ABSTRACT
Due to their high specific strength and stiffness, fibre-reinforced composite materials are being increasingly used in structural applications where a high level of performance is important (e.g. aerospace, automotive, offshore structures, etc.). Performance in service of these composites is affected by multi-mechanism damage evolution under loading and environmental conditions. For instance, carbon fibre-reinforced cross-ply laminates demonstrate a wide spectrum of failure phenomena under tensile fatigue, with matrix cracking and delamination being initiated at relatively early stages of loading/service. These damage mechanisms can result in significant deterioration of the residual stiffness and load-bearing capacity of composite components and should be thoroughly investigated. The delamination failure mechanism is studied in this paper for a double cantilever beam loaded in Mode I. The microstructural randomness of laminates that is responsible for non-uniform distributions of stresses in them even under uniform loading conditions is accounted for. This is achieved by introducing random fracture properties within the layer of cohesive elements, used to model delamination in laminates. Several sensitivity studies are performed with a view to analyse the effects of parameters of the cohesive law on the damage propagation in laminates.

1. INTRODUCTION
1.1 Matrix cracking and delamination: effect of microstructure
The first stage of failure development in CFRP [0\textdegree/90\textdegree]_n-composites under tensile fatigue conditions is characterised by a multiple generation and growth of transverse cracks in their inner 90\textdegree layers. The place of the matrix crack’s arrest at the interface between 0\textdegree and 90\textdegree layers is characterised by considerable stress concentration and often serves as a nucleus for initiation of delamination along such interfaces. The latter is due to the mismatch of the Poisson’s ratios and in-plane shear stiffness between differently oriented plies.

In most micromechanical analyses of fibre-reinforced composite materials, an assumption of spatial periodicity of their properties is employed: it is considered that the material has an ordered (deterministic) distribution of fibres. Such periodic distributions called regular fibre packing are usually assumed, based typically, on a square or hexagonal layout used in a representative volume element (RVE). Employing this periodicity assumption, effective properties for composite materials can be computed, and various homogenization theories and two-scale methods have been developed under this assumption. These theories have been applied to simulate fibre-reinforced composite materials with linear [1] and non-linear behaviours [2]. Still, conventional fibre-reinforced composites are far from being ordered materials since fibres are usually randomly distributed in the matrix. This can be easily noticed even with simple optic microscopy as shown in Fig. 1.

Wang et al. [3] were among the first to consider the location of matrix cracks as a statistic quantity in their models. They presented a stochastic simulation model for the growth of multiple matrix cracks in cross-ply laminates subjected to both static and fatigue loads. Fukunaga et al. [4] combined the shear-lag approach with stochastic analyses to study progressive matrix cracking in carbon fibre cross-ply laminates. Baxevanakis et al. [5]
demonstrated a high level of spatial non-uniformity for a composite by applying the image analysis technique to cross-sectional areas of T300/914 specimens. In this composite with an average volume fraction of fibres 55.9%, the local minimum and maximum levels of the observed volume fractions were 15% and 85%, respectively. Such dispersion and scatter undermines any use of schemes, based on ideal periodic or quasi-periodic arrangements of fibres, to estimate the effective properties and, especially, failure parameters of such composites. Bulsara et al. [6] presented their study of the RVE size appropriate for characterization of initiation of damage under a tensile load normal to fibres in a unidirectional composite. This work revealed that the scatter among random realizations of RVE varies with the RVE size and the appropriate size of the RVE should be linked to the lowest scatter. Silberschmidt [7] has established that cross-ply carbon–epoxy laminates demonstrate a considerable extent of randomness in distributions of transverse cracks in 90° layers and studied the effect of microstructure randomness in the distribution of matrix cracks in these laminates. Solutions for sets of cracks with minimum and maximum spacings were obtained, which in a majority of cases provide lower and upper bounds, respectively, for residual macroscopic properties of laminates. It was concluded that the study of the behaviour of specimens with multiple matrix cracks cannot be always reduced to analysis of a single unit cell under the assumption of equal crack spacing. Trias et al. [8] showed by comparing stress and strain distributions obtained with periodic and random models for a carbon fibre-reinforced polymer that former models can be used to assess effective properties but latter ones must be considered for simulations of local phenomena such as local damage or matrix cracking. Silberschmidt [9] further contributed to this topic by presenting a lattice model to study damage and fracture evolution in laminates, linking microstructural randomness with macroscopic properties and demonstrating that a random character of the fibres’ distribution results in fluctuations of local elastic moduli in composites, the bounds and type of distribution of which depend on the characteristic length scale. Hence, an adequate model of real fibre-reinforced laminates presupposes an account for their microstructural randomness. Such an approach could be based on numerical – finite-element (FE) – simulations with introduction of respective descriptions for damage/cracking evolution. The latter can be achieved with the use of cohesive zone (CZ) elements.
1.2 Cohesive zone modelling

The analysis of fracture development has changed considerably in the last few decades. Instead of conventional approaches of fracture mechanics, various predictive schemes such as cohesive zone models or traction-separation models are now increasingly being used. Cohesive zone models (CZM) are established as relatively simple schemes, which are easier to implement into FE codes, and as a result, CZ elements are now available in numerous commercial FE packages.

Among various damage models, the cohesive model (CM) seems particularly attractive for practical application since it can be incorporated effectively in computational FE codes. Several schemes have been developed. Barenblatt [10], who formulated the fundamental idea for a cohesive zone model, investigated the fracture of brittle materials by defining the traction along the crack path as a function of the crack tip distance along the crack front. Needleman [11; 12] was one of the first to use polynomial and exponential types of traction-separation relations to simulate the particle debonding in metal matrices. There, an exponential fit is used for a normal traction while a trigonometric one for a shear traction. Tvergaard [13] also extended the Needleman’s model [11] of pure normal separation to mixed-mode loading.

So, cohesive zone modelling is widely used and has been found very useful in tackling problems of non-linear fracture mechanics. This method is used as a first stage of our analysis of two-mode damage i.e. matrix cracking and delamination in CFRP laminates in this paper. Its aim is to analyze the delamination in unidirectional laminates accounting for the effect of spatial non-uniformity of microstructure in terms of random CZ elements.

2. DELAMINATION IN CFRPS: COHESIVE ZONE MODELLING IN A DCB SPECIMEN

The delamination failure in fibre-reinforced composites, is often studied by employing the double-cantilever beam (DCB) specimen[14], shown in Figure 2. A double cantilever beam test is utilized to determine mode I (tensile-opening) interlaminar fracture toughness, \( G_{\text{IC}} \) of the material under consideration. This test, based on a special type of linear-elastic fracture-mechanics (LEFM) specimen, is incorporated in several international test standards. This study employs the standard test method given in ISO 15024 [15]. In the first part of our analysis, the assumption of the homogenized material is used, i.e. its properties within the entire cohesive layer are the same. The subsequent part of the study is aimed at highlighting the effect of microstructural randomness in the laminates, based on a comparison of the results for both parts.

The test specimen was made of a unidirectional carbon fibre-reinforced laminate based upon an epoxy matrix. The volume fraction of the thermosetting matrix was nominally 35% and the carbon fibres were T400 [16]. The specimen is 125 mm long, 20 mm wide, with twenty four 0.125 mm thick plies – 12 on each side of the layer of cohesive elements that contains an initial crack at the edge with length of 30 mm. The material has the following properties: the longitudinal modulus in the fibre direction \( E_{11} = 137 \text{ GPa} \) and the moduli transverse to the fibre direction \( E_{22} = E_{33} = 8 \text{ GPa} \); the shear moduli \( G_{12} = G_{13} = 4 \text{ GPa} \) and \( G_{32} = 3.2 \text{ GPa} \); the Poisson’s ratios \( \nu_{12} = \nu_{13} = 0.31 \) and \( \nu_{23} = 0.52 \).

A delamination growth can be simulated with finite elements by placing interfacial decohesion elements between the composite layers. A decohesion failure criterion that combines aspects of strength-based analysis and fracture mechanics is used to simulate debonding by reducing the stiffness of the softening element. The interface is described
by constitutive equations, which relate the applied stress $\sigma$, normal to the interface, to the relative displacement at the interface $\delta$ given by a bilinear traction-separation law. The graph of this law can be subdivided into two main parts (Fig. 2) described by the following constitutive equations [17]:

Elastic part:  
$$\sigma = k_1 \delta \quad \text{if} \quad \delta < \delta_c ; \quad (1)$$

Softening part:  
$$\sigma = (1-D)k_1 \delta \quad \text{if} \quad \delta_c < \delta < \delta_t ; \quad (2)$$

where $D$ is a variable characterizing damage, which has a zero value in the virgin (undamaged) state and attains a value of unity when the material is fully damaged. The area under the curve is the fracture energy $G_F$ for a particular mode and is defined as:

$$G_F = \int_0^{\delta_t} \sigma(\delta)d\delta , \quad (3)$$

where $\sigma(\delta)$ is the traction and $\delta_t$ is the maximum crack separation when the softening stress is equal to zero.

A two-dimensional model has been developed for this study using finite element software ABAQUS and is schematically shown in Figure 3. The CFRP material is presented as a continuum that has anisotropic properties while the interface properties are

Figure 2: Bilinear traction-separation law

Figure 3: Double cantilever beam model
governed by a bilinear traction separation law (Fig. 2). The fracture energy of 0.257 kJ/m² [14] is assigned to cohesive zone elements. The initial stiffness of the traction-separation law is 10000 N/mm³ and the tripping traction is 50 MPa.

In order to model the progressive damage and failure using a CZM approach, a pre-defined crack path has to be provided as input to the model. The cohesive zone model simulates the macroscopic damage along this path in terms of a traction-separation response between initially coincident nodes on either side of the pre-defined crack. The analysis is performed for a displacement-control loading; a displacement $u$ is applied at the ends of the double cantilever beam as shown in Fig. 3.

The continuum model is meshed using 8-node bi-quadratic plane strain quadrilateral elements (CPE8R) while 4-node two-dimensional cohesive elements (COH2D4) are used for the cohesive zone. The thickness of the cohesive layer is taken as 75 μm that represents the resin-rich area within different lamina of the laminated composite. The edge length used for meshing is 0.075 mm (75 μm). The element size within the cohesive layer is 0.075 mm x 0.075 mm. The cohesive elements behave as stiff springs until a critical (maximum) traction (shown by $\sigma_{\text{max}}$ on the traction-separation law in Fig. 3) is reached. The spring unloads progressively after this point, and this unloading process dissipates energy. The nodes start to separate upon unloading and then finally de-bond totally. In such a way, the crack propagation and the failure evolution can be simulated. The parameters that govern the traction-separation law are the maximum traction ($\sigma_{\text{max}}$) and the energy of separation per unit area ($G_f$).

To implement crack growth simulations with the use of a damage mechanics approach, the constitutive equations introduce a damage parameter $D$ as a function of stresses and strain. The value of this parameter varies from zero (virgin state) to unity (fully damaged state). Crack propagation is represented in terms of element softening or removal when the damage parameter reaches a critical value. In ABAQUS this parameter is named the scalar stiffness degradation variable (SDEG).

A load-displacement plot for the DCB with similar CZ elements is presented in Figure 4 which gives a load of 53 N for the onset of damage. The value of the corresponding
displacement \( u \) at this load is found out to be 1.28 mm. The variation in damage within the cohesive elements of layer is shown in Figure 5 starting from a virgin state \((D = 0)\) and attaining a damaged at \( D = 1 \) in the vicinity of the crack tip.

3. EFFECT OF RANDOMNESS ON DELAMINATION IN LAMINATES
As mentioned earlier, laminated composites exhibit randomness at the level of their microstructure that affects the mechanical properties and, hence, the damage initiation and evolution process. The next stage of our study is linked with an account of this microstructural randomness that is responsible for non-uniform distributions of stresses in real composites. This is achieved by introduction of random fracture properties within the layer of cohesive elements in the DCB specimen. A value of 50\% scatter of properties is selected as a starting point to underline the effect

Figure 6: Cohesive laws with traction and initial stiffness variation for the same level of \( G_c \) (a) and variation of \( G_c \) for the same level of \( \sigma_{\text{max}} \) (b)
of this material stochastic nature on the course of damage initiation and propagation. This is achieved in the first phase by introducing random cohesive zone properties of the tripping traction but retaining the constant level of the fracture energy $G_F$. This, in its turn, affects the opening critical ($\delta_c$) and final ($\delta_f$) displacements and, hence, the initial stiffness of the cohesive law within the cohesive layer. The cohesive layer between the continuum elements has been divided into small windows of 185 $\mu$m x 75 $\mu$m. Three sets of fracture properties are introduced, one being the same as used in the previous model without randomness in material’s parameters. Two other sets correspond to the tripping traction of 0.5 $\sigma_{\text{max}}$ and 1.5 $\sigma_{\text{max}}$, respectively. The total energy levels are the same for both cases.

The respective variations of traction-separation plots can be seen in Figure 6a. The effect of varying the fracture energy (Mode-I fracture energy in this case) (Fig. 6b) will also be investigated. A load-displacement plot for the case with the random distribution of the tripping traction is shown in Figure 7. Here, the load for the onset of damage is 61 N; the value of the corresponding displacement $u$ is 0.9 mm. This plot clearly demonstrates the difference in levels of the failure load (i.e., damage initiation point) as compared to the case of constant properties within the cohesive layer (Fig. 4). The post-critical behaviour

![Figure 7: Force-displacement diagram for DCB with varying tripping traction within the cohesive layer (a) and its declining part (b)](image7.png)

![Figure 8: Variation in damage along the cohesive layer for different statistical realisations of cohesive properties](image8.png)
of the specimen after the onset of delamination is presented in Fig. 7(b) at the enlarged scale. As the DCB specimen is opening, the prescribed displacement is resisted or supported by the neighbouring cohesive elements, depending on their properties. To elaborate, a stronger section of the cohesive layer near a weaker section will additionally resist to the applied displacement and hence increase the force. This is evident from the graph in Fig. 7(b) where the force after starting to decrease from the damage initiation demonstrates non-monotonous behaviour. To further highlight the effect of the spatial non-uniformity on the damage in CFRP laminates, three statistical realisations of spatial distributions of properties are used for the same half-scatter level that was employed in the previous case. The calculated levels of damage plotted along the length of the cohesive zone are shown in Figure 8 demonstrating their variation due to the spatial non-uniformity of local properties. These diagrams demonstrate different values of damage for the same locations of the delamination zone as a result of introduced randomness; positions of weak and strong elements can be identified in these plots. Finally, another case with random fracture properties (traction and hence the initial stiffness) has been studied for an increased level of scatter, with $\sigma_{\text{max}}$ changing within the band $[0.25\sigma_{\text{max}}, 1.75\sigma_{\text{max}}]$. The variation in the force with displacement is more pronounced here as can be seen in Figure 9. A large scatter results in transformation of not only the part of the graph after the point corresponding to the maximum force that describes a post-critical behaviour; the presence of weak elements results in a deviation from the straight line for the loading (i.e. pre-critical) portion of the curve.

4. CONCLUSIONS

The underlying objective of this study is to model the delamination failure mechanism experienced by laminated composites in order to highlight the effect of microstructural randomness of laminates on this mechanism. A standard DCB specimen with various types of spatial distributions of properties of CZ elements has been analysed. The results

![Graph showing force-displacement for a DCB with varying tripping traction within the cohesive layer](image)
for the model with periodic and random fracture properties have been obtained, and the comparison of results shows the variation in the overall force in the DCB specimen and the damage variation due to this incorporation of such randomness. The obtained data for forces and corresponding displacements demonstrate a difference in the results of the two analyses models, i.e. the one assuming a uniform distribution, in which the material properties are constant along the cohesive layer, and the other, using random distributions, in which a variation/scatter of cohesive properties is introduced with different scatter bands applied. These results emphasise the importance of taking into account the microstructural randomness exhibited by CFRP laminates. The first stage deals only with the effect of variation in the tripping traction and initial stiffness of the cohesive law. Further work will include the effect of local fluctuations in the level of fracture energy.

REFERENCES


