m+1} = N_i^m + \Delta N_i; \quad \sigma_i^k = Q^{s\,k}_{ij}\varepsilon_j^k;$$

$$A^s_{ij} = \sum_k \{[T_{ir}^{-1}(\theta^k)Q^{s\,k}_{rl}T_{lj}(\theta^k)]\delta^k\}; \; Q^{s\,k}_{rl} = \begin{Bmatrix} E_1^s & \nu_1^s E_2^s & 0 \\ \nu_2^s E_1^s & E_2^s & 0 \\ 0 & 0 & 2G_{12}^s \end{Bmatrix}^k, \tag{1}$$

$$\begin{pmatrix} E_1^s, \nu_1^s \\ E_2^s, \nu_2^s \\ G_{12}^s \end{pmatrix} = \begin{pmatrix} E_1, \nu_1 \\ E_2, \nu_2 \\ G_{12} \end{pmatrix} \left[1 - \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \right]; \; \bar{\sigma}_i = \bar{N}_i / \sum_k \delta^k,$$

where $\bar{N}$ and A - vector of load and matrix of laminate stiffness, σ, ε - vectors of lamina stress and strain, s and k- symbols of secant modulus and number of lamina, Q - matrix of lamina elastic modules, E, G and ν - Young's modulus, shear modulus and Poisson's ratio,

1 and 2 - longitudinal and transverse directions of lamina, θ - angle between axis #1 of lamina and global axis #1 of laminate.

For modelling each lamina is the parallel ensemble of geometrically equal cells consisting of fibres imbedded into matrix, which work independently and can separately damaged in three forms, three specific lamina damage functions (fibre damage ω_1 reliant of 'fibre' stress σ_1, transverse ω_2 and shear damages ω_3 which can only increase from 0 to 1 and reliant of transverse and shear stress of lamina σ_2 and σ_3, e.g. relative damages). Estimation of micro damages bases on Gaussian law of cell strength dispersion, Eg.2, Figure 1:

$$\omega_i(\sigma_i) = \frac{1}{\sqrt{2\pi S_i}} \int_0^{p_i} \exp\left[-\frac{(x-M_i)^2}{2S_i^2}\right] dx, \qquad (2)$$
$$p_i = \sigma_i/(1-\zeta_i\omega_i), \quad \zeta_1 = \zeta_2 = 1, \quad \zeta_3 = 0.$$

For engineering applications in this work we assumed that S_i/M_i=0,2. Parameters M (five numbers) can be derived from lamina strength characteristics as:

$$M_{1t(c)}=F_{1t(c)}/0{,}672; \quad M_{2t(c)}=F_{2t(c)}/0{,}672; \quad M_3=F_3/1{,}328. \qquad (3)$$

Here subscripts *t* or *c* mean *tension* or *compression*, *1,2* and *3* mean longitudinal, transverse and shear.

The parallel work of cell's ensemble gives simple linear law for secant modules versus relative damages ω, Eq.4 (see Figure 1, also):

$$E_1^s = E_1(1-\omega_1); \quad E_2^s = E_2(1-\omega_2); \quad G_{12}^s = G_{12}(1-\omega_3);$$
$$\nu_1^s = \nu_1(1-\omega_1); \quad \nu_2^s = \nu_2(1-\omega_2). \qquad (4)$$

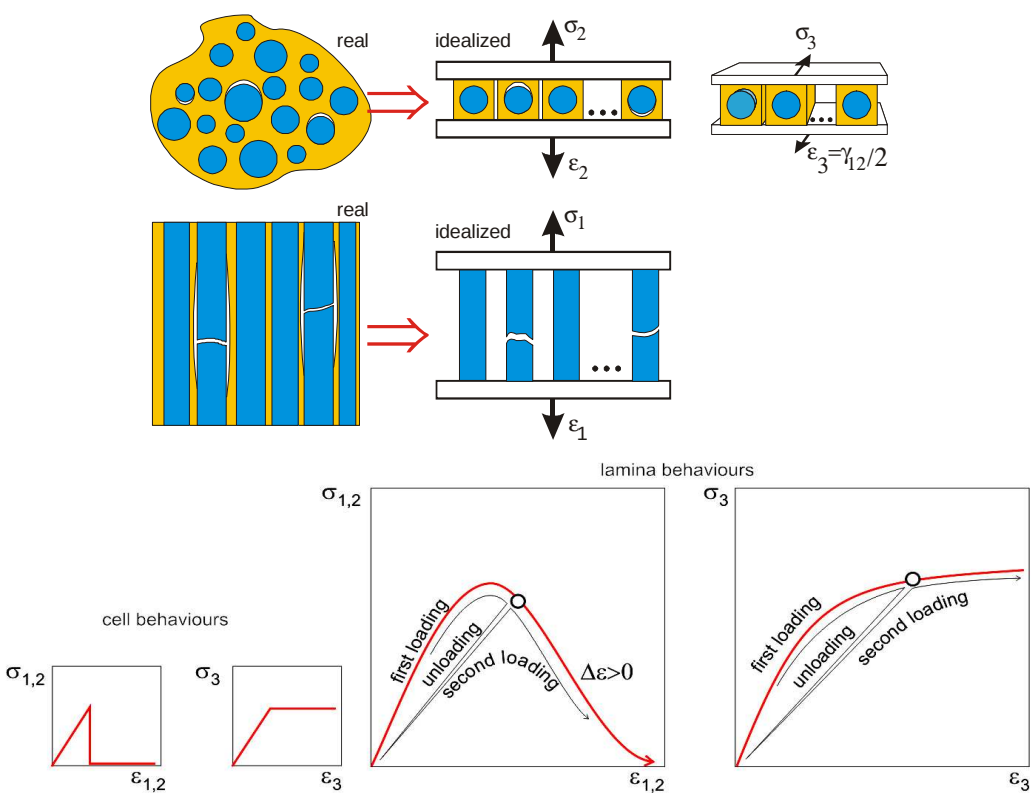

Figure 1. Idealization of structure and behaviours of cell for longitudinal, transverse and shear deformation (for loading Δε > 0, for unloading Δε < 0).

The lamina stress-strain curve has one smooth function for both increasing and decreasing (post-failure) parts under 'rigid' or 'deformation' loading of laminate. It is true due to ideal bonding between layers.

Failure load of composite laminate is the load step before of unlimited increasing of laminate deformations in active loading directions:

$$\bar{\varepsilon}_j (if\ \ N_j \neq 0) \rightarrow \infty. \qquad (4)$$

It caused by decreasing secant modules of laminate down to zero, Eq.3. Artificially we limited maximum of ω by value 0.999999 to get absolute stability during calculations.

At the end of modifications of classical laminate analysis there is changing of fibres orientation θ^k into lamina under large shear deformation before failure ('scissoring' effect):

$$\theta_{i+1}^k = \theta_i^k + 2\varepsilon_{3i}^k, \qquad (5)$$

where i - number of iteration and k - number of lamina.

The inclusion of any of these forms of nonlinear behavior into the laminate analysis requires the introduction of iterative numerical method of solution.

Proposed iterative algorithm is absolutely stable and fast due to use the secant modules of lamina and laminates too. This algorithm was put into easy-to-use MathCAD processor to have clear and convenient code 'SCM.mcd' with all visible formulas and graphical illustrations.

3. COMPARISON OF CURRENT THEORY WITH WWFE EXPERIMENTS

The WWFE test data [3] were presented for the tube type samples under axial tension, torsion, internal or external pressure, and axial compression, with and without liner, Table 1.

In 'blind' phase of our investigation we used original lamina data to predict biaxial deformation and strength. In 'tune' phase we took into account all presented in WWFE data concerning curing, variation of volume fraction of fibers, etc. to correct mechanical behaviors of lamina (estimation of new elastic modules and strength with initial self-balanced 'thermal' stresses).

Table 1.
Summary of the laminates and loading cases

Loading case	Laminate lay-up	Material	Description of loading cases
1	0	E-glass/LY556/HT907/DY063	Biaxial failure stress envelope under transverse and shear loading (σ_y vs. τ_{xy})
2	0	T300/BSL914C	Biaxial failure stress envelope under longitudinal and shear loading (σ_x vs. τ_{xy})
3	0	E-glass/MY750/HY917/DY063	Biaxial failure stress envelope under longitudinal and transverse loading (σ_y vs. σ_x)
4	90/±30/90	E-glass/LY556/HT907/DY063	Biaxial failure stress envelope (σ_y vs. σ_x)
5	90/±30/90	E-glass/LY556/HT907/DY063	Biaxial failure stress envelope (σ_x vs. τ_{xy})
6	±55	E-glass/MY750/HY917/DY063	Biaxial failure stress envelope (σ_y vs. σ_x)
7	0/±45/90	AS4/3501-6	Biaxial failure stress envelope (σ_y vs. σ_x)
8	0/90	E-glass/MY750/HY917/DY063	Stress–strain curve under uniaxial tensile loading for (σ_y : σ_x =0:1)
9	±45	E-glass/MY750/HY917/DY063	Stress–strain curves for (σ_y : σ_x =1:1)

10	±45	E-glass/MY750/HY917/DY063	Stress–strain curves for (σ_y : σ_x =1:-1)
11	±55	E-glass/MY750/HY917/DY063	Stress–strain curves under uniaxial tensile loading for (σ_y : σ_x =1:0)
12	±55	E-glass/MY750/HY917/DY063	Stress–strain curves for (σ_y : σ_x =2:1)
13	0/±45/90	AS4/3501-6	Stress–strain curves under uniaxial tensile loading in *y* direction (σ_y : σ_x =1:0)
14	0/±45/90	AS4/3501-6	Stress–strain curves for (σ_y : σ_x =2:1)

Typical diagrams for all loading cases are shown below (hereinafter blue line - current calculation after 'tune' phase) which were put over graphs of [3-5].

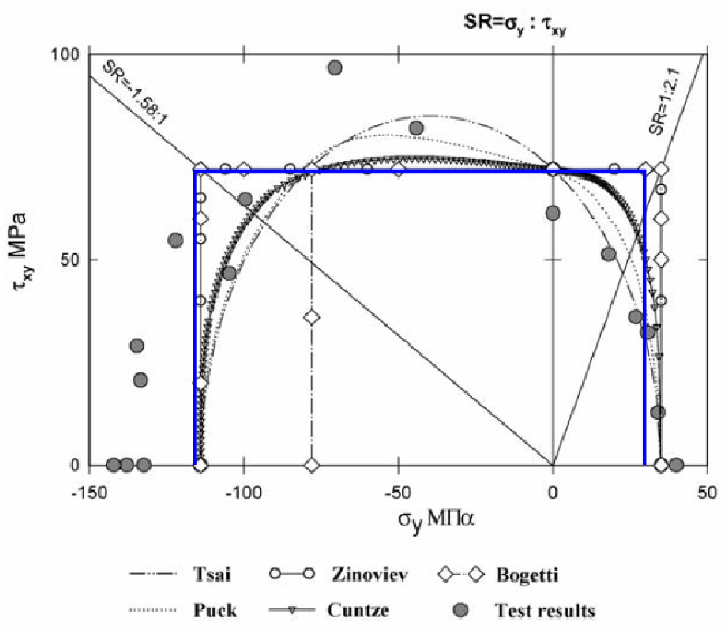

Figure 2. Case #1.

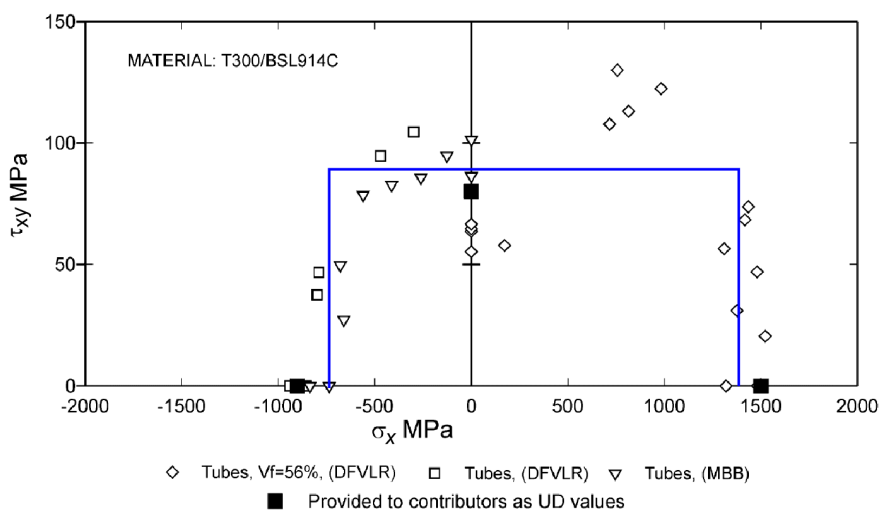

Figure 3. Case #2.

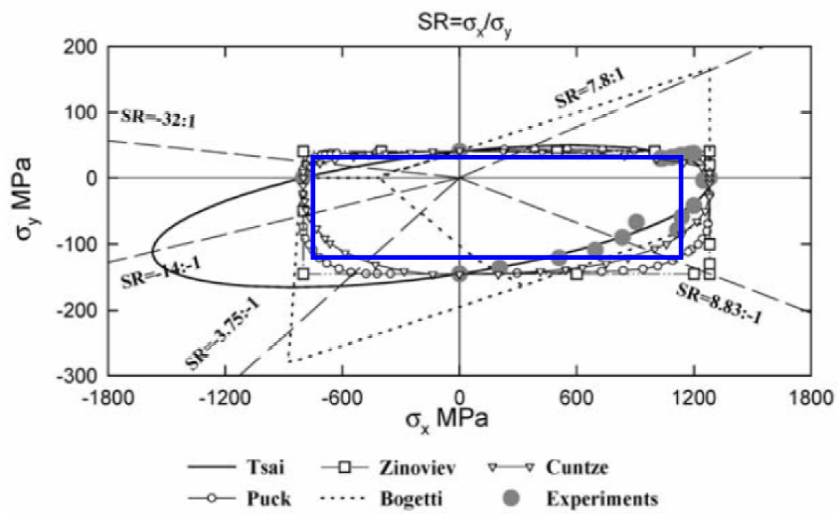

Figure 4. Case #3.

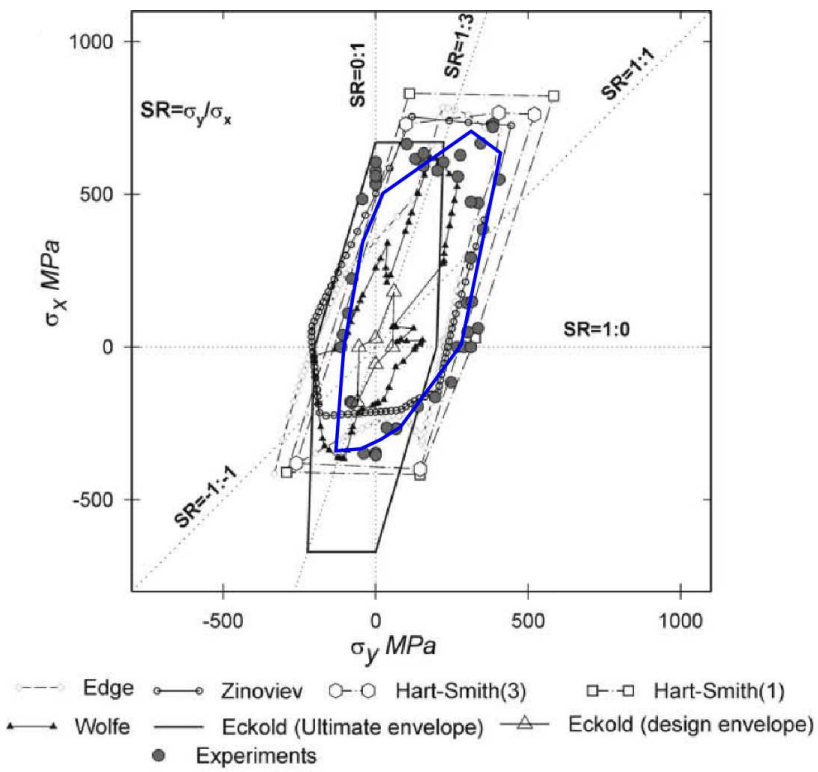

Figure 5. Case #4.

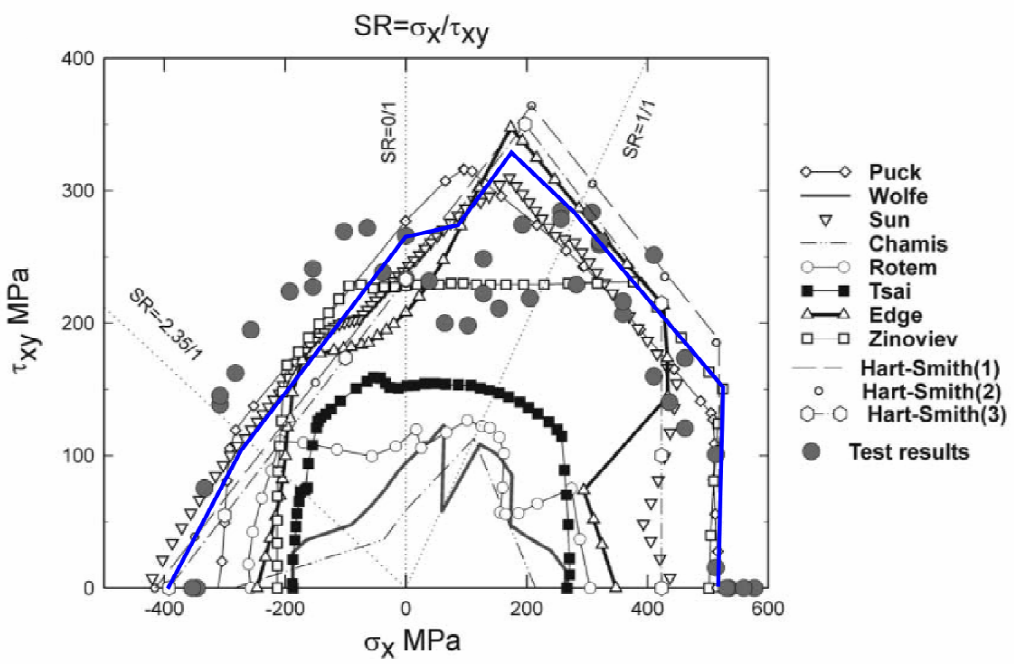

Figure 6. Case #5.

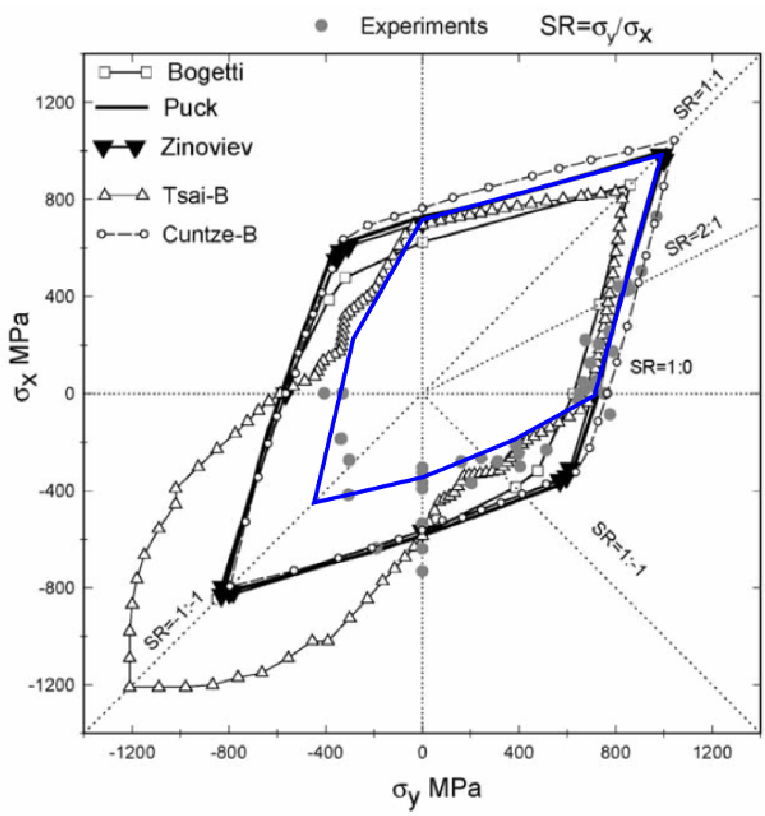

Figure 7. Case #6.

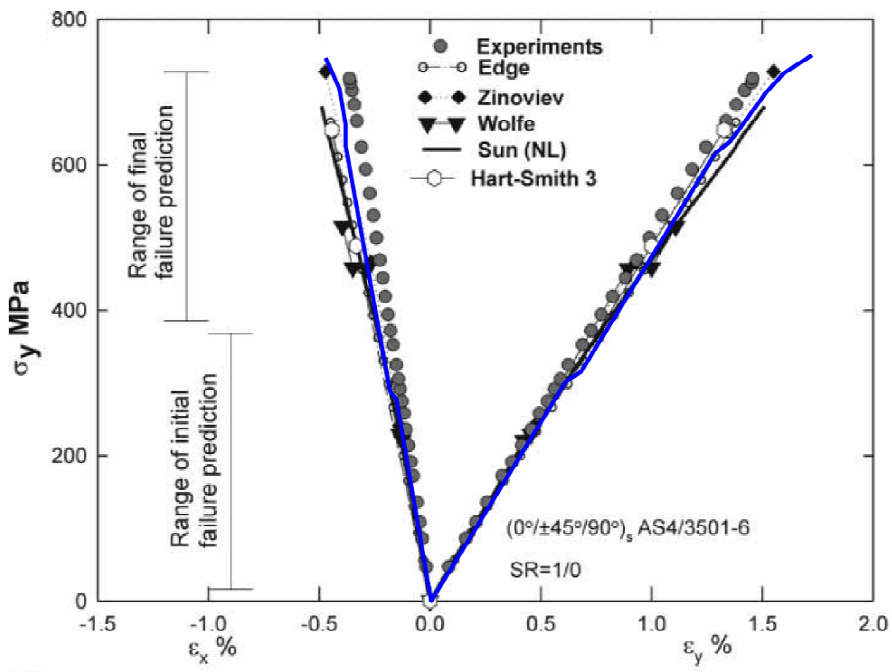

Figure 8. Case #7.

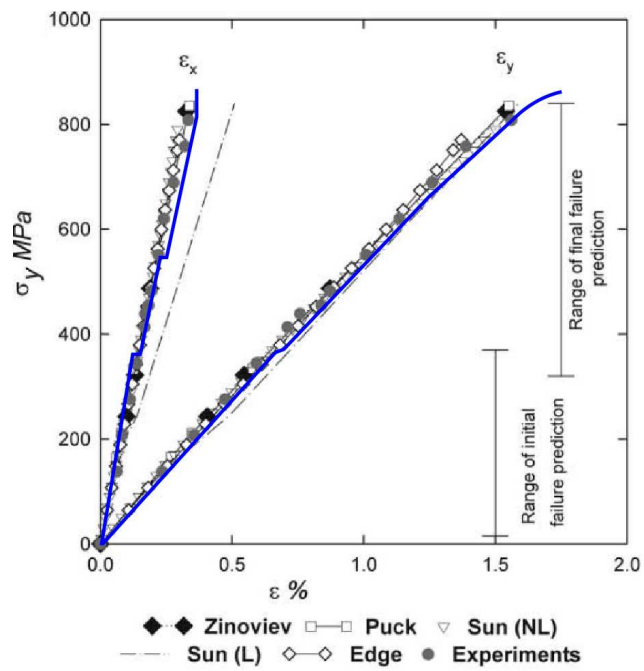

Figure 9. Case #8.

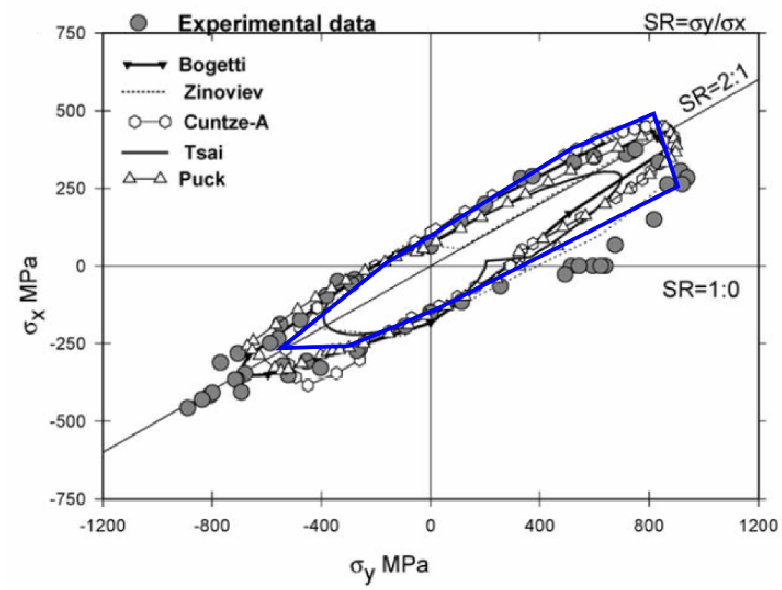

Figure 10. Case #9.

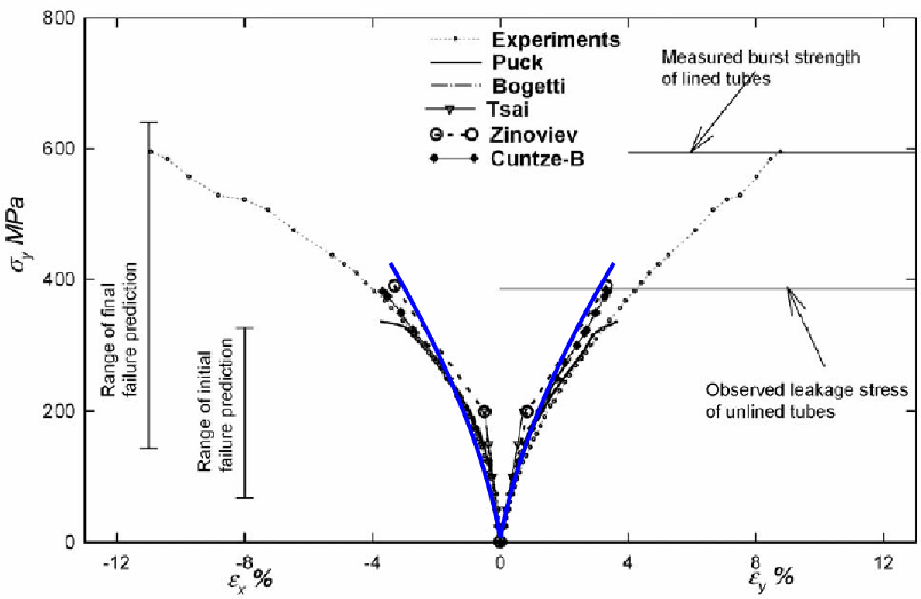

Figure 11. Case #10.

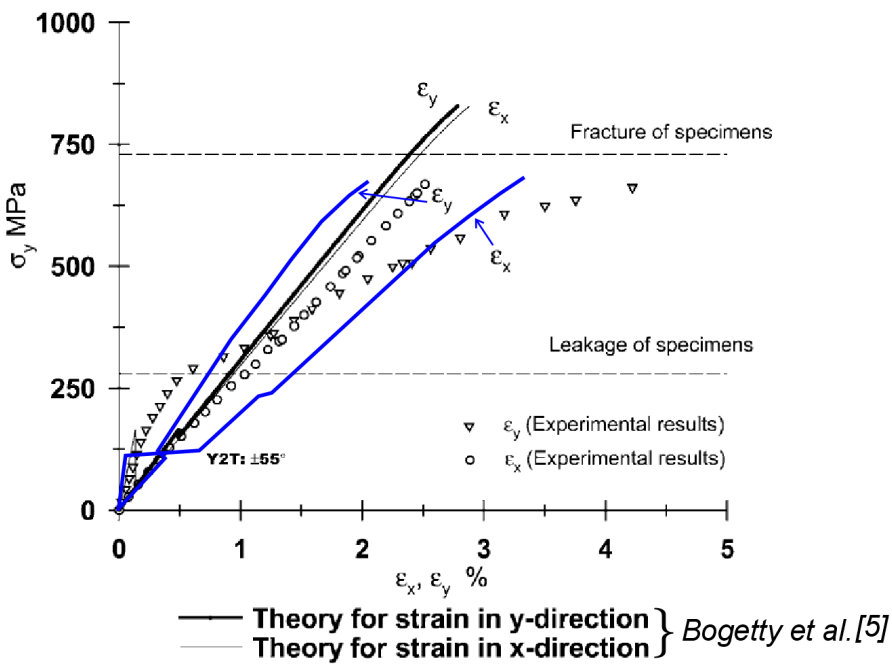

Figure 12. Case #11.

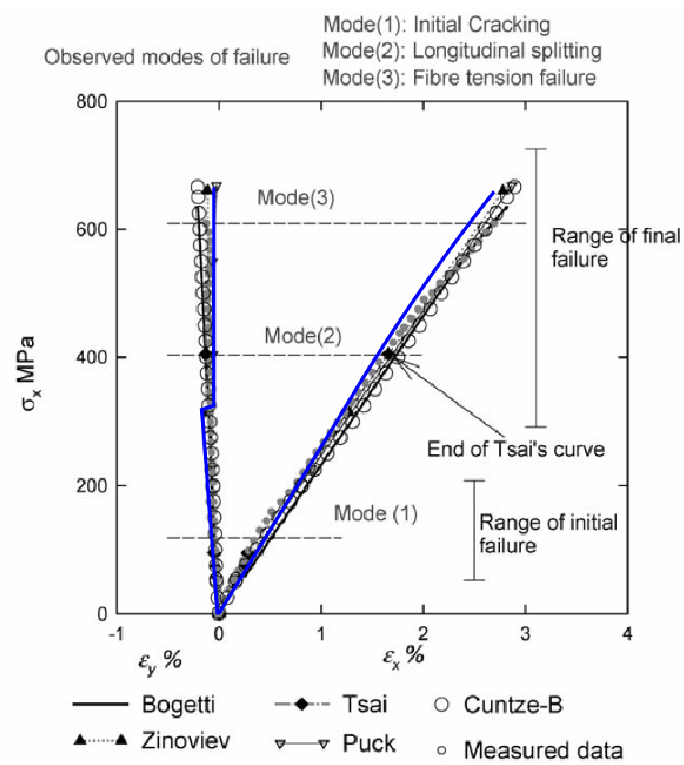

Figure 13. Case #12.

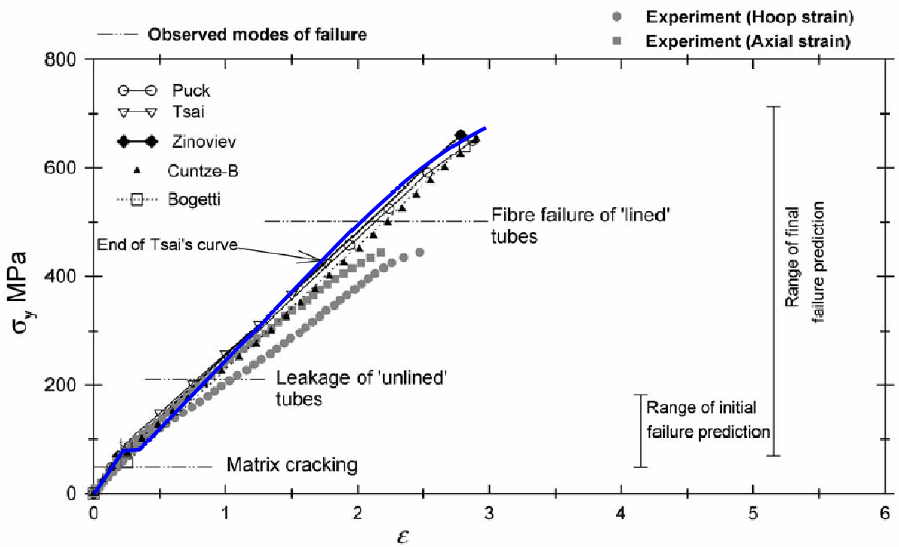

Figure 14. Case #13.

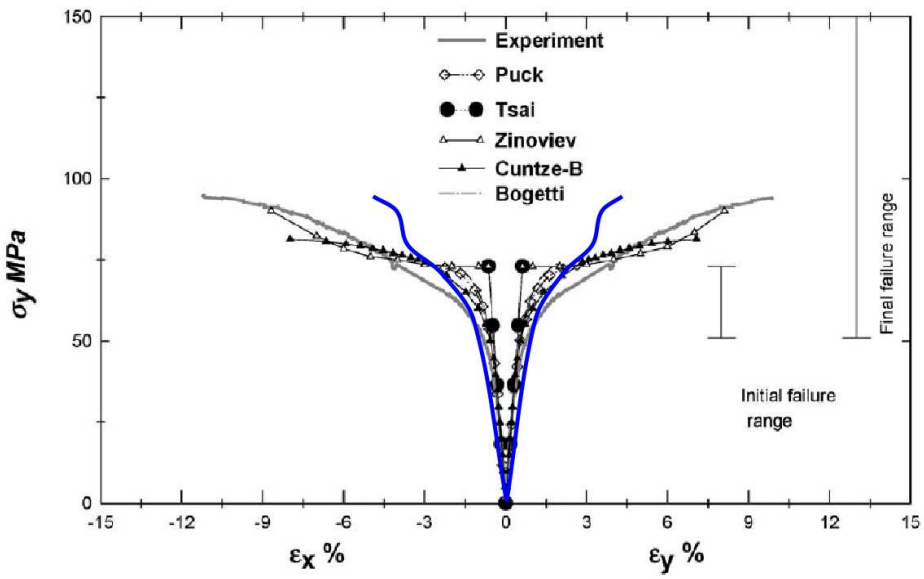

Figure 15. Case #14.

3. CONCLUSIONS

Now we can say that our predictions of strength are much better than best predictions of WWFE (Zinoviev, Bogetti, Puck, Cuntze and Tsai). We have average difference between strength estimations and experimental results less than 10% in 13 of 14 test cases (more than 92% of success). By the way in WWFE it was good if you had that difference less than 50 % in 75% of test cases only.

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